The effect of the polarization of a plasma on its kinetic properties in the presence of a strong electric field

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The fluctuations in a plasma in the presence of a strong electric field are computed with allowance for the polarization of the plasma. In a high-frequency field, the difficulty in taking the polarization into account is due to the nonstationarity of the fluctuation process. An expression with allowance for the polarization of the plasma is found for the collision integral in a strong field. The conductivity of the plasma is computed on the basis of the obtained kinetic equation. It is shown that at field frequencies close to the Langmuir frequency the conductivity becomes field dependent at comparatively low intensities.

The kinetic equation for a plasma in a strong electric field has been considered by Silin and Balescu [1] in perturbation-theory approximation in the interaction. In zero field, this equation coincides with the Landau kinetic equation.

Allowance for the effects of the polarization of the plasma in the presence of a strong high-frequency field has so far not been carried out. This is due to the fact that the fluctuation process, which determines the collision integral, is, owing to the influence of the field, nonstationary, as a result of which the problem of deriving the kinetic equation becomes significantly complicated. In a weak field, allowance for the plasma polarization leads, as is well known, to the Balescu-Lenard collision integral [2,3]. A computation of the fluctuations in a plasma in the presence of a high-frequency electric field has been carried out by Bychenkov et al. [4], but their calculations are not quite consistent. The investigation of the fluctuation processes in a plasma located in a strong electric field is the aim of the present paper.

In the case of high field frequencies when the frequency ω_0 is of the order of, or higher than, the Langmuir frequency, the problem is solved in the zeroth approximation in the electron-to-ion-mass ratio. We derive in this approximation an expression for the nonstationary spectral density of the fluctuations in the field, and determine the form of the collision integral that, in a weak field, coincides with the Balescu-Lenard integral. On the basis of the obtained kinetic equation, we compute the electrical conductivity in the strong field with allowance for the polarization. In a weak field, the expression for the equilibrium-state conductivity coincides with the expression obtained in [5] by Pepel' and Éliashberg. In the limiting case of very high-intensity fields, but without allowance for polarization, the results agree with the results obtained in [6] by Silin. It is shown that for $\omega \gtrsim \omega_L$ the conductivity is field dependent even at comparatively weak fields when the work done by the field over a mean free path is comparable to the mean kinetic energy.

In the zeroth approximation in the ratio m_e/m_i (m_e and m_i are the electron and ion masses respectively), the plasma is stable in a high-frequency field. For the finite mass ratio the appearance of an aperiodic instability is possible in an isothermal plasma (see [7], Sec. 8). In a nonisothermal plasma a parametric instability is possible ([7], Secs. 9 and 10). An instability obtains with respect to perturbations of wavelength much longer than the Debye radius.

There arises in connection with the possibility of the appearance of an instability the following question: Is the manifestation of the nonlinear field dependence of the conductivity found in the approximation $m_e/m_i = 0$ possible? As noted above, the nonlinear dependence is manifested already at weak fields for which $eEl \sim \kappa T$ (l is the mean free path). This condition can be written in the form $E^2/n\kappa T \sim \mu^2$ (μ is the plasma parameter). The critical field for the aperiodic instability is given by the condition $E^2/n\kappa T \gtrsim \mu$. Thus, for $\mu \ll 1$ the nonlinearity appears earlier than the aperiodic instability. In a nonisothermal plasma the parametric instability sets in when $E^2/n\kappa T \gtrsim (m_e/m_i)^{1/2}\mu$. If $\mu \ll (m_e/m_i)^{1/2}$, then the conductivity of the nonisothermal plasma begins to be field dependent at fields at which the plasma is still stable.

The fluctuations and the collision integral in the low-frequency ($\omega_0 \ll \omega_L$) case are also computed for an arbitrary field. The corresponding expression for the conductivity is derived. The conductivity in this case begins to be field dependent at substantially higher fields (the work done by the field over a distance of the order of r_D should be comparable to κT).

1. THE BASIC EQUATIONS. THE SPECTRAL DENSITIES OF THE FLUCTUATION SOURCES

As the basic equations let us use the system of equations for the phase densities in the coordinate and momentum space of the individual components of the plasma

$$N_a(x,t)=\sum_{1\leq i\leq N}\delta(x-x_{ia}(t)),\qquad x=(\mathbf{r},\mathbf{p}),$$

and the microscopic intensity $\mathbf{E}^M$ of the electric field. The equation for the distribution function $f_a = \langle N_a\rangle/n_a$ (n_a is the mean concentration) of the spatially homogeneous plasma has the form

$$\left(\frac{\partial}{\partial t}+e_a\mathbf{E}(t)\frac{\partial}{\partial \mathbf{p}}\right)f_a(\mathbf{p},t)=-\frac{e_a}{n_a}\frac{\partial}{\partial \mathbf{p}}\langle\delta N_a\delta\mathbf{E}\rangle=I_a(\mathbf{p},t). \qquad (1.1)$$

