Absorption of first sound in concentrated superfluid He³-He⁴ solutions

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The temperature, frequency, and concentration dependences of the absorption coefficient of first sound are investigated in superfluid He³-He⁴ solutions with 6.3–31.4% He³ at sound frequencies 5.00, 9.65, and 15.00 MHz in the temperature range 0.4–1.6 °K. It is shown that the contribution of second viscosity to the absorption decreases with increasing He³ concentration, and the role of first viscosity and diffusion is predominant. The quadratic dependence of the absorption coefficient on frequency is found not to hold at low temperatures.

1. INTRODUCTION

As is well known, the absorption of sound in a superfluid liquid is a quite unique phenomenon. Even the first experiments, carried out in HeII,[1] showed that in spite of its superfluidity, helium is one of the liquids that are strong absorbers of sound. Here the chief dissipative process is the second viscosity, which is associated with relaxation processes in the liquid. It has been established that the absorption maximum of first sound, which occurs in HeII at a temperature of ~0.9°K, is due to the establishment of an energy equilibrium between the phonon and roton gases.[2] The numerous experimental data available in this range are in excellent agreement with the predictions of theory.

The character of the relaxation processes, and consequently the absorption of sound in superfluid helium, change markedly in the presence of He³ impurities [3-6]. However, at the present time, studies of the absorption of first sound in He³-He⁴ solutions have not matched the completeness of the data for HeII. The only exceptions are weak He³ solutions in superfluid He⁴ in the low-temperature region, where the impurity excitations experience a Fermi degeneracy.[7-10] The possibility of exact accounting for the He³-He³ and He³-phonon interactions in such solutions has made it possible to solve the problem of sound absorption by means of the kinetic equations for the range of temperatures below 0.6°K. It has been established here that at T < 0.3°K the sound absorption is due in practice only to impurities and is connected with the viscosity of He³. In the range of temperatures above 0.3°K, a large contribution to sound absorption is also made by thermal phonons interacting with He³ quasiparticles. The calculation that has been carried out was found to be in excellent agreement with the experimental data.

For concentrated He³-He⁴ solutions, such a complete theory is still lacking; here experimental data have appeared only recently[5-6] and are in the stage of accumulation. In the present study, which is an extension of [5-6], the absorption coefficient of first sound was measured in concentrated He³-He⁴ solutions in the superfluid region for various vibration frequencies.

2. FEATURES OF THE METHOD

We used the pulsed ultrasonic method to measure the sound absorption coefficient. Rf pulses of controlled amplitude, duration, and carrier frequency were applied to a piezoelectric quartz crystal, which served simultaneously as the source and the detector of the oscilla- tions.[11] The measurements were carried out at vibration frequencies of 5.00, 9.65 and 15.00 MHz.

In the region of relatively high temperatures (T > 1.4°K), absolute measurements of the sound absorption coefficient were carried out in a test cell with variable length, as described in [11]. At T < 1.4°K, the absorption coefficient was determined by a relative method with normalization in the high-temperature region.

Solutions with He³ molar concentrations of 6.3, 11.0, 19.7 and 31.4%, and also with pure He⁴, were studied. The solutions were prepared by mixing pure gaseous isotopes in a special apparatus. The error in the concentration determination did not exceed 0.1% of He³.

Temperatures below ~1.4°K were obtained in an He³ refrigerator in which the vapors above the liquid He³ were evacuated by an adsorption pump, located in another dewar. The vapor pumping rate was regulated by two special valves, which were used simultaneously for maintaining constant temperature. All the measurements were carried our after holding the apparatus in the corresponding isothermal state until the amplitude of the sound pulses did not change with time. The value of the temperature was determined from the vapor pressure of He³ (T62 scale) with account of thermomolecular corrections.

Special attention was given to the absence of skewness between the planes of the piezoelectric crystal and the reflector. The parallelism adjustment was carried out in a calibration experiment with He⁴ at T ≈ 1.95°K, when the absorption of first sound is at minimum. Here we looked for the maximum number of echo signals, which should also be accompanied by the maximum amplitude. A check for parallelism was made after each experiment with the solution.

