Graviton emission in collisions of high-energy hadrons

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An expression has been obtained for the probability of emission of soft gravitons in elastic and inelastic collisions of hadrons with energies much larger than their masses. An estimate is obtained for the fraction of the energy transfered into gravitons.

In the present note we consider the problem of emission of gravitons in hadron collisions at energies $E \gg m$ (m is a characteristic hadron mass of the order of 1 GeV; everywhere $h = c = 1$). Such radiation can be essential during the early stages of evolution of the hot (big bang) Universe[1,2].

Let us consider the process of emission of a soft graviton in an N-particle collision. In the linearized theory in first order the matrix element for this process is of the form[3,4]

$$M=M_0\sqrt{8\pi k}\sum_{\alpha=1}^{N}\eta_0 p_{\alpha i}p_{\alpha j}\overline{h_{ij}^{*}}\frac{1}{(p_\alpha q)}, \tag{1}$$

where M_0 is the matrix element for the same process without graviton emission; the gravitational constant is $k = 6\times 10^{-39}/m_N^2$ (m_N is the nucleon mass); $q = (\omega, \mathbf{q})$ and $\overline{h}_{ij}$ are respectively the four-momentum and the tensor wave function of the graviton: $\overline{h}_{ik}\overline{h}_{ik}^{*} = 1$, $\overline{h}_{ik} = \overline{h}_{ki}$, $\overline{h}_{ii} = 0$; $\eta_\alpha = +1(-1)$ for an incoming (outgoing) particle numbered α and $p_\alpha = (E_\alpha, \mathbf{p}_\alpha)$ is its momentum.

If one considers graviton helicity states corresponding to its spin projections ± 2 on the direction of motion, then $\overline{h}_{ik}$ can be written in the form $\overline{h}_{ik}^{\pm} = e_i^{\pm}e_k^{\pm}$, where $e^{\pm} = (e_1 \pm ie_2)/\sqrt{2}$, $e_0^{\pm} = 0$, $\mathbf{e}_1\cdot\mathbf{q} = \mathbf{e}_2\cdot\mathbf{q} = \mathbf{e}_1\cdot\mathbf{e}_2 = 0$, $e_1^2 = e_2^2 = 1$. Then in this 3-transverse gauge the differential cross section for the emission of a graviton in the frequency interval $(\omega, \omega + d\omega)$ and in the solid angle $d\Omega$ is

$$d\sigma=d\sigma_0\frac{k}{2\pi^2}\frac{d\omega}{\omega}d\Omega|R|^2; \tag{2}$$

here $d\sigma_0$ is the cross section for the formation of the given hadron configuration and

$$R=\sum_\alpha\eta_\alpha\frac{(\mathbf{p}_\alpha\mathbf{e}^{\pm})^2}{E_\alpha[1-\mathbf{v}_\alpha\mathbf{n}]}, \tag{3}$$

where $\mathbf{v}_\alpha$ is the particle velocity and $\mathbf{n} = \mathbf{q}/\omega$.

From (2) and (3) it follows directly that, in distinction from bremsstrahlung of photons, the emission probability remains finite also in the ultrarelativistic limit as $E \to \infty$ (which is equivalent to taking the limit $m = 0$ and $|\mathbf{v}_\alpha| = 1$ for all α). Indeed, $|\mathbf{p}_\alpha\cdot\mathbf{e}| = E_\alpha \sin\vartheta_\alpha$ (where ϑ_α is the angle between the momenta $\mathbf{p}_\alpha$ and $\mathbf{q}$) and $1 - \mathbf{v}_\alpha\cdot\mathbf{n}$ is of the order ϑ_α as $\vartheta_\alpha \to 0$, therefore each term in the sum in (3) is finite. The absence of a divergence with respect to the angle in the cross section for graviton emission by particles with mass $m = 0$ has been noted before[3], but in the gauge used in that paper it was valid only for the sum as a whole.

Let now all $m_\alpha = 0$ and the configuration p_α, q be flat, i.e., let all $\mathbf{p}_\alpha$ and $\mathbf{q}$ be in the same plane. Then in the soft-graviton approximation energy and momentum conservation imply

$$\sum_\alpha\eta_\alpha E_\alpha=0,\quad \sum_\alpha\eta_\alpha E_\alpha\cos\vartheta_\alpha=0.$$

Selecting now the vector $\mathbf{e}_1$ in the plane determined by the vectors $\mathbf{p}_\alpha$ and $\mathbf{q}$, we obtain for both helicities

$$|R|=\sum_\alpha\eta_\alpha E_\alpha\frac{\sin^2\vartheta_\alpha}{1-\cos\vartheta_\alpha}=\sum_\alpha\eta_\alpha E_\alpha(1+\cos\vartheta_\alpha)=0. \tag{4}$$

