Spectral and angular distribution of bremsstrahlung produced by high energy electrons in a dense medium

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The spectral-angular distribution of bremsstrahlung produced by an ultrarelativistic electron in a absorbing medium is considered. It is shown that the properties of the medium lead to larger effective emission angles than those predicted by the Bethe–Heitler theory. Analytic expressions for the bremsstrahlung spectral-angular distribution are derived by taking into account multiple scattering with variable mean-square angle, variable polarization of the medium, and variable absorption of the virtual quanta. The effect of the boundary of the material on the spectral and angular distribution of the bremsstrahlung emitted into vacuum by a sufficiently thick layer of strongly absorbing matter is investigated.

INTRODUCTION

As is well known, the effect produced by bremsstrahlung of a high-energy electron in a condensed medium[1-8] leads to a change in the energy spectrum of the radiation. Landau and Pomeranchuk[1] were the first to point out that the effect of multiple scattering of the electron should lead to an increase in the effective bremsstrahlung angles in comparison with the characteristic Bethe-Heitler emission angles. According to[1], when the effective multiple scattering appears, the effective angles are determined by the relation $\theta_{\text{eff}} \approx (q_0/\omega)^{1/4}$, where ω is the emission frequency and $4q_0$ is the mean squared multiple-scattering angle per unit length[1)].

The influence of the polarization of the medium on the spectral and angular distributions of the bremsstrahlung were considered by Ter-Mikaélyan[2, 9]. It follows from the results of these papers that the effective emission angles, with allowance for the effect of the polarization of the medium in the region of relatively large frequencies $\omega \gg 1$, are determined by the relation $\theta_{\text{eff}} \approx \omega_0/\omega$, where ω_0 is the plasma frequency, and greatly exceed the effective Bethe-Heitler emission angles. The spectral-angular bremsstrahlung distribution with simultaneous allowance for the multiple scattering of the electron and for the polarization of the medium was subsequently obtained by Gol'dman[10] and by Pafomov[11] by solving the kinetic equation for the classical distribution function of the electrons in matter. The results of these studies confirm the estimates of the effective emission angles obtained by Landau and Pomeranchuk[1], and agree with the results of Ter-Mikaélyan[2, 9].

The influence of multiple scattering on the angular distribution of the bremsstrahlung was considered also by Kalashnikov and Ryazanov[12], who used a solution they obtained for the Dirac equation in the field of randomly distributed centers. Their results also confirm the qualitative estimates of θ_{eff} given in the paper of Landau and Pomeranchuk[1], and agree with the result of Gol'dman[10].

Galitskiĭ and Gurevich[13] have shown how the effective bremsstrahlung angles of a fast electron in a dense medium can be obtained on the basis of a qualitative treatment that uses the concept of the coherent radiation length. In the same paper they estimated the effective bremsstrahlung angles in the case when the effect of absorption of virtual quanta becomes appreciable ($\theta_{\text{eff}} \approx (\omega L_C)^{-1/2}$, where L_C is the mean free path of the quanta in the medium).

It was shown in[7, 8], that the form of the energy spectrum of the bremsstrahlung in an unbounded absorbing medium depends significantly on the method of separating the energy losses to bremsstrahlung from the total energy loss. It is therefore natural to expect the angular distribution of the bremsstrahlung to be also dependent on the concrete method of separating the energy losses. The change that takes place in the mean-squared multiple scattering angle[2, 7], an effect that should influence the angular distribution of the bremsstrahlung, has never been considered before. In this paper we use classical electrodynamics to calculate the spectral-angular distribution of the bremsstrahlung of a high-energy electron in an infinite medium, with allowance of all the known effects that take place in the medium, including the aforementioned change in the constant of the multiple scattering and the effective absorption of the virtual quanta. We first consider the total energy losses. The separation of the energy losses defined as bremsstrahlung proper from the total losses will be carried out by the method used in[7, 8].

SPECTRAL-ANGULAR DISTRIBUTION OF THE DENSITY OF THE ENERGY LOSSES AND OF THE BREMSSTRAHLUNG

We use the results of[7], assuming that all the conditions for their applicability are satisfied. The energy loss of an electron at the frequency $\omega \ll E_0$, as it moves through a layer of matter with complex dielectric constant $\epsilon(\omega) = \epsilon'(\omega) + i\epsilon''(\omega)$ and of thickness $T \gg L$ (L is the radiation unit of length and E_0 is the initial energy of the electron) can be represented according to[7] in the form

$$E_\omega = \frac{e^2\omega^3}{\pi^3}\operatorname{Re}\iint\int_0^T\int_0^{T-t}\varphi(\xi,t)\frac{\varepsilon''(\omega)\,\xi\eta\, w^{(-k)}(\eta-\xi,t,t+\tau)\,e^{-i\omega\tau}}{(k^2-\varepsilon'\omega^2)^2+\omega^2L_c^{-2}}\,d\tau\,dt\,d\xi\,d\eta\,d\mathbf{k}. \tag{1}$$