The value of the acoustic path was determined by measurement of the velocity of first sound in this same experiment, and the error in its determination amounted to 0.2–0.3%. The total error in the determination of the absorption coefficient of first sound did not exceed 2.5%.

3. TEMPERATURE AND CONCENTRATION DEPENDENCES

The general form of the temperature dependence of the absorption coefficient of first sound in superfluid He³-He⁴ solutions is shown in Fig. 1, and smoothed values, corresponding to different frequencies, temperatures and concentrations, are given in Table I. These results show that the sound absorption decreases sig-
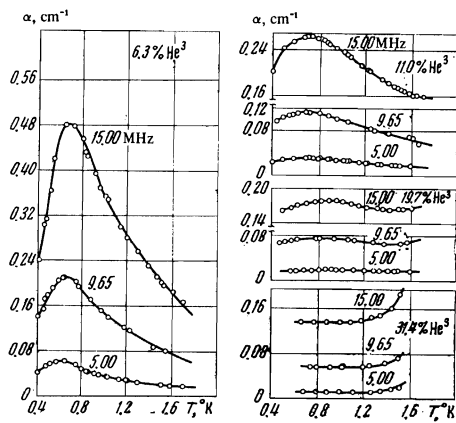
FIG. 1. Dependence of the absorption coefficient of first sound in superfluid He³-He⁴ solutions on the temperature. (He³ the concentrations and the frequencies are indicated on the plots.)

TABLE I. Smoothed values of the first-sound absorption coefficient (cm⁻¹) in He³–He⁴ solutions

T, °K	6.3% He³			11.0% He³			19.7% He³			31.4% He³		
	5.00 MHz	9.65 MHz	15.00 MHz	5.00 MHz	9.65 MHz	15.00 MHz	5.00 MHz	9.65 MHz	15.00 MHz	5.00 MHz	9.65 MHz	15.00 MHz
0.4	0.0400	0.131	0.225	0.0257	--	0.207	0.0180	0.0675	0.161	--	--	--
0.5	0.0535	0.175	0.340	0.0290	0.103	0.241	0.0180	0.0703	0.169	--	--	--
0.6	0.0610	0.207	0.453	0.0304	0.113	0.260	0.0183	0.0703	0.169	--	--	--
0.7	0.0580	0.209	0.491	0.0310	0.113	0.268	0.0191	0.0730	0.173	0.0154	0.0574	0.139
0.8	0.0515	0.190	0.463	0.0302	0.113	0.266	0.0195	0.0746	0.176	0.0154	0.0574	0.139
0.9	0.0450	0.168	0.410	0.0282	0.103	0.255	0.0195	0.0735	0.177	0.0154	0.0574	0.139
1.0	0.0400	0.148	0.360	0.0270	0.103	0.242	0.0191	0.0710	0.177	0.0154	0.0574	0.139
1.1	0.0350	0.132	0.320	0.0250	0.0940	0.228	0.0187	0.0692	0.169	0.0155	0.0575	0.140
1.2	0.0315	0.119	0.286	0.0235	0.0880	0.213	0.0184	0.0680	0.165	0.0157	0.0585	0.142
1.3	0.0280	0.106	0.259	0.0220	0.0809	0.199	0.0179	0.0665	0.161	0.0162	0.0600	0.146
1.4	0.0250	0.096	0.232	0.0204	0.0760	0.186	0.0176	0.0655	0.159	0.0173	0.0640	0.156
1.5	0.0220	0.086	0.206	0.0190	0.0715	0.173	0.0175	0.0650	0.158	0.0200	0.0740	0.180
1.6	0.0200	0.077	0.186	0.0180	0.0680	0.164	0.0177	0.0658	0.160	--	--	--

nificantly with increasing He³ concentration. We note that at temperatures above the λ point of the solutions, the absorption increases with increasing He³ concentration, as was shown in [5-6]. This is due to the following circumstance. If we use the usual hydrodynamic expression for the first-sound absorption,

$$\alpha = \frac{2\pi^2 f^2}{\rho u_1^3}\,\tilde{\alpha}, \qquad (1)$$

(f is the frequency of vibration, ρ the density of the liquid, u_1 the speed of first sound, $\tilde{\alpha}$ the dissipative part due to first viscosity, heat conduction, second viscosity, and diffusion), then (see [6]) the quantity $\tilde{\alpha}$ is practically independent of the He³ concentration at $T > T_\lambda$. The increase of α with concentration in this case is connected only with the decrease in the density of the solutions and the speed of sound. At $T < T_\lambda$, the principal role in the concentration dependence is increasingly played by the dissipative processes described by the quantity $\tilde{\alpha}$.