In the c.m.s. of the colliding particles two jets are formed in which the longitudinal momenta are large (of the order E, where E are the energies of the particles 1 and 2), and the transverse momenta are small (of the order $\mu = m_\pi$). We choose the z axis along the direction of $\mathbf{p}_1$, and let ϑ be the angle between $\mathbf{q}$ and the z axis, and ϑ_α the angle of the α-th hadron with the z axis. Owing to the smallness of the transverse momenta the angles ϑ_α are close to 0 or π. If one integrates the emission cross section with respect to the angle ϑ, the regions $\vartheta \sim 0$ and $\vartheta \sim \pi$ give contributions of the same order and therefore it suffices to consider a region of ϑ near 0. Let $1 \gg \vartheta \gg \vartheta_\alpha$, then R can be expanded in powers of ϑ. Since $R = 0$ when all ϑ_α are 0 or π, we have from (4)

$$R\sim E\vartheta_s f(\vartheta), \tag{5}$$

where ϑ_s is the average scattering angle and $f(\vartheta)$ is a function of that angle. At $\vartheta \sim \vartheta_s$ we have $R \sim E$, therefore $f(\vartheta)$ must increase as $1/\vartheta$ when ϑ tends to zero.

Summing over the graviton helicities and integrating over the angles ϑ near 0 and π we obtain

$$d\sigma' = 4C\frac{k}{\pi}\frac{d\omega}{\omega}d\sigma_0 E^2\vartheta_s^2\int_{\vartheta_s}^{\vartheta\sim 1}\frac{d\vartheta}{\vartheta},$$

where $d\sigma'$ is the cross section for the emission of a graviton in the interval $d\omega$ for a given configuration of hadrons, and C is a constant on the order of unity. Integrating over the configurations we obtain finally

$$d\sigma'' = \frac{4Ck}{\pi}E^2\vartheta_s^2\sigma_0\frac{d\omega}{\omega}\ln\frac{1}{\vartheta_s},$$

where $d\sigma''$ is the total cross section for the emission of a graviton in the interval $d\omega$.

Recognizing that ϑ_s is of the order of μ/E, where μ is the pion mass, we obtain

$$d\sigma'' = \frac{4kC\mu^2}{\pi}\sigma_0\frac{d\omega}{\omega}\ln\frac{E}{\mu}. \tag{6}$$

The estimate (6) can also be obtained in another manner. After summing over the polarizations the expression (2) can be integrated with respect to the emission angles of the graviton for arbitrary m_α[3]. Passing to the limit of vanishing mass yields (cf. also the Appendix)

$$d\sigma'=d\sigma_0\frac{2k}{\pi}\frac{d\omega}{\omega}\sum_{\alpha,\beta}\eta_\alpha\eta_\beta(p_\alpha p_\beta)\ln(p_\alpha p_\beta). \tag{7}$$

We note that when the energy scale changes the expression (7) remains unchanged, in spite of the fact that the quantity under the logarithm is dimensional. Indeed, upon the substitution $p_\alpha = \lambda p'_\alpha$ a term proportional to

$$\sum_{\alpha,\beta} \eta_\alpha\eta_\beta (p_\alpha p_\beta)\ln\lambda^2$$

is added, but this term vanishes on account of four-momentum conservation

$$\sum \eta_\alpha p_\alpha = \sum \eta_\beta p_\beta = 0.$$

Taking into account the conservation laws, one can rewrite the sum in (7) in the form

$$\sum \eta_\alpha\eta_\beta E_\alpha E_\beta (1-\cos\vartheta_{\alpha\beta})\ln\frac{1-\cos\vartheta_{\alpha\beta}}{2}, \qquad (8)$$

where $\vartheta_{\alpha\beta}$ is the angle between the momentum p_α and p_β.

Returning to the jet configuration and noting that $|\vartheta_{\alpha\beta}| \ll 1$ (or $|\vartheta_{\alpha\beta} - \pi| \ll 1$ if the particles belong to different jets), we obtain

$$d\sigma' = d\sigma_0 \frac{k}{\pi}\frac{d\omega}{\omega}\sum{}' \eta_\alpha\eta_\beta E_\alpha E_\beta \vartheta_{\alpha\beta}^2 \ln\frac{\vartheta_{\alpha\beta}^2}{4} + O(\vartheta_s^2), \qquad (9)$$

where the primed sum is over hadrons with parallel momenta.

The integration over the configurations, recognizing that the effective angle of separation of the particles is μ/E, again leads to the estimate (6).

As an example we consider the case of elastic scattering of hadrons. Let the scattering angle be ϑ_0. Then we obtain from Eq. (8)

$$d\sigma' = d\sigma_0 \frac{2k}{\pi}\frac{d\omega}{\omega} 2E^2\left[(1-\cos\vartheta_0)\ln\frac{2}{1-\cos\vartheta_0} + (1+\cos\vartheta_0)\ln\frac{2}{1+\cos\vartheta_0}\right]. \qquad (10)$$

For $\vartheta_0 \ll 1$ the second term will tend to zero like ϑ_0^2 and it can be neglected, since it does not contain the factor $\ln(1/\vartheta_0)$; then

$$d\sigma' = d\sigma_0 \frac{2k}{\pi}\frac{d\omega}{\omega} E^2\vartheta_0^2 \ln\frac{4}{\vartheta_0^2}. \qquad (11)$$

The expression (9) again leads to the estimate (6) for the graviton emission cross section in the interval $(\omega, \omega + d\omega)$.