Here $\varphi(\xi, t)$ is the probability density of electron scattering through an angle $\xi - \xi_0$ within a time t, $\xi_0 = \mathbf{v}_0/v_0 - \mathbf{n}$, $\xi = \mathbf{v}_1/v_1 - \mathbf{n}$, $\eta = \mathbf{v}_2/v_2 - \mathbf{n}$; $\mathbf{v}_0$, $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities (assumed to be random quantities) at the initial instant and at the instants t and $t + \tau$, respectively; $\mathbf{n} = \mathbf{k}/k$ is a unit vector in the photon emission direction, $\epsilon''(\omega) = 1/L_C\omega$, and L_C is the quantum-absorption length. The spatial Fourier component of the correlation function $w^{(-k)}$ satisfies the equation

$$\frac{\partial w^{(-k)}}{\partial\tau}-ik\left[1-\left(\frac{1}{2}E^2\right)e^{2(t+\tau)/L}-\frac{\eta^2}{2}\right]w^{(-k)}=q_0e^{2(t+\tau)/L}\Delta_\eta w^{(-k)}, \quad (2)$$

where $4q_0$ is the mean square of the angle of multiple scattering of an electron electron with energy E_0 per unit path, and $w^{(-k)}|_{\tau=0}=\delta(\xi-\eta)$.

The subsequent transformations of (1) differ from the corresponding transformations used in[7] in that no integration over the emission angles of the quanta is carried out in our case[2)]. It follows from (1) that the energy loss E_ω depends in fact on the quantity

$$\tilde{w}=\int\xi\varphi(\xi-\xi_0,t)w^{(-k)}d\xi.$$

The function $\tilde{w}$ satisfies Eq. (2) with initial condition $\tilde{w}|_{\tau=0}=\eta\varphi(\eta-\xi_0,t)$. At sufficiently large distances $x=t+\tau$ from the foward boundary of the medium ($x\gg l_{coh}$), which will be considered from now on[3)], the solution of the equation for the function $\tilde{w}$ can be obtained by the procedure used in[6].

We introduce the angle $\theta_e=\eta-\xi_0$ of the electron deviation from the initial direction of motion, and the quantum emission angle $\theta_\gamma=-\xi_0$ relative to the direction of the initial velocity v_0. The expression for the angular density of the electron energy loss to emission of quanta of frequency ω ($d\Omega=2\pi\theta_\gamma d\theta_\gamma$) as the electron passes through a layer of matter of thickness $T\gg l_{coh}$ can be represented in the form

$$\frac{dE_\omega}{d\Omega}=\int_0^T\int\varphi(\theta_e,x)\frac{dI(z,\omega,x)}{dz}\frac{1}{\pi}\left(\frac{\omega}{q}\right)^{1/2}d\theta_e\,dx,$$

$$\frac{dI(z,\omega,x)}{dz}=\frac{2e^2\omega^2}{\pi^2L_c}\operatorname{Re}\int_0^\infty\frac{i}{\lambda L}\cdot$$

$$\cdot\int_0^\infty e^{-2is(k)\tau}\frac{d}{d\tau}\exp\left\{\frac{i^{3/2}z}{2^{3/2}}\frac{\psi_2(\lambda,\tau)}{\psi_1(\lambda,\tau)}\right\}\frac{d\tau\,dk}{(k^2-\varepsilon'\omega^2)^2+\omega^2L_c^{-2}} \quad (3)$$

$$z=\sqrt{\omega/q}(\theta_e-\theta_\gamma)^2,\quad \psi_1(\lambda,\tau)=H_0^{(1)}(\beta)H_1^{(2)}(\delta)-H_0^{(2)}(\beta)H_1^{(1)}(\delta),$$

$$\psi_2(\lambda,\tau)=H_0^{(2)}(\beta)H_0^{(1)}(\delta)-H_0^{(1)}(\beta)H_0^{(2)}(\delta),$$

$$\delta=2^{1/2}i^{3/2}\lambda^{-1},\quad \beta=\delta e^{-\lambda\tau/2};$$

$$\lambda=1/L\sqrt{qk},\quad q=q_0e^{2x/L},\quad s(k)=[\omega-kv(x)]/4\sqrt{qk},$$

$H_\nu^{(i)}$ is a Hankel function of index ν and order i.