Here $\mathbf{E}(t)$ is the external electric field. For concreteness let us assume it has the form

$$\mathbf{E}(t)=\mathbf{E}\sin\omega_0 t. \qquad (1.2)$$

It follows from (1.1) that to determine the collision integral we must express the correlations of the fluctuations δN_a and $\delta\mathbf{E}$ in terms of the functions f_a. In the first approximation in the plasma parameter (in the polarization approximation) the equations for the functions δN_a

and $\delta\mathbf{E}$ can be written in the form [8]

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+e_a\mathbf{E}(t)\frac{\partial}{\partial\mathbf{p}}\right)(\delta N_a-\delta N_a^S)=-e_a n_a\delta\mathbf{E}\frac{\partial f_a}{\partial\mathbf{p}}, \quad (1.3)$$

$$\operatorname{div}\delta\mathbf{E}=4\pi\sum_a e_a\int\delta N_a(\mathbf{r},\mathbf{p},t)\,d\mathbf{p}. \quad (1.4)$$

The Eq. (1.3) determines the induced part $\delta N_a - \delta N_a^S$ of the function δN_a that is proportional to $\delta\mathbf{E}$. The correlation of the fluctuations δN_a^S in the source satisfies the equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+e_a\mathbf{E}(t)\frac{\partial}{\partial\mathbf{p}}\right)\langle\delta N_a\delta N_b\rangle^S_{x,t,x',t'}=0, \quad (1.5)$$

which is solved with the initial condition

$$\langle\delta N_a\delta N_b\rangle_{x,t,x',t'}|_{t=t'}\equiv\langle\delta N_a\delta N_b\rangle_{x,x',t'}=n_a\delta_{ab}\delta(x-x')f_a(x',t'). \quad (1.6)$$

From Eq. (1.1) for the function $f_a(\mathbf{p}, t)$ let us go over to the equation for the slowly varying function

$$F_a(\mathbf{P},t)=f_a\Big(\mathbf{P}+e_a\int_{-\infty}^{t}\mathbf{E}(t')\,dt',t\Big)=f_a\Big(\mathbf{P}-\frac{e_a}{\omega_0}\mathbf{E}\cos\omega_0 t,t\Big). \quad (1.7)$$

From Eq. (1.1) follows the equation for the function $F_a(\mathbf{P}, t)$:

$$\frac{\partial F_a}{\partial t}=I_a(\mathbf{P}(t),t)\equiv I_a(\mathbf{P},t) \quad (1.8)$$

where

$$\mathbf{P}(t)\equiv\mathbf{P}-(e_a/\omega_0)\mathbf{E}\cos\omega_0 t.$$

Let us represent the expression for the collision integral I_a in the form

$$I_a(\mathbf{P},t)=\frac{e_a}{(2\pi)^3 n_a}\frac{\partial}{\partial\mathbf{P}}\int\operatorname{Re}(\delta N_a\delta\mathbf{E})_{\mathbf{k},\mathbf{P}(t)}\,d\mathbf{k}. \quad (1.9)$$

It can be seen from the expressions (1.8) and (1.9) that in the first approximation in the plasma parameter μ the derivative of the distribution function is of the order of μF_a, as a result of which the fast (vibrational) contributions to the functions F_a are small when $\mu \ll 1$. The slowly varying part of the distribution function is the dominant part. It can be separated out from the distribution function by averaging the latter over the period $2\pi/\omega_0$. Below, unless otherwise stated, under F_a and I_a we shall understand the slowly varying parts of these functions.

It follows from Eqs. (1.3) and (1.4) that the spectral densities of the fluctuations δN_a and δE can be expressed in terms of the spectral functions of the fluctuations δN_a^S. To find the spectral function $(\delta N_a\delta N_b)^S$, we turn to Eq. (1.5). From it follows the solution

$$(\delta N_a\delta N_b)^S_{x,t,x',t-\tau}=n_a\delta_{ab}\delta\Big(\mathbf{p}-\mathbf{p}'-e_a\int_{t-\tau}^{t}\mathbf{E}(t')\,dt'\Big)\delta\Big(\mathbf{r}-\mathbf{r}'-\mathbf{v}\tau$$

$$-\frac{e_a}{m_a}\int_{t-\tau}^{t}(t-\tau-t')\mathbf{E}(t')\,dt'\Big)f_a\Big(\mathbf{p}-e_a\int_{t-\tau}^{t}\mathbf{E}(t')\,dt',t-\tau\Big). \quad (1.10)$$

Recognizing that according to (1.7)

$$f\Big(\mathbf{p}-e_a\int_{t-\tau}^{t}\mathbf{E}(t')\,dt',t-\tau\Big)=F_a(\mathbf{P},t-\tau), \quad (1.11)$$

we can express the correlations (1.10) in terms of the slowly varying functions F_a.