A comparison of the graphs shown in Fig. 1 shows that the very character of the temperature dependence of the sound absorption changes with increasing He³ concentration. The absorption maximum, which occurs in the range ~0.9°K in He⁴ and weak He³–He⁴ solutions, decreases gradually with increase in the concentration and is completely absent in the solution with 31.4% He³. Some growth of the absorption coefficient of the 31.4% solution is due to the effect of the λ transition, where the absorption goes through a sharp maximum connected with a phase transition of the second kind.[6]

The location of the low-temperature sound absorption maximum in He⁴ is connected, as is well known,[2] with

scattering of phonons by rotons, which determines the second viscosity in this temperature region. A maximum occurs when the characteristic relaxation time τ_{ph-r} becomes $\sim 1/2\pi f$. The addition of He³ impurities speeds up the establishment of equilibrium of elementary excitations in the gas, thereby decreasing the value of the second viscosity and, consequently, the sound absorption in the solution. In weak solutions, the impurities decrease, τ_{ph-r} [1)] and therefore the maximum absorption shifts in the direction of lower temperatures, for which the condition $\tau_{ph-r} \sim 1/2\pi f$ is again satisfied.

However, upon a further increase in the He³ concentration, beginning with ~6% He³, the position of the maximum begins to shift in the direction of higher temperatures. Such a nonmonotonic dependence of the temperature of the maximum T_{max} on the He³ concentration is shown in Fig. 2 for the two frequencies studied: 9.65 MHz and 15.00 MHz. The result obtained evidently indicates that the character of scattering of phonons by rotons changes sharply with increase in the He³ concentration in the presence of impurity excitations. It is not excluded that for concentrated solutions the principal role will be played by some new relaxation process, whose characteristic relaxation time does not depend on temperature and concentration in the same way as τ_{ph-r}.

In the temperature range above the maximum, the dependence of the sound absorption coefficient on the temperature can be approximated by a power law $\alpha \sim T^{-n}$, where the exponent n does not depend on the frequency of the oscillations and is determined only by the He³ concentration. The values of n for different solutions are given in Table II, where, in addition to the experimental data of the present study, we have also given the data of [4].

In the temperature range below the maximum, the character of the dependence of α on T is determined not only by the concentration of the solution, but also by the frequency of vibration.

4. FREQUENCY DEPENDENCE

In accord with expression (1), the sound absorption should be proportional to the square of the frequency. This conclusion is valid in the hydrodynamic approximation, when $\omega\tau \ll 1$, where $\omega = 2\pi f$, and τ is some relaxation time that characterizes the interaction of elementary excitations in the solution. The presence of maxima on the temperature curve of sound absorption in solutions means that the condition $\omega\tau \ll 1$ is not satisfied everywhere, i.e., in such cases, it is impossible to use Eq. (1) and it is necessary to carry out the analysis on the basis of the kinetic equations.
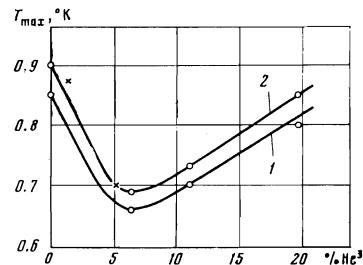
FIG. 2. Concentration dependence of the temperature of the sound-absorption maximum: O—present study; X—[4]. Curve 1 corresponds to a frequency of 9.65 MHz, curve 2 to 15.00 MHz.
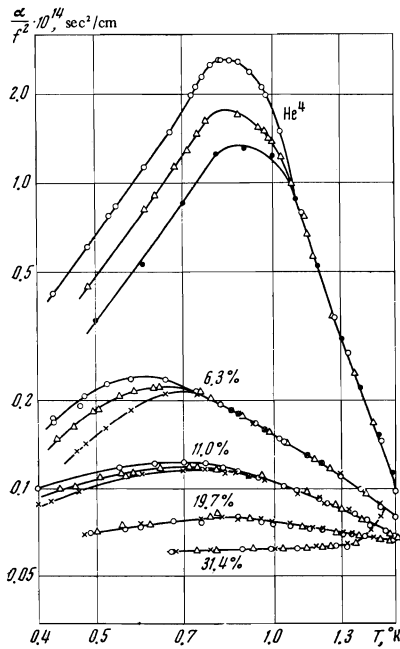
FIG. 3. Temperature dependence of α/f^2. Concentrations are indicated in the figure; O—5.00 MHz, △—9.65 MHz, X—15.00 MHz, ●—14.4 MHz (data of [15]).