The function dI/dz in (3) depends on the angle between the direction of the electron velocity at the depth x and the quantum emission angle. The dependence on the depth of penetration of the electron into the medium $x\gg l_{coh}$, enters only via the electron energy $E(x)$. It can be assumed that the function dI/dz describes the spectral-angular distribution of the quanta emitted by an electron of energy $E(x)$, and thus determines the influence of the effects of a medium in the "elementary act" of the radiation. In particular, it takes into account the "nontrivial" effect of multiple scattering of the electron over the coherence length (see[12]). The factor $\varphi(\theta_e, x)$ is the electron distribution function with respect to the deviation angles at the depth x. This function describes the "trivial" influence of the multiple scattering of the electrons prior to the "instant of emission" on the angular distribution of the quanta. In the region of emission frequencies and electron energies in which multiple scattering over the coherent radiation length becomes significant, the nontrivial influence of the multiple scattering can be separated only arbitrarily, since the effective emission angles are in this case of the same order of magnitude as the angle of multiple scattering of the electron over the coherence length l_s (see below). In the case of a rarefied medium, the influence of the multiple scattering over the coherence length of the radiation is inessential and dI/dz reduces to an expression describing the spectral-angular distribution of the bremsstrahlung intensity after Bethe-Heitler. On the other hand, the influence of the multiple scattering remains trivial as before.[4]

After integration with respect to the absolute magnitude of the wave vector **k**, we can represent dI/dz in the form

$$\frac{dI(\omega,z,x)}{dz}=-\frac{e^2}{\pi l_s}\operatorname{Im}F_s+\frac{e^2}{\pi L_c}\frac{z/8}{(z/8+s_1)^2+(\lambda_s^{(c)}/8)^2}, \quad (4)$$

$$F_s\equiv F_s(s_1,\lambda_s,\lambda_s^{(c)},z)$$

$$=\int_0^\infty\exp\{-2i(s_1-i\lambda_s^{(c)}/8)\tau\}\frac{d}{d\tau}\left\{\exp\left[\frac{i^{3/2}z}{2^{3/2}}\frac{\psi_2(\lambda_s,\tau)}{\psi_1(\lambda_s,\tau)}\right]-e^{-iz\tau/4}\right\}d\tau, \quad (4')$$

$$l_s=(q\omega)^{-1/2},\quad \lambda_s=l_s/L,\quad \lambda_s^{(c)}=l_s/L_c,\quad s_1=s(\omega\sqrt{\varepsilon'}).$$

The second term in (4), multiplied by the solid-angle element $dz=(\omega/q)^{1/2}d\Omega/\pi$, is practically independent of the multiple-scattering angle $q_0^{1/2}$ and describes processes connected with the uniform motion of the electron. In the case when the mean free path of the virtual quanta L_c is due to their absorption in the cluster of the formation of electron-positron pairs, this term describes the angular distribution of the electron-positron pairs produced directly by the fast electron. In a nonabsorbing medium ($\lambda_s(c)\to 0$), in the region of atomic frequencies where the condition $v\sqrt{\epsilon'}\geq 1$ can be satisfied, the second term in (4) yields the well known expression for the angular distribution of the Cerenkov radiation

$$\frac{dI_C}{d\Omega}=\frac{e^2}{\pi^2}(v\sqrt{\varepsilon'}-1)\delta\left(1-\frac{(\theta_e-\theta_\gamma)^2}{2}-\frac{1}{v\sqrt{\varepsilon'}}\right),$$
$$1-v\ll 1,\quad \varepsilon'-1\ll 1.$$

The first term in (4) vanishes as $q\to 0$ and describes, in accordance with the accepted definition, the angular distribution of the bremsstrahlung with allowance for the effects of the medium in the elementary radiation act. In essence, Eq. (4) is the most general result. We shall show that by using (4) we can obtain the known results concerning the influence of the medium on the spectral-angular distribution of the bremsstrahlung as certain limiting cases. At the same time, we shall spell out concretely the new results that are contained in formula (4).

ANGULAR DISTRIBUTION OF BREMSSTRAHLUNG IN A NONABSORBING MEDIUM

We consider the angular distributions of the bremsstrahlung intensity in a nonabsorbing medium ($\lambda_s^{(c)}=0$) in the frequency region where the condition for the Cerenkov radiation cannot be satisfied ($s_1>0$) without taking into account the change of the mean-squared scattering angle over the coherence length. Using the asymptotic equation

$$\frac{i^{3/2}}{2^{3/2}}\frac{\psi_2(\lambda_s,\tau)}{\psi_1(\lambda_s,\tau)}\approx\frac{1+i}{4}\operatorname{th}\frac{1+i}{2}\tau,\quad \lambda_s\ll 1, \quad (5)$$

we obtain the angular distribution of the bremsstrahlung intensity with allowance for the effect of the multiple scattering and the effect of the polarization of the medium, which agrees with Gol'dman's result[10]

$$\frac{dI(z,\omega,x)}{dz}=\frac{e^2}{\pi l_s}\operatorname{Re} i\int_0^\infty e^{-2is_1\tau}\frac{d}{d\tau}\exp\left[\frac{1+i}{4}z\operatorname{th}\frac{1+i}{2}\tau\right]d\tau. \quad (6)$$