For the ideal plasma (in the zeroth approximation in the retardation of the function F_a [9]), we obtain from (1.10) and (1.11) the following expression for the spectral density of the source of the fluctuations δN_a:

$$(\delta N_a\delta N_b)^S_{\omega,\mathbf{k},\mathbf{p},\mathbf{p}',t}=n_a\delta_{ab}2\operatorname{Re}\int_0^{\infty}\delta\Big(\mathbf{p}-\mathbf{p}'-e_a\int_{t-\tau}^{t}\mathbf{E}(t')\,dt'\Big)\cdot$$

$$\times\exp\Big\{-\Delta\tau+i(\omega-\mathbf{k}\mathbf{v})\tau-i\frac{e_a}{m_a}\int_{t-\tau}^{t}(t-\tau-t')\mathbf{k}\mathbf{E}(t')\,dt'\Big\}\,d\tau F_a(\mathbf{P},t). \quad (1.12)$$

We shall need for the determination of the collision integral the expressions for the simpler spectral functions $(\delta\mathbf{E}\delta\mathbf{E})^S$ and $(\delta N_a\delta\mathbf{E})^S$. For $\mathbf{E}(t)$ of the form given in (1.2) we find from (1.12) that

$$(\delta N_a\delta\mathbf{E})^S_{\omega,\mathbf{k},\mathbf{P}(t)}=i\frac{4\pi e_a n_a}{k^2}\mathbf{k}2\operatorname{Re}\int_0^{\infty}d\tau\exp\{-\Delta\tau+i(\omega-\mathbf{k}\mathbf{v})\tau \quad (1.13)$$

$$-ia_a[\sin\omega_0(t-\tau)-\sin\omega_0 t]\}F_a(\mathbf{P},t).$$

To find from this the spectral density of the fluctuations in the field, we must take into account the fact that

$$\delta\mathbf{E}^S(\omega,\mathbf{k})=-\frac{i\mathbf{k}}{k^2}\sum_a 4\pi e_a\int\delta N_a^S(\omega,\mathbf{k},\mathbf{p})\,d\mathbf{p}. \quad (1.14)$$

In the expression (1.13) we have used the notation

$$a_a=e_a\mathbf{k}\mathbf{E}/m_a\omega_0^2. \quad (1.15)$$

2. THE SPECTRAL DENSITY OF THE FIELD FLUCTUATIONS

Let us now consider the spectral densities of the fluctuations δN_a and δE at frequencies $\omega_0 \gtrsim \omega_e$ (ω_e is the Langmuir frequency for the electrons). From Eqs. (1.3) and (1.4) follows the integral equation for the Fourier component $\delta\mathbf{E}(\mathbf{k}, t)$ of the field fluctuation:

$$\delta\mathbf{E}(\mathbf{k},t)-i\sum_b\frac{4\pi e_b^2 n_b}{k^2}\int\int_0^{\infty}\exp\{-\Delta\tau-i\mathbf{k}\mathbf{V}\tau-ia_b[\sin\omega_0(t-\tau) \quad (2.1)$$

$$-\sin\omega_0 t]\}\delta\mathbf{E}(\mathbf{k},t-\tau)\left(\mathbf{k}\frac{\partial F_b(\mathbf{P},t-\tau)}{\partial\mathbf{P}}\right)d\tau\,d\mathbf{P}=\delta\mathbf{E}^S(\mathbf{k},t),$$

where $\mathbf{V} = \mathbf{P}/m_a$. Below, it will be more convenient to use in place of $\delta\mathbf{E}(\mathbf{k}, t)$ the functions

$$\delta\rho_a(\mathbf{k},t)=\exp(-ia_a\sin\omega_0 t)\,i(\mathbf{k}\delta\mathbf{E}(\mathbf{k},t))/4\pi. \quad (2.2)$$

Hence

$$\delta\rho_a(\omega,\mathbf{k})=\sum_n J_n(a_a)\,i(\mathbf{k}\delta\mathbf{E}(\omega-n\omega_0,\mathbf{k}))/4\pi. \quad (2.3)$$

From (2.1)–(2.3) follows an infinite system of algebraic equations for the functions $\delta\rho_a(\omega, \mathbf{k})$:

$$\delta\rho_a(\omega,\mathbf{k})+4\pi\alpha_a(\omega,\mathbf{k})\sum_b\sum_{l=-\infty}^{\infty}J_l(a_{ab})\delta\rho_b(\omega-l\omega_0,\mathbf{k})=\delta\rho_a^S(\omega,\mathbf{k}), \quad (2.4)$$

where α_a is the polarizability (see below (2.8)). We have used here the notation

$$a_{ab}=\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)\frac{\mathbf{k}\mathbf{E}}{\omega_0^2}. \quad (2.5)$$