TABLE II

% He^3	n	T, °K	% He^3	n	T, °K
0	6.0±0.1	1.1—1.6	6.3	1.26±0.05	0.8—1.4
0.32 [4]	4.8±0.1	1.1—1.6	11.0	0.82±0.05	1.0—1.6
1.6 [4]	3.2±0.1	1.0—1.6	19.7	0.24±0.05	1.0—1.4
5.2 [4]	1.6±0.05	0.9—1.5			

To establish the frequency dependence of sound absorption, it is convenient to use the quantity α/f^2, which is not a function of the frequency of the oscillations in the case of validity of Eq. (1). Figure 3 shows this quantity as a function of temperature at various concentrations and frequencies. It follows from the plots that the condition $\alpha \sim f^2$ is satisfied for the concentrated solutions (19.7 and 31.4% He^3) at all the investigated frequencies. This condition is violated for very weak solutions of He^3 in He^4 at low temperatures, beginning with the region near the absorption maximum. Here the sound absorption increases with frequency more slowly than the quadratic law, so that the quantity α/f^2 falls off with the frequency.

If we represent the frequency dependence of the absorption of first sound in solutions at low temperatures in the form $\alpha \sim f^m$, it then turns out that the exponent m depends not only on the temperature, but also on the concentration of He^3. As the He^3 concentration in the solution decreases, a steady transition is observed from the square-law frequency dependence of the absorption to $\alpha \sim f^{3/2}$. The value of m = 3/2 for He^4 is in excellent agreement with the data of the work of Jeffers and Wheatney.[13]

The position of the sound-absorption maximum also changes with change in the frequency of oscillation (see Figs. 1 and 2). If we assume that the maximum is due to a relaxation process with characteristic time τ, then the sound absorption in the region of the maximum can be written as usual in the form

$$\alpha = A(T, P)\frac{\omega^2\tau}{1+\omega^2\tau^2}, \qquad (2)$$

where A(T, P) is a function of pressure and temperature, and does not depend on the frequency. In accord with (2), the absorption maximum occurs at a temperature for which $\omega\tau \sim 1$. As follows from Figs. 1 and 2, the maximum shifts in the direction of higher temperatures with increase in the frequency for all solutions studied. This means that the relaxation time of the process that leads to the absorption maximum in the solutions decreases with increase in the temperature. The value of the relaxation time τ near the absorption maximum amounts to ~10^{-8} sec.

5. DISSIPATIVE PROCESSES IN SUPERFLUID SOLUTIONS

Sound absorption in superfluid He^3-He^4 solutions is produced by various irreversible processes, which are described by the coefficients of first viscosity η, second viscosity ζ_1, ζ_2 and ζ_3, thermal conductivity κ, and diffusion D. However, the character of these processes is found to be different above and below the absorption maximum. In the high-temperature region, the characteristic relaxation times are much smaller than the period of the sound wave and, as has already been pointed out, the sound absorption is proportional to the square of the frequency, while the kinetic coefficients do not depend on the frequency. In the temperature range below the maximum, the hydrodynamic treatment is not applicable and the dissipative processes here have a more complicated character.

a) High temperatures.