The angular distribution of the bremsstrahlung intensity at a depth T relative to the initial direction of the velocity, with allowanxe for the total effect of multiple scattering (trivial and nontrivial) and the polarization of the medium (see (3)) takes the form

$$\frac{dE_\omega}{d\Omega}=\frac{e^2\omega}{\pi^2}\operatorname{Im}\int_0^T\int_0^1\exp\left[-\frac{(\theta_\gamma/\theta_s)^2(1+i)\alpha}{1+(\theta_x/\theta_s)^2(1+i)\alpha}\right] \cdot\frac{d}{d\alpha}\left(\frac{1-\alpha}{1+\alpha}\right)^\sigma\frac{d\alpha\,dx}{1+(\theta_x/\theta_s)^2(1+i)\alpha}, \quad (7)$$

where $\theta_s^2 = 4\sqrt{\omega/q}$ is the mean-squared angle of multiple scattering over the coherence length l_s; $\theta_s^2 = 2q_0L(e^{2x/L}-1)$ is the mean-squared multiple-scattering angle at the depth x; $\sigma = (1+i)s_1$; $\varphi(\theta_e, x) = (\pi\theta_x)^{-2}\exp(\theta_e/\theta_x)^2$ is the angular distribution function of the electrons at the depth x. Expression (7) agrees with the corresponding result of Pafomov[11] (see also[9]) for the angular distribution of unpolarized bremsstrahlung quanta in an unbounded medium.

In that region of emission frequencies and electron energies where the polarization of the medium has no effect and an important role is played by the multiple scattering and by the change of the mean-squared scattering angle ($s_1 \to 0$), the angular distribution of the bremsstrahlung intensity per unit electron path can be represented in the form

$$\frac{dI_\tau(z,\omega,x)}{dz}=\frac{e^2}{\pi l_s}\operatorname{Re} i\exp\left[\frac{1-i}{4}\frac{J_0(2^{1/2}i^{3/2}\lambda_s^{-1})}{J_1(2^{1/2}i^{3/2}\lambda_s^{-1})}z\right]. \quad (8)$$

If we neglect the change of the mean-squared scattering angle over the coherence length ($\lambda_s \ll 1$), then the angular distribution of the bremsstrahlung in the region where multiple scattering predominates in the elementary radiation act takes the form

$$\frac{dI_\tau(z,\omega,x)}{dz}=\frac{e^2}{\pi l_s}e^{-z/4}\sin\frac{z}{4}. \quad (9)$$

Expression (9) can also be obtained directly from formula (6) at $s_1 = 0$. The maximum of the angular distribution of the intensity of the bremsstrahlung at the frequency ω according to (9) occurs at the angles

$$\theta_{eff}\approx 2(q/\omega)^{1/4}. \quad (10)$$

As already noted, the result (10) confirms the quantitative estimates obtained for the angular distribution by Landau and Pomeranchuk[1] and by Galitskiĭ and Gurevich[13] (for a nonabsorbing medium), while the angular distribution (9) agrees with the corresponding result of Kalashnikov and Ryazanov[12], obtained by another method.

The angular distribution of the bremsstrahlung at the depth $T \gg l_{coh}$ of the medium relative to the initial electron momentum without allowance for the polarization of the medium is obtained by using (7) as $s_1 \to 0$:

$$\frac{dE_\omega^{(T)}}{d\Omega}=-\frac{e^2}{2\pi^2}\left(\frac{\omega}{q}\right)^{1/2}\operatorname{Ei}(-\theta_\gamma^2/4qT),$$

where Ei(y) is the integral exponential function of y.

Thus, in our case when multiple scattering exerts the overwhelming influence on the bremsstrahlung ($s_1 \to 0$) in a nonabsorbing medium ($\lambda_s^{(c)} = 0$), without allowance for the loss of energy by the electron $\lambda_s = 0$, the angular distribution of the bremsstrahlung intensity at the depth $T \gg l_s$ is determined mainly by the trivial influence of the multiple scattering. This result is natural (cf.[14]), since the effective emission angles in the elementary act (10) are significantly smaller than the electron multiple-scattering angle θ_T at the depth $T(\theta_{eff}^2/\theta_T^2 \approx l_s/T \ll 1)$.

In the other limiting case, when $\lambda_s \gg 1$, an important role is assumed by the change in the mean-squared scattering angle over the coherence length. Expanding (at $\lambda_s \gg 1$) the Bessel functions in (8) in a series, we obtain the spectral-angular distribution of the bremsstrahlung, which takes into account the effective multiple scattering with a mean-squared scattering angle that varies strongly over the coherence length l_s:

$$\frac{dI_\tau(z,\omega,x)}{dz}=\frac{e^2}{\pi l_s}\exp\left(-\frac{\lambda_s\tau}{2}\right)\sin\frac{\lambda_s^{-1}z}{8}.$$

The effective angles are determined then by the relation

$$\theta_{eff}\approx 2^{1/2}\lambda_s^{-1/2}(q/\omega)^{1/4}=2^{1/2}E_s/E, \qquad E_s=4\pi\cdot 137. \quad (11)$$

The effective emission angles (11) greatly exceed the characteristic Bethe-Heitler emission angles ($\theta_{eff} \approx E^{-1}$), but are much smaller than the effective emission angles with allowance for the usual multiple-scattering effect (10). We note that the angles (11) agree in order of magnitude with the mean-squared scattering angle of the electron over the radiation unit lengths L, which in this case determines the coherent radiation length.