The system (2.4) corresponds to the system (2) of Silin's paper [10]. The system (2.4) for $\omega_0 \sim \omega_e$ can be solved, using perturbation theory with m_e/m_i as the perturbation parameter. In the zeroth approximation, i.e., for $m_i = \infty$, we find from (2.4) that

$$\delta\rho_e(\omega,\mathbf{k})=\sum_n\sum_m J_n(a_e)J_m(a_e)\left[\frac{\delta\rho_e^S(\omega+(n-m)\omega_0,\mathbf{k})}{\varepsilon(\omega+n\omega_0,\mathbf{k})}\right.$$

$$\left.-\frac{\varepsilon(\omega,\mathbf{k})-1}{\varepsilon(\omega,\mathbf{k})}\delta\rho_i^S(\omega+(n-m)\omega_0,\mathbf{k})\right], \quad (2.6)$$

$$\delta\rho_i(\omega,\mathbf{k})=\delta\rho_i^S(\omega,\mathbf{k}). \quad (2.7)$$

Here

$$\varepsilon(\omega,\mathbf{k})=1+\frac{4\pi e^2 n_e}{k^2}\int\frac{\mathbf{k}\partial F_e(\mathbf{P},t)/\partial\mathbf{P}}{\omega-\mathbf{k}\mathbf{V}+i\Delta}d\mathbf{P} \quad (2.8)$$

is the permittivity in the approximation $m_i = \infty$.

From (2.6), (2.7), and (2.2) follows the solution to Eq. (2.1) in the approximation $m_i = \infty$:

$$\delta\mathbf{E}(k,t)=\int_0^\infty\left(\frac{1}{\varepsilon}\right)_{k\tau}\exp\{-ia_e[\sin\omega_0(t-\tau)-\sin\omega_0 t]\}\delta\mathbf{E}^S(\mathbf{k},t-\tau)\,d\tau. \quad (2.9)$$

Thus, we have expressed $\delta\mathbf{E}(\mathbf{k}, t)$ in terms of $\delta\mathbf{E}^S$. In (2.9)

$$\left(\frac{1}{\varepsilon}\right)_{\mathbf{k},\tau}=\int_0^\infty\frac{1}{\varepsilon(\omega,\mathbf{k})}e^{i\omega\tau}\,d\omega. \quad (2.10)$$

From Eq. (2.9) and the expression (1.13) we find the spectral density of the fluctuations in the field:

$$(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},t}=\frac{(4\pi)^2e^2}{k^2}\left\{\int d\mathbf{P}\frac{n_eF_e(\mathbf{P})}{|\varepsilon(\mathbf{kV},\mathbf{k})|^2}+\int_0^\infty d\tau\int_0^\infty d\tau'\,d\mathbf{P}\right.$$

$$\left.\times\left(\frac{1}{\varepsilon}\right)_{\mathbf{k},\tau}\left(\frac{1}{\varepsilon}\right)^*_{\mathbf{k},\tau'}\exp\{i\mathbf{kV}(\tau-\tau')-ia_e[\sin\omega_0(t-\tau)-\sin\omega_0 t]\}n_iF_i(\mathbf{P},t)\right\}. \quad (2.11)$$

The first term in this expression determines the contribution of the electrons, the second term that of the ions. The contribution of the ions to the spectral density depends explicitly on the fast time. From (1.19) we find the averaged (over the period) spectral density:

$$(\delta\mathbf{E}\delta\mathbf{E})_{\mathbf{k},t}=\frac{(4\pi)^2e^2}{k^2}\left\{\int d\mathbf{P}\frac{n_eF_e(\mathbf{P})}{|\varepsilon(\mathbf{kV},\mathbf{k})|^2}+\sum_n J_n^2(a_e)\frac{n_i}{|\varepsilon(n\omega_0,\mathbf{k})|^2}\right\}. \quad (2.12)$$

Here we have used the fact that $F_i(\mathbf{V}) = \delta(\mathbf{V})$. The expression (2.12) determines in the zeroth approximation in m_e/m_i the distribution of the electromagnetic fluctuations in the plasma in the presence of a strong high-frequency field ($\omega_0 \gtrsim \omega_e$). Only the second term, which determines the ionic contribution, explicitly depends on the field. It has a resonance character.