The expression (6) is applicable for graviton frequencies $\omega \ll m$, for which the cross section for the emission of gravitons factorizes with the separation of the factor R. As can be seen from (6), in this region the graviton emission cross section does not increase with the hadron energy, as assumed in Matzner's paper[2], where it was assumed that the probability for emission of a graviton is proportional to E^2, where E is the hadron energy.

As was remarked by Ya. B. Zel'dovich (cf., e.g.[1]), this estimate does not take into account the fact that hadron scattering occurs basically at small angles, which should diminish the emission considerably. As can be seen from the preceding discussion, the same is valid also for the graviton emission in processes with multiple hadron production, owing to the smallness of the angles at which the hadrons are emitted relative to one another. We note that the estimate (6) at the constant C on the order of unity is too large, since cancellations of terms of different signs may occur in the sum (9).

For the case of emission of hard gravitons there is no reason for a factorization of the graviton emission cross section. One can assume that at $E \gg m$ the graviton emission cross section does not depend at all on the hadron masses. Then dimensional considerations imply that up to logarithmic factors the cross section of the process with graviton emission σ_g is proportional to Ck (where C is a numerical constant) and does not depend on the initial hadron energy. The relative probability for the emission of a graviton in hadronic collisions equals σ_g/σ_0. This means that if the average energy transferred to the gravitons in one collision is written in the form $\Delta E = PE$, then since $\sigma_0 \sim \pi/\mu^2$, we have

$$P \sim k\mu^2, \qquad (12)$$

whereas Matzner's estimate[2] corresponds to

$$P \sim kE^2. \qquad (13)$$

If the estimate (13) were correct this would lead to the establishment of a thermal equilibrium between the gravitons and matter in the hot model of the Universe. This is no longer so for p corresponding to (12).

We are grateful to Ya. B. Zel'dovich, who called our attention to the problem discussed in this paper.

APPENDIX

The expression (7) can be obtained directly by integrating Eq. (2) with respect to the emission angles of the graviton. Summation over the polarizations in (3) and taking the limit $E \to \infty$ leads us to the following formula for the differential cross section:

$$d\sigma = d\sigma_0 \frac{k}{2\pi^2}\frac{d\omega}{\omega} d\Omega \sum_{\alpha,\beta} \eta_\alpha\eta_\beta E_\alpha E_\beta A_{\alpha\beta}, \qquad (A.1)$$

where

$$A_{\alpha\beta} = \frac{2[\mathbf{n}_\alpha\mathbf{n}_\beta - (\mathbf{n}_\alpha\mathbf{n})(\mathbf{n}_\beta\mathbf{n})]^2 - [\mathbf{n}_\alpha\times\mathbf{n}]^2[\mathbf{n}_\beta\times\mathbf{n}]^2}{2[1-\mathbf{n}_\alpha\mathbf{n}][1-\mathbf{n}_\beta\mathbf{n}]}.$$

and $\mathbf{n}_\alpha = \mathbf{p}_\alpha/E$.

After decomposing $A_{\alpha\beta}$ into partial fractions and integrating over the azimuthal angle, taking into account the energy and momentum conservation, this expression leads to the following equation for the graviton distribution:

$$d\sigma = d\sigma_0 \frac{k}{2\pi}\frac{d\omega}{\omega}\sin\vartheta\, d\vartheta \sum_{\alpha,\beta} \eta_\alpha\eta_\beta E_\alpha E_\beta [\mathrm{sign}(\cos\vartheta_\alpha - \cos\vartheta) \qquad (A.2)$$
$$+ \mathrm{sign}(\cos\vartheta_\beta - \cos\vartheta)](1-\cos\vartheta_{\alpha\beta})[\cos\vartheta(1+\cos\vartheta_{\alpha\beta}) - (\cos\vartheta_\alpha + \cos\vartheta_\beta)]\cdot$$
$$\times[\cos^2\vartheta(1-\cos\vartheta_{\alpha\beta}) - 2\cos\vartheta(\cos\vartheta_\alpha + \cos\vartheta_\beta) + 1 - \cos\vartheta_{\alpha\beta} + 2\cos\vartheta_\alpha\cos\vartheta_\beta]^{-1},$$

where sign x is the sign of x.

Integrating (A.2) further over the angle leads to Eq. (7). In the simplest case of two-particle scattering the expression (A.2) simplifies. If we restrict our attention to small angle scattering ($\vartheta_0 \ll 1$), we obtain for $\vartheta_0/2 \ll \vartheta \ll \pi - \vartheta_0/2$

$$d\sigma = d\sigma_0 \frac{4k}{\pi}\frac{d\omega}{\omega} E^2\vartheta_0^2 \frac{d\vartheta}{\sin\vartheta}. \qquad (A.3)$$

This result agrees with the estimate for the cross section $d\sigma'$ given above.

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