As shown in[7], in ordinary media the effect of the change of the multiple-scattering angle becomes manifest, generally speaking, simultaneously with the effect of the absorption of the quanta in the process of production of electron-positron pairs (at $\omega \gg 1$). The reason is that significant losses of the electron energy occur over a length that is comparable with the absorption length of the virtual quanta. Let us examine the limiting case of very large electron energies: $E \gg E_c$, where $E_c \approx 9L/E_s^2$ ($E_c \approx 4\times 10^{13}$ for lead). The characteristic length L over which the electron loses an appreciable fraction of its energy is increased by approximately $\sqrt{E/E_c}$ times as a result of the influence of multiple scattering on the emission energy spectrum[5)]. For the same reason, the mean free path of quanta with energy larger than E_c, according to Migdal[3], begins to depend on the frequency in accordance with the law $L_c(\omega) \sim L_c\sqrt{\omega/E_c}$. At these high energies but relatively low frequencies $\omega \ll E$, the absorption of the virtual quanta can occur over much shorter lengths than L. As a result the change of the mean squared electron scattering angle becomes negligible because of the stronger influence of the absorption of the virtual quanta ($\lambda_s \ll \lambda_s^{(c)}$). In this case when calculating the angular distribution (4) for the function $\psi_2(\lambda_s, \tau)/\psi_1(\lambda_s, \tau)$, we can use the asymptotic value (5). In the region of the overwhelming influence of the absorption of the virtual quanta we have $\lambda_s^{(c)}/8 \gg \max\{1, s_1\}$ and the hyperbolic tangent (see (5)) in the integrand of (4) can be expanded in powers of τ. As a result, the spectral-angular distribution of the bremsstrahlung takes the form

$$\frac{dI_\tau(z,\omega,x)}{dz}=-\frac{e^2}{4\pi L_c}\operatorname{Im}\int_0^\infty\exp\left\{-(\lambda_s^{(c)}+iz)\frac{\tau}{4}\right\} \times\left[\exp\left\{z\left(-\frac{\tau^3}{24}+i\frac{\tau^5}{80}\right)\right\}-1\right]d\tau. \quad (12)$$

The effective bremsstrahlung angles in the case of the overwhelming influence of the absorption of virtual quanta are determined, according to (12), by the relation

$$\theta_{eff} \approx (\omega L_c)^{-1/2}. \tag{12'}$$

It follows from the expression given above (the second term of (4)) the angular distribution of the electron-positron pairs is characterized in this case by the same effective angles.

The angular distribution of the bremsstrahlung quanta in an infinite absorbing medium ($\lambda_s^{(c)} \gtrsim 1$) can be determined only from the angular distribution of the electron-positron pairs produced by them. On the other hand, electron-positron pairs can be produced directly by a fast electron. Therefore, as already noted above, the spectral-angular distribution of bremsstrahlung in an absorbing medium depends on the method of separating the total energy losses (4) into bremsstrahlung proper and direct pair production. Galitskiĭ and Yakimets[4], for example, used another method of separating the energy losses in an infinite absorbing medium of ($\lambda_s^{(c)}/8 \gg \max\{1, s_1\}$). This method corresponds to an angular distribution of the bremsstrahlung in the form

$$\frac{dI_\tau(z,\omega,x)}{dz} = \frac{e^2}{8\pi L_c}\frac{z/8}{(z/8)^2+(\lambda_s^{(c)}/8)^2}\Theta(1-z/8),$$

where $\Theta(z)$ is the Heaviside unit function. In particular, it is seen that the effective emission angles do not change when account is taken of the virtual quanta in comparison with the effective angles (10) determined by the multiple scattering (see[7,8]) concerning the choice of the method of separating the energy losses.

The angular distribution of the bremsstrahlung, the spectrum of which is influenced not only by the absorption of the virtual quanta but also by the change of the multiple-scattering constant over the coherence length, can be obtained also from expression (4). Analysis shows that in the limiting case of a strong influence of the effects of the change of the multiple-scattering constants and of the absorption of the virtual quanta ($\lambda_s \approx \lambda_s^{(c)} \gg 1$), allowance for the change in the mean-squared multiple-scattering angle over the coherence length does not lead to a noticeable change in the effective bremsstrahlung angles in comparison with (12′), although the energy spectrum of the radiation is appreciably altered in this case[7].