3. THE COLLISION INTEGRAL FOR $\omega_0 \gtrsim \omega_e$

From Eqs. (1.3) and (1.4) we find the equation for the Fourier component of δN_a:

$$\delta N_a\left(\mathbf{k},\mathbf{p}-\frac{e_aE}{\omega_0}\cos\omega_0 t,t\right)=\delta N_a^S\left(\mathbf{k},\mathbf{p}-\frac{e_a}{\omega_0}\mathbf{E}\cos\omega_0 t,t\right)$$

$$-e_an_a\int_0^\infty d\tau\exp\{-\Delta\tau-i\mathbf{kv}\tau-ia_e[\sin\omega_0(t-\tau)$$

$$-\sin\omega_0 t]\}\delta\mathbf{E}(k,t-\tau)\frac{\partial F_a(\mathbf{P},t-\tau)}{\partial\mathbf{P}}. \quad (3.1)$$

Substituting into this expression the expression (2.9), which is the solution to Eq. (2.1) for $m_i = \infty$, we express the functions $\delta N_a(\mathbf{k}, \mathbf{p}, t)$ in terms of δN_a^S and $\delta\mathbf{E}^S$. Using the expressions (1.12) and (1.13), we can derive from (3.1) and (2.9) an expression for the spectral density $(\delta N_a\delta\mathbf{E})_{\mathbf{k},\mathbf{p},t}$.

Let us substitute this expression into (1.9) and average it over the period $2\pi/\omega_0$. We obtain as a result an expression for I_e. We can represent it in the form

$$I_e=I_{ee}+I_{ei},\quad \partial F_e/\partial t=I_e. \quad (3.2)$$

Here I_{ee} is the electron-electron collision integral:

$$I_{ee}=2e^4n\frac{\partial}{\partial P_\alpha}\int\frac{k_\alpha k_\beta}{k^4}\frac{\delta(\mathbf{kV}-\mathbf{kV}')}{|\varepsilon(\mathbf{kV},\mathbf{k})|^2}\left\{\frac{\partial F_e}{\partial P_\beta}F_e(P')-\frac{\partial F_e}{\partial P'_\beta}F_a(P)\right\}d\mathbf{P}'\,d\mathbf{k}. \quad (3.3)$$

This expression coincides in form with the Balescu-Lenard collision integral[2,3].

The electron-ion collision integral I_{ei} is determined by the expression

$$I_{ei}=2e_i^2e^2n_i\frac{\partial}{\partial P_\alpha}\sum_{n,m=-\infty}^{\infty}\int d\mathbf{k}\frac{k_\alpha k_\beta}{k^4}J_n(a_e)J_m(a_e)$$
$$\times\frac{\delta(\mathbf{kV}-\mathbf{kV}')}{\varepsilon(m\omega_0,\mathbf{k})\varepsilon^*(n\omega_0,\mathbf{k})}e^{i(n-m)\omega_0 t}\frac{\partial F_e(\mathbf{P},t)}{\partial P_\beta}. \quad (3.4)$$

This expression explicitly depends on the fast time, and contains all the harmonics of the external-field frequency ω_0. Averaging (3.4) over the period $2\pi/\omega_0$, we obtain

$$I_{ei}=2e^2e_i^2n_i\frac{\partial}{\partial P_\alpha}\int\frac{k_\alpha k_\beta}{k^4}\sum_n J_n^2(a_e)\frac{\delta(n\omega_0-\mathbf{kV})}{|\varepsilon(n\omega_0,\mathbf{k})|^2}\frac{\partial F_e}{\partial P_\beta}d\mathbf{k}. \quad (3.4')$$

If we neglect the polarization in the expressions (3.3) and (3.4), then they coincide (in the approximation $m_i = \infty$) with the Landau collision integral with allowance for the influence of the field. The collision integral I_e possesses the properties

$$n_e\int\varphi(\mathbf{P})I_e\,d\mathbf{P}=0\quad\text{for}\quad\varphi=1,\mathbf{P}, \quad (3.5)$$

which guarantee the fulfillment of the laws of conservation of the particle number and the momentum of the plasma.

Let us consider the energy-balance equation. For an ideal plasma we find from Eq. (1.1) that

$$\frac{\partial}{\partial t}\sum_a n_a\int\frac{p^2}{2m_a}f_a\,d\mathbf{p}=\mathbf{jE}. \quad (3.6)$$

Using the definition (1.7) for the function F_a, and noting that according to (1.8) the dominant contribution is made by the mean value of the function F_a over the period $2\pi/\omega_0$, we find from (3.6) the averaged energy-balance equation:

$$\frac{\partial}{\partial t}\sum_a n_a\int\frac{P^2}{2m_a}F_a\,d\mathbf{P}=\mathbf{jE}. \quad (3.7)$$

The collision integral I_{ee} does not make any contribution to the energy-balance equation; therefore, it follows from (3.7) and the kinetic equation (3.3) that

$$n_e\int\frac{P^2}{2m_e}I_{ei}\,d\mathbf{P}=\mathbf{jE}\equiv\frac{\sigma E^2}{2}, \quad (3.8)$$

where σ is the electrical conductivity at the frequency ω_0.