In the range of temperatures above ~1°K, the absorption of first sound in the solutions investigated is described by Eq. (1), where the value of $\tilde{\alpha}$ was calculated in the hydrodynamic approximation by Karchava and Sanikidze [14] for solutions with arbitrary He^3 content. Inasmuch as the different dissipation mechanisms enter additively into the absorption coefficient, they can be considered independently. The dissipative part of α due to first and second viscosities has, according to [14], the form

$$\tilde{\alpha}(\eta, \zeta) = \tilde{\alpha}(\eta) + \tilde{\alpha}(\zeta_1, \zeta_2, \zeta_3) = \frac{1}{1+\rho_s\bar{c}^2/\rho_n}\left[\left(\frac{4}{3}\eta+\zeta_2\right)\left(1+\frac{\rho_s}{\rho_n}\bar{c}\right)^2\right.$$

$$\left. +2\rho\frac{\rho_s}{\rho_n}\zeta_1\bar{c}\left(1-\frac{\rho_s}{\rho_n}\bar{c}\right)+\rho^2\left(\frac{\rho_s}{\rho_n}\right)^2\bar{c}^2\zeta_3\right], \qquad (3)$$

$$\bar{c} \equiv \frac{c}{\rho}\frac{\partial\rho}{\partial c},$$

where η denotes the viscosity of the normal component of the solutions, ρ_s, ρ_n, and ρ are the densities of the superfluid and normal components and the total density of the liquid, respectively, and c is the weight concentration of He^3 in the solution.

It is a significant fact here that the absorption of first sound in concentrated solutions is connected not only with the two coefficients η and ζ_2, as in the case of pure He^4; all four viscosity coefficients contribute. This is due to the fact that in the wave of first sound, relative motion takes place in the solution between the normal and superfluid components (even upon neglect of the thermal expansion of the helium); ($v_n - v_s \neq 0$).

The processes of diffusion, pressure diffusion and

thermal diffusion of impurities due to the presence of concentration, pressure and temperature gradients in the sound wave, are also specific for solutions. The contribution of these processes to the absorption of first sound was calculated in [14]. Here, as estimates show, the basic contribution to α is made by the term associated with the pressure diffusion of the impurities:

$$\tilde{\alpha}(D) = D\bar{c}^2\rho u_1^2 \left[c^2 \frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\left(1+\frac{\rho_*}{\rho_n}\bar{c}^2\right)\right]^{-1}, \qquad (4)$$

where the potential Z is connected with the chemical potentials of He^3 and He^4 in the solution: $Z = \rho(\mu_3 - \mu_4)$.

Another dissipative process which produces sound absorption in the liquid is heat conduction. However, in superfluid solutions, as in HeII, $\tilde{\alpha}(\kappa)$ turns out to be a negligible quantity in the considered range of temperatures, because of the small difference of the specific heats of helium at constant volume and at constant pressure. From the estimates, $\tilde{\alpha}(\zeta)$ amounts to less than 1% of the total absorption.

The experimental data obtained on the absorption of first sound in solutions can be compared with Eqs. (3)-(4). Such a comparison makes it possible, first of all, to estimate the contribution of second viscosity $\tilde{\alpha}(\zeta)$ to the sound absorption. Unfortunately, inasmuch as there is no independent method of determination of ζ_1, ζ_2 and ζ_3, it is customary to use for their determination data on the excess sound absorption produced by relaxation processes in the gas of quasiparticles, which in macroscopic terms is equivalent to the presence of second viscosity.[2] Thus the quantity $\tilde{\alpha}(\zeta)$ in our case can be defined as the difference between $\tilde{\alpha}$ and $\tilde{\alpha}(\eta) + \tilde{\alpha}(D)$.

To determine $\tilde{\alpha}(\eta)$, we used experimental data on the density of the normal component [15] and the viscosity [16] of the solutions. A calculation of the contribution of diffusion to the sound absorption has been carried out with account of experimental data [17] on the diffusion coefficient, and the derivative $c^2(\partial/\partial c)\cdot(Z/\rho)$ was calculated in the approximation of ideal solutions:[18]

$$c^2 \frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right) = kT\left(\frac{c^3}{m_4(1-c)} + \frac{c}{m_3}\right), \qquad (5)$$

where m_3 and m_4 are respectively the masses of the He^3 and He^4 atoms and k is the Boltzmann constant.