SPECTRAL-ANGULAR DISTRIBUTION OF RADIATION EMITTED INTO VACUUM

We now consider the radiation observed in vacuum when an electron of energy E is emitted from a medium into vacuum. The layer of matter will be assumed to be thick ($T \gg L_c$) and to absorb the radiation strongly ($l_{coh} \approx L_c$); the interface between the medium and the vacuum is assumed to be plane. As noted above, expression (4) for the spectral-angular distribution of the energy losses of the electron is valid for sufficiently large distances from the boundary of the medium in comparison with the coherence length of the bremsstrahlung l_{coh}, where the influence of the boundary can be neglected. On the other hand, the spectral-angular distribution of the radiation emitted into the vacuum from a strongly absorbing substance is determined precisely by the radiation of the electron near the boundary of the substance, since the radiation from the internal layer is absorbed in the substance itself. Thus, expression (4) is not valid in this case. The spectral-angular distribution $dQ_\omega/d\Omega$ of the radiation produced near the boundary of the medium, with allowance for all the considered effects of the medium, can be obtained in analogy with the procedure used, for example, by Gol'dman[10]. However, averaging over the trajectories of the electron in the medium should now be carried out with the aid of the function $w^{(-k)}$ (see (22)) which takes into account the change of the mean-squared multiple-scattering angle. In addition, it is necessary to take into account also the absorption of the virtual quanta ($\epsilon''(\omega)$). As a result, $dQ_\omega/d\Omega$ can be represented in the form

$$\frac{dQ_\omega}{d\Omega} = \frac{e^2}{\pi^2}\left\{\frac{\theta_\gamma^2}{(\theta_\gamma^2+E^{-2})^2} + \frac{2\,\mathrm{Re}\,F}{\theta_\gamma^2+E^{-2}} - \omega\frac{\partial}{\partial L_c^{-1}}\mathrm{Im}\,F\right\},$$

$$F \equiv F(s_1,\lambda_s,\lambda_s^{(c)},z) = \int_0^\infty \exp\{-2i(s_1 - i\lambda_s^{(c)}/8)\tau\} \tag{13}$$

$$\times\frac{d}{d\tau}\exp\left[\frac{i^{3/2}z}{2^{3/2}}\frac{\psi_2(\lambda_s,\tau)}{\psi_1(\lambda_s,\tau)}\right]d\tau.$$

The quantum emission angle θ_γ is reckoned from the direction of the electron velocity at the instant of the emission to the vacuum and all the quantities that depend on the electron energy are taken at the energy possessed by the electron on the boundary of the medium.

The first term in (13) is due to that section of the electron trajectory in vacuum which is caused by the interference of the fields connected with the motion of the electron in the vacuum and in the medium. The third term, as shown by Gol'dman[10], is due to the "macroscopic" renormalization of the electron charge in the medium.

Expression (13) can be represented in the form of a sum of two terms:

$$dQ_\omega/d\Omega = dQ_\omega^{(0)}/d\Omega + dQ_\omega^{(s)}/d\Omega,$$

which describe two different types of radiation (cf. (4)). The expression

$$\frac{dQ_\omega^{(0)}}{d\Omega} = \frac{e^2}{\pi^2}\mathrm{Re}\left\{\frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2}\left[\frac{1}{\theta_\gamma^2+\alpha_2} - \frac{1}{\theta_\gamma^2+\alpha_1}\right] - \frac{\alpha_1}{(\theta_\gamma^2+\alpha_1)^2} - \frac{\alpha_2}{(\theta_\gamma^2+\alpha_2)^2}\right\},$$

$$\alpha_1 = E^{-2}, \quad \alpha_2 = (E^{-2}+\omega_0^2/\omega^2) - i(\omega L_c)^{-1}, \tag{14}$$

does not depend on the multiple-scattering angle $q^{1/2}$ and is the spectral-angular distribution of the transition radiation[6]. The remaining part of the expression (13) takes the form

$$\frac{dQ_\omega^{(s)}}{d\Omega} = \frac{e^2}{\pi^2}\left\{\frac{2\,\mathrm{Re}\,F_s}{\theta_\gamma^2+\alpha_1} - \left(\frac{\omega}{q}\right)^{1/2}\frac{\partial}{\partial\lambda_s^{(c)}}\mathrm{Im}\,F_s\right\}, \tag{14'}$$

where $F_s(s_1, \lambda_s, \lambda_s^{(c)}, z)$ is determined by formula (4′). The quantities $dQ_\omega^{(s)}/d\Omega$ vanish as $q \to 0$, so that the function $dQ_\omega^{(s)}/d\Omega$ can be conditionally attributed to the bremsstrahlung produced near the boundary of the medium.

Expression (13) is the most general result for the spectral-angular distribution of the radiation produced at the boundary of the medium. In particular, for a non-absorbing medium ($\lambda_s^{(c)} = 0$), and without allowance for the change in the mean-square scattering angle ($\lambda_s = 0$), it coincides with the corresponding result obtained by Gol'dman[10].