4. CALCULATION OF THE ELECTRICAL CONDUCTIVITY

Let us substitute into (3.8) the expression (3.4) for the collision integral and use as the function $F_e(P)$ the Maxwell distribution function. As a result, we obtain the following expression for σ:

$$\frac{\sigma(E)E^2}{2}=\frac{4\sqrt{2\pi}\,e^2n_e}{(\varkappa T_e)^{3/2}}\sum_a e_a^2n_a\omega_0^2m_e^{1/2}\sum_{n=-\infty}^{\infty}\int_0^{k_{max}}\frac{dk}{k^3}\exp[-n^2\gamma(\omega_0,k)].$$
$$\times\int_0^1 dx\frac{n^2J_n^2(a_ex)}{|\varepsilon(n\omega_0,k)|^2},$$
$$a_e=e_ekE/m_e\omega_0^2,\quad m_e\omega_0^2/2\varkappa T_ek^2=\gamma(\omega_0,k). \quad (4.1)$$

It follows from this expression that the field can be considered weak if $a_e \ll 1$ in the entire k-integration domain, i.e., if

$$a_e=ekE/m_e\omega_0^2\ll 1\quad\text{for}\quad k_{min}<k<k_{max}. \quad (4.2)$$

Let us consider as an example the case when $\omega_0 = \omega_e$. Since $k_{max}/k_{min} \sim 1/\mu \sim r_D\kappa T_e/e^2$ (where μ is the plasma parameter), the condition for the field-intensity E to be low can be written in the form

$$eEl/\varkappa T\ll 1,\quad l=r_D/\mu. \quad (4.3)$$

Thus, for $\omega_0 \sim \omega_e$ the dependence of the conductivity on the field begins to be felt at fairly weak fields, at which the work done by the field over the mean free path l is comparable to κT.

At low frequencies ($\omega_0 \ll \omega_e$) the field can be considered to be weak (see Sec. 6) when

$$eEr_D/\varkappa T \ll 1. \tag{4.4}$$

For weak fields (i.e., for fields satisfying the condition (4.3)) the expression for the conductivity assumes the form

$$\sigma=\frac{\sqrt{2\pi}}{3\pi}\frac{\omega_e^2}{\omega_0^2}\frac{e^2}{(\varkappa T_e)^{3/2}}\frac{1}{m_e^{1/2}}\sum_a e_a^2 n_a \int_0^{k_{max}}\frac{dk}{k}\frac{\exp[-\gamma(\omega_0,k)]}{|\varepsilon(\omega_0,k)|^2}. \tag{4.5}$$

This expression coincides with the expression obtained by Perel' and Éliashberg in [5], where they compute the conductivity for the equilibrium plasma with allowance for the polarization. If we neglect the polarization in the expression (4.5) (i.e., if we set $\epsilon = 1$), then the expression coincides with the one obtained by Silin in [6]. For $\omega_e \gg \omega_0$ the polarization in (4.5) does not play any role.

The integral is truncated at small k because of the exponential factor under the integral sign ($k_{min} \sim \omega_0/v_T$). In the opposite limiting case, when $\omega_0 \ll \omega_e$, the expression (4.5) assumes the form

$$\sigma=\frac{\sqrt{2\pi}}{3\pi}\frac{\omega_e^2}{\omega_0^2}\frac{e^2}{(\varkappa T_e)^{3/2}}\frac{1}{m_e^{1/2}}\sum_a e_a^2 n_a \int_0^{k_{max}}\frac{dk}{k}\frac{1}{1+r_D^2k^2}. \tag{4.6}$$

Let us consider the case of strong fields (i.e., the case when $a_e \gg 1$). From (4.1) we find the following expression:

$$\sigma=\frac{8\sqrt{2\pi}\,en_e}{(\varkappa T_e)^{3/2}}\sum_a e_a^2 n_a m_e^{1/2}\frac{\omega_0^4}{E^3}\left[A(\omega_0)+B(\omega_0)\ln\frac{k_{max}eE}{m_e\omega_0^2}\right]. \tag{4.7}$$

Here we have introduced the notation:

$$A(\omega_0)=\sum_n\int_0^{k_{max}}\frac{dk}{k^4}\frac{\exp[-n^2\gamma(\omega_0,k)]}{|\varepsilon(n\omega_0,k)|^2}n^2\left\{\int_0^1 J_n^2(y)\,dy+\int_1^\infty\left[J_n^2(y)-\frac{1}{\pi y}\right]dy+\frac{1}{\pi}\ln\frac{k}{k_{max}}\right\},$$

$$B(\omega_0)=\sum_n\int_0^{k_{max}}\frac{dk}{k^4}\frac{\exp[-n^2\gamma(\omega_0,k)]}{|\varepsilon(n\omega_0,k)|^2}\frac{n^2}{\pi}.$$

The strong-field conductivity calculation carried out without allowance for the polarization in Silin's paper [6] leads to the same field dependence. Allowance for the polarization (for $\omega < \omega_e$) leads to the convergence of the k integrals in the expressions for $A(\omega_0)$ and $B(\omega_0)$.