The typical concentration dependences obtained for the dissipative losses associated with first viscosity, diffusion, and second viscosity are shown in Fig. 4. It is seen from the graphs that the contribution of first viscosity and diffusion depends weakly on the He^3 concentration, while the contribution of second viscosity is determined by the concentration of the solution to a significant degree. While the second viscosity is the chief dissipative process for He^4 in weak solutions of He^3-He^4, its contribution becomes very small in concentrated solutions and the principal role in the absorption of first sound is played by the first viscosity and diffusion.

The relation between the contributions to the absorption of second and first viscosities depends not only on the concentration but also on the temperature. This is illustrated in Fig. 5. The ratio $\tilde{\alpha}(\zeta)/\tilde{\alpha}(\eta)$ is especially sensitive to the temperature for weak He^3 solutions in He^4, but for concentrated solutions, it is practically independent of the temperature and becomes smaller than unity.
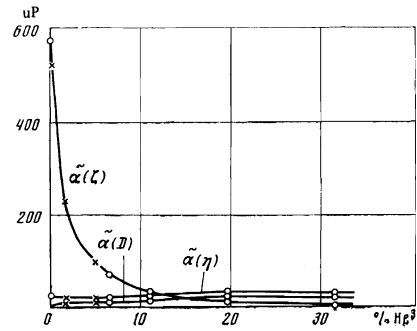

FIG. 4. Contribution of various dissipative processes to sound absorption: first viscosity $(\tilde{\alpha}(\eta))$, diffusion $(\tilde{\alpha}(D))$ and second viscosity $(\tilde{\alpha}(\zeta))$ as functions of the He^3 content in the solution. Temperature 1.2°K; O—present study; X–[4].
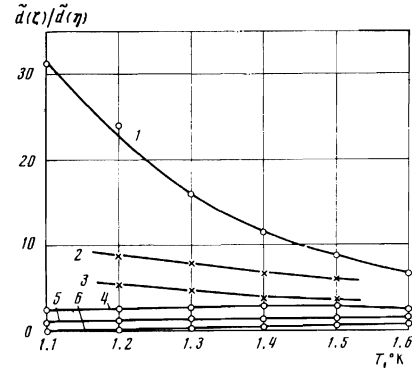

FIG. 5. Temperature dependence of the ratio $\tilde{\alpha}(\zeta)/\tilde{\alpha}(\eta)$: O—present study; X–[4]; 1–He^4; 2–1.6% He^3; 3–5.2% He^3; 4–6.3% He^3; 5–11.0% He^3; 6–19.7% He^3.

The sharp decrease in the second viscosity in weak He^3 solutions in He^4 has been explained by Andreev,[19] who proposed that, just as in the case of He^4 at high temperatures, the processes of establishment of equilibrium in the number of phonons and rotons are the slow processes. The effect of small impurities on the rate of change of the number of phonons and rotons was established in an examination of the two most probable processes: the three-phonon process on the impurity and the process of roton-phonon transformation on the impurity. A comparison of the experimental data with this calculation [5] shows that agreement is observed only for solutions containing He^3 up to ~6%. At higher concentrations, other relaxation processes probably play the basic role. We note that, beginning with ~6% He^3, as has already been noted, a shift of the sound-absorption maximum in the direction of higher temperatures has also been observed (see Fig. 2).

b) Low temperatures.

In the region of temperatures below the maximum, where the sound absorption is not proportional to the square of the frequency, the problem of the effect of the different processes on sound absorption is much more complicated than it is in the hydrodynamic region. This problem has been solved only for temperatures below ~0.6°K [9,10] under the conditions that one can neglect the phonon-phonon collision integral in the kinetic equation for the phonons, and that the principal role is played by He^3-phonon scattering. At the higher temperatures corresponding to the experimental data of the present study, contributions to the sound absorption are made

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by interactions of rotons, phonons and impurity excitations. For this region, unfortunately, there is as yet no microscopic theory analogous to the theory of [2] which applies to pure He^4.

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[1] In the solutions, the time τ_{ph-r} characterizes the phonon-roton scattering with participation of the impurities.

[2] To determine each of the coefficients of second viscosity ζ_1, ζ_2, and ζ_3, it is also necessary to use data on the absorption of second and fourth sounds in solutions. At the present time, these data are lacking.

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