The analysis of the angular distribution (14′) reduces to an investigation of the function $F_s(s_1, \lambda_s, \lambda_s^{(c)}, z)$ at different ratios of the parameters, an investigation carried out above. Without dwelling on a detailed analysis of the angular distributions of the transition radiation (14) and the bremsstrahlung (14′), let us examine the examine the spectral distribution of the corresponding types of radiation. Integrating (14) and (14′) over the emission angle of the quanta, we obtain

$$Q_\omega = Q_\omega^{(0)} + Q_\omega^{(s)},$$
$$Q_\omega^{(0)} = \frac{e^2}{\pi} \operatorname{Re}\left\{\frac{\alpha_2+\alpha_1}{\alpha_2-\alpha_1}\ln\frac{\alpha_2}{\alpha_1} - 2\right\},$$
$$Q_\omega^{(s)} = \frac{e^2}{\pi}\left\{2\operatorname{Re} G_1 - \operatorname{Im}\frac{\partial}{\partial\lambda_s^{(c)}} G_2\right\}, \tag{15}$$
$$G_1 = \int_0^\infty e^{-(1+i)\mu\tau/2}\frac{d}{d\tau}\{e^{2is_2\tau}\operatorname{Ei}(-2is_2\tau) - e^{s_2\chi(\tau)}\operatorname{Ei}(-s_2\chi(\tau))\}d\tau,$$
$$G_2 = 2i\mu\int_0^\infty e^{-(1+i)\mu\tau/2}\left[\frac{\psi_1(\lambda_s,\tau)}{\psi_2(\lambda_s,\tau)} - \frac{2^{1/2}}{i^{1/2}\tau}\right]d\tau, \quad \mu = 2(1+i)\left(s_1 + \frac{i\lambda_s}{8}\right),$$
$$\chi(\tau) = -(2i)^{3/2}\psi_2(\lambda_s,\tau)/\psi_1(\lambda_s,\tau), \quad s_2 = \omega^{3/2}/8E^2q^{1/2}.$$

If we disregard the changes of the mean-squared scattering angle ($\lambda_s = 0$), we can put

$$\chi(\tau) = 2(1+i)\operatorname{th}\left[\frac{(1+i)\tau}{2}\right], \quad G_2 = \frac{4\mu}{1+i}\left[\ln\frac{\mu}{2} - \psi\left(\frac{\mu}{2}\right) - \frac{1}{\mu}\right],$$

where $\psi(x)$ is the logarithmic derivative of the Γ function. In this case the spectral distribution of the radiation Q_ω (15) coincides with the corresponding result of Ternovskiĭ[15].

The general expression (15) for the spectrum of the considered radiation can be greatly simplified in two limiting cases, which explain the meaning of the subdivision of the total radiation into a sum of two terms. For a nonabsorbing medium, $Q_\omega^{(0)}$ is given by

$$Q_\omega^{(0)} = \frac{e^2}{\pi}\left\{\left(1+\frac{2\omega^2}{\omega_0^2E^2}\right)\ln\left(1+\frac{\omega_0^2E^2}{\omega^2}\right) - 2\right\},$$

which coincides with the known result from the intensity of the transition radiation of an ultrarelativistic particle (see, e.g., the paper by Garibyan[16]). The spectrum of the bremsstrahlung $Q_\omega^{(s)}$ produced near the boundary of a nonabsorbing substance in the region of the overwhelming influence of multiple scattering ($s_1 \ll 1 + \lambda_s^{(c)}$) coincides in this case with Gol'dman's expression[10]

$$Q_\omega^{(s)} = \frac{e^2}{\pi}\ln\frac{E^2}{\gamma}\left(\frac{2q}{\omega}\right)^{1/2}, \quad \gamma \approx 1.78.$$

In the other limiting case of a strong influence of the absorption ($\lambda_s^{(c)} \gg s_1$), the intensity of the transition radiation takes the form

$$Q_\omega^{(0)} = \frac{e^2}{\pi}\left[\ln\frac{E^2}{\omega L_c} - 2\right]. \tag{15′}$$

The intensity of the "bremsstrahlung" in the frequency region where besides the strong effective absorption of virtual quanta, it is necessary to take into account only the influence of multiple scattering of an electron with a constant mean-squared angle ($s_1 \ll - + \lambda_s^{(c)}/8$), takes the form

$$Q_\omega^{(s)} = -\frac{19\cdot 2^7 e^2}{3\pi} q^2\omega^2 L_c^4. \tag{15″}$$

As follows from (15″), the quantity $Q_\omega^{(s)}$ assumes in this case negative values, and the absolute value of its intensity is much lower than that of the transition radiation proper. A physical meaning is possessed, strictly speaking, only by the total intensity of the radiation, which is always positive. The negative values of $Q^{(s)}$ can be naturally interpreted, following the terminology of Garibyan and Pomeranchuk[17], as a correction to the intensity of the transition radiation as a result of the influence of multiple scattering, and not as "negative" bremsstrahlung[7].