For $\omega \gg \omega_0$ and $A \ll B\ln(k_{max}eE/m\omega_0^2)$ the expression (4.7) assumes the form

$$\sigma=\frac{32\sqrt{\pi}}{\pi}en_e\sum_a e_a^2 n_a\frac{\omega_0}{E^3}\ln\frac{k_{max}eE}{m_e\omega_0^2}\left\{\frac{\sqrt{\pi}}{2}\operatorname{erf}x-xe^{-x^2}\right\}\Big|_x^\infty,\quad x=\gamma^{1/2}(\omega_0,k_{max}). \tag{4.8}$$

Notice that under the strong-field condition, when the expressions (4.7) and (4.8) are valid, $eEr_D/\kappa T_e \gg 1$, and the limitations on the field are very rigid. The conductivity then begins to be field dependent at considerably weaker fields when $eEr_D/\kappa T \ll 1$, but $eEl/\kappa T_e \sim 1$. This follows from the criterion for a weak field (see (4.2) and (4.3)).

5. THE COLLISION INTEGRAL FOR A STRONG FIELD WITH $\omega_0 \ll \omega_e$

Let us consider the influence of a strong monochromatic field of the form

$$\mathbf{E}(t)=\mathbf{E}\cos\omega_0 t \tag{5.1}$$

on the collision integral. Such a form is convenient in that it allows us to also consider the case of the constant field (i.e., the $\omega_0 = 0$ case). For $\omega_0 \ll \omega_e$ we can expand the expression for the spectral density (1.12) of the source in powers of $\omega_0\tau$ ($\tau < 1/v_{Te}k_{min} \sim r_D/v_{Te}$) and retain the linear and quadratic terms. The equations for the fluctuations $\delta N_a(\mathbf{k}, \mathbf{p}, t)$ and $\delta\mathbf{E}(\mathbf{k}, t)$ get simplified accordingly. For example,

$$\delta N_a(\mathbf{k},\mathbf{p},t)=\delta N_a^{S}(\mathbf{k},\mathbf{p},t)-e_an_a\int_0^\infty\exp\left\{-\Delta\tau-i\mathbf{kv}\tau+ia_e(t)\frac{\omega_0^2\tau^2}{2}\right\}\delta\mathbf{E}(\mathbf{k},t-\tau)\frac{\partial f_a(\mathbf{p}-e_a\mathbf{E}(t)\tau,t-\tau)}{\partial\mathbf{p}}d\tau. \tag{5.2}$$

Here we have used the notation

$$a_e(t)=e_akE\cos\omega_0t/m_a\omega_0^2. \tag{5.3}$$

The collision integral is determined by the spectral density $(\delta N_a\delta\mathbf{E})_{\omega,\mathbf{k},\mathbf{p}}$ for the frequency region

$$\omega>v_{Te}k_{min}\sim\omega_e,\quad k>k_{min}\sim 1/r_D. \tag{5.4}$$

Taking this and the fact that $\omega_0 \ll \omega_e$ into account, we can, in the zeroth approximation, neglect in (5.2) the explicit time dependence through $\mathbf{E}(t)$, as well as the dependence of the distribution functions f_a on the time. We then arrive at the following expression for the collision integral:

$$J_a(\mathbf{p},t)=\sum_b\frac{2e_a^2e_b^2n_an_b}{\pi^2}\frac{\partial}{\partial p_i}\int d\omega d\mathbf{k}d\mathbf{p}'\frac{k_ik_j}{k^4|\varepsilon(\omega,\mathbf{k})|^2}\times\int_0^\infty d\tau\int_0^\infty d\tau'\cos\left[(\omega-\mathbf{kv})\tau+a_a(t)\frac{\omega_0^2\tau^2}{2}\right]\cos\left[(\omega-\mathbf{kv}')\tau'+a_b(t)\frac{\omega_0^2\tau'^2}{2}\right]\left\{\frac{\partial f_a}{\partial p_j}f_b-\frac{\partial f_b}{\partial p_j'}f_a\right\}_{\mathbf{p}(\tau),\mathbf{p}'(\tau'),t};$$

$$\mathbf{p}_a(\tau)=\mathbf{p}-e_a\mathbf{E}(t)\tau,\quad \mathbf{p}_b'(\tau')=\mathbf{p}'-e_b\mathbf{E}(t)\tau'. \tag{5.5}$$

The permittivity is determined by the expression

$$\varepsilon(\omega,k)=1-i\sum_a\frac{4\pi e_a^2n_a}{k^2}\int\int_0^\infty\exp\left[-\Delta\tau+i(\omega-\mathbf{kv})\tau+ia_a(t)\frac{\omega_0^2\tau^2}{2}\right]\left(\mathbf{k}\frac{\partial f_a}{\partial\mathbf{p}}\right)_{\mathbf{p}(\tau),t}d\tau d\mathbf{p}. \tag{5.6}$$

For $\mathbf{E}(t) = 0$ the expression (5.5) coincides with the Balescu-Lenard integral.