In the considered case when a strong absorption effect exists, one can neglect the radiation produced in the interior of the medium at distances greatly exceeding L_C. Therefore the intensity of the emission emitted to the vacuum from a sufficiently thick ($T \gg L_C$) layer of the medium is determined by the sum of (15′) and (15″). As follows from (13), this radiation is directed mainly at an angle $\theta_{eff} \approx (\omega L_C)^{-1/2}$ to the electron emission direction.

Just as in the case of an infinite medium[7], allowance for the change in the mean-squared angle of multiple scattering does not change the frequency dependence of the spectrum (15″), or the effective emission angles corresponding to this spectrum.

DISCUSSION OF RESULTS

It follows from the results that the effect of absorption of the virtual quanta appears for the spectral-angular distribution of the bremsstrahlung together with the effect of the change in the mean squared scattering angle in the frequency and energy region defined by the relations (cf.[13])

$$L\omega_0^2 \leqslant \omega \leqslant L\omega_0^2E^2/E_L, \quad E \sim E_L = 4L\omega_0 E_s.$$

The change of the mean-squared scattering angle can greatly influence the spectral-angular distribution of the bremsstrahlung, but the effective emission angles $\theta_{eff} \approx (\omega L_C)^{-1/2}$ are determined mainly by the absorption of the virtual quanta. With increasing electron energy $E \gg E_L$, the effect of the change of the mean-squared scattering angle becomes inessential for the spectral-angular distribution of the emission in the entire frequency region. The spectrum absorption of the virtual quanta becomes manifest in the following regions of frequency and energies[8)]:

$$E_L \ll E \leqslant (\tilde{E}E_c)^{1/2}, \quad L\omega_0^2 \leqslant \omega \leqslant L\omega_0^2E^2/E_L^2;$$
$$E \geqslant (\tilde{E}E_c)^{1/2}, \quad \tilde{E} \equiv 16LE_s^2, \quad L\omega_0^2 \leqslant \omega \leqslant E(E_c/\tilde{E})^{1/2}.$$

At frequencies ω exceeding the upper limits of the indicated inequalities and at the corresponding energies, a more important role is played by the usual multiple scattering effect $\theta_{eff} \approx 2(q/\omega)^{1/4}$, and at frequencies $\omega \ll L\omega_0^2$, the effective polarization of the medium predominates ($\theta_{eff} \approx \omega_0/\omega$).

All the effects of a dense medium lead to an increase of the effective bremsstrahlung angles in comparison with the effective angles of radiation from isolated atoms ($\theta_{eff} \approx E^{-1}$). The largest increase in the effective emission angles is due to the absorption of virtual quanta.

The spectral-angular distribution of the bremsstrahlung emitted to vacuum from a sufficiently thick layer of a medium is determined by the effective medium and by the multiple scattering of the electron in the entire layer of the medium. In the absence of effects of the medium, shifts result remains valid for this distribution[4]. This result takes into account only the trivial influence of the multiple scattering on the angular distribution of the bremsstrahlung, which depends on the ratio of the effective emission angle in the elementary act ($\theta_{eff} \approx E^{-1}$) to the electron multiple scattering angle in the layer of the medium. The multiple scattering changes significantly the magnitude and frequency dependence of the intensity of the bremsstrahlung emitted into the vacuum. The effective emission angles are determined mainly by the angle of the multiple scattering of the electron in the layer of the medium, which in this case ($T \gg l_s$) always exceeds the emission angle in the elementary act (see (7)).

The radiation emitted into vacuum from a strongly absorbing substance ($l_{coh} \sim L_c$) is in the main transition radiation. The bremsstrahlung in this case can be separated from the total radiation only purely arbitrarily, and therefore cannot be regarded independently of the transition radiation. In the case of a strongly absorbing medium, there is also lost the direct connection between the spectrum of the bremsstrahlung produced far from the boundary of the medium[7,8] and the spectrum of the bremsstrahlung (see (15)) emitted into the vacuum, which is formed near the boundary of the medium.

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1)We use a system of units with $\hbar = m = c = 1$.

2)If this integration is carried out and we use the condition for the normalization of the probability density $\varphi(\xi, t)$, then we obtain expression (5) of [7] for E_ω.

3)The coherent bremsstrahlung length $l_{coh} = \min \{l_s, l_s/s_1, L_c\}$ and the quantities l_s and s_1 are defined below.

4)The spectral-angular distribution of the bremsstrahlung (3) was calculated for this case by Schiff [14].

5)This fact was first pointed out by Landau and Pomeranchuk [1].

6)According to the definition assumed by us, transition radiation is the radiation produced when the electron moves uniformly from the medium into the vacuum, even though actually it experiences an appreciable multiple scattering.

7)A similar situation arises when bremsstrahlung is considered in the region of atomic frequencies, where Cerenkov radiation can exist [8].

8)For lead we have $L\omega_0^2 \approx 10^8$ eV, $E_L \approx 10^{14}$ eV, $E_c \cong 4 \cdot 10^{13}$ eV, $\widetilde{E} \approx 2 \cdot 10^{20}$ eV.

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