The collision integral (5.5) possesses the properties

$$I(t)=\sum_a n_a\int\varphi_a(\mathbf{p})J_a(\mathbf{p},t)\,d\mathbf{p}=0;\quad \varphi_a=1,\mathbf{p},p^2/2m_a. \tag{5.7}$$

The collision integral can be substantially simplified if, following [9], we introduce an effective potential, taking into account the contribution of the averaged permittivity:

$$\langle|\varepsilon(\omega,\mathbf{k})|^{-2}\rangle^{(\omega)}=\left(\sum_b e_b^2n_b\right)^{-1}\int\frac{d\omega}{2\pi}\sum_a\frac{e_a^2n_a}{|\varepsilon(\omega,\mathbf{k})|^2}\times 2\int_0^\infty d\tau\int d\mathbf{p}\cos\left[(\omega-\mathbf{kv})\tau-a_a(t)\frac{\omega_0^2\tau^2}{2}\right]f_a(\mathbf{p},t). \tag{5.8}$$

For $\mathbf{E}(t) = 0$

$$\langle|\varepsilon(\omega,\mathbf{k})|^{-2}\rangle^{(\omega)}=\left(\sum_b e_b^2n_b\right)^{-1}\sum_a e_a^2n_a\int\frac{f_a(\mathbf{p},t)}{|\varepsilon(\mathbf{kv},\mathbf{k})|^2}d\mathbf{p}. \tag{5.9}$$

In the approximation (5.8), the collision integral assumes the form

$$J_a(\mathbf{p},t)=\sum_b\frac{2e_a^2e_b^2n_b}{\pi}\frac{\partial}{\partial p_i}\int d\mathbf{k}d\mathbf{p}'\int_0^\infty d\tau\frac{k_ik_j}{k^4}\langle|\varepsilon(\omega,\mathbf{k})|^{-2}\rangle\cos\left[(\mathbf{kv}-\mathbf{kv}')\tau-a_{ab}(t)\frac{\omega_0^2\tau^2}{2}\right]\left\{\frac{\partial f_a}{\partial p_j}f_b-\frac{\partial f_b}{\partial p_j'}f_a\right\}_{\mathbf{p}_a(\tau),\mathbf{p}_b(\tau)}. \tag{5.10}$$

Here

$$a_{ab}(t)=\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)\frac{\mathbf{kE}\cos\omega_0t}{\omega_0^2}.$$

6. THE ELECTRICAL CONDUCTIVITY

Using the kinetic equation with the collision integral (5.10), we obtain the following expression for the current:

$$\frac{\partial \mathbf{j}}{\partial t}+\nu\mathbf{j}=\frac{e^2 n}{\mu}\mathbf{E}(t),\quad \nu=\sum_{ab}\frac{2e_a^2e_b^2n_an_b}{\varkappa T(e_an_a-e_bn_b)}\left(\frac{e_a}{m_a}-\frac{e_b}{m_b}\right)I_{ab},\tag{6.1}$$

where

$$I_{ab}=\frac{1}{\pi}\int_0^\infty d\tau\int d\mathbf{k}\,d\mathbf{p}\,d\mathbf{p}'\frac{(\mathbf{kj})^2}{k^4j^2}\langle|\varepsilon(\omega,\mathbf{k})|^{-2}\rangle^{(\omega)}$$

$$\times\cos\left[(\mathbf{kv}-\mathbf{kv}')\tau+a_{ab}(t)\frac{\omega_0^2\tau^2}{2}\right]f_a(\mathbf{p},t)f_b(\mathbf{p}',t);\tag{6.2}$$

$$f_a(\mathbf{p},t)=\frac{1}{(2\pi m_a\varkappa T)^{3/2}}\exp\left[-\frac{(\mathbf{p}-m_a\mathbf{u}_a(t))^2}{2m_a\varkappa T}\right].\tag{6.3}$$

It follows from the expression (6.2) that the electric field is weak if

$$a_{ab}(t)\,\omega_0^2\tau^2/2\sim eEr_D/\varkappa T\ll 1.\tag{6.4}$$

Comparing the inequalities (4.3) and (6.4), we see that at low frequencies (i.e., for $\omega_0 \ll \omega_e$) the field can be considered to be weak in a considerably wider range of E values.

For a weak field the expression for ν in the formula (6.1) for the conductivity assumes the form

$$\nu=\frac{4\sqrt{2\pi}}{3}\frac{e^4n}{\mu^{1/2}(\varkappa T)^{3/2}}\ln(1+r_D^2k_{max}^2)^{1/2}.\tag{6.5}$$

This result coincides with the result obtained earlier in[9].

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