Electromagnetic generation of sound in metals located in a magnetic field

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The amplitude u of a sound wave excited by an electromagnetic wave incident on the surface of a metallic sample is calculated. The case of a strong magnetic field and short wavelengths $l \gg r \gg k_s^{-1}$ is considered [l is the electron mean free path, r is the Larmor orbit radius, and k_s^{-1} is the sound wavelength (divided by 2π)]. Oscillations of u corresponding to geometric resonance are possible in a magnetic field parallel to the surface. In the presence of open electron trajectories, resonance oscillations should be possible. Resonance oscillations corresponding to certain selected points on the Fermi surface may arise in an inclined magnetic field. The case of a magnetic field perpendicular to the sample surface is considered for a metal with a spherical Fermi surface.

Investigations of kinetic phenomena in metals under conditions of strong spatial dispersion are important in that they allow us to infer properties of different groups of electrons on the Fermi surface from various features of the kinetic characteristics. In the problem of electromagnetic generation of sound, such a condition is smallness of the sound wave in comparison with the characteristic scales of electron displacements (free path length l, Larmor radius r). Analysis of the generation problem in the absence of a magnetic field has, however, shown that the amplitude of the sound wave u^∞ is determined by the averaged contribution of all the electrons on the Fermi surface.[1] The situation changes materially in the presence of a strong constant magnetic field $\mathbf{H}_0$, as a consequence of the action of which certain groups of electrons excited by the electromagnetic wave in the skin layer congregate in narrow layers at definite distances from the surface—distances that depend on H_0. For this reason, the force exciting the sound is modulated spatially with a period characteristic of the electron distribution. In our paper,[2] we have shown that for the case of H_0 parallel to the surface and closed electron orbits, the value of u^∞ undergoes oscillations of the geometric-resonance type,[3] which depend on the ratio of the Larmor diameter and the sound wavelength. It is important that the amplitude of the oscillations depends not only on the averaged value of the deformation potential $\hat{\lambda}(\mathbf{p})$, but also on its local value (which characterizes definite points on the Fermi surface).

In the present work, we consider the behavior of u^∞ for arbitrary orientations of the field $\mathbf{H}_0$ and different forms of the Fermi surfaces. For open orbits, the picture changes qualitatively, because the characteristic period with which the electron distribution is modulated is the value of the drift displacement of the electrons perpendicular to the directions of openness and of the field H_0. In short, the so-called resonance oscillations [4], which are very significant in magnitude and which depend on the ratio of the electron-displacement length and the sound wave length, should be observed for u^∞. A similar situation may also exist for closed orbits in a field that is inclined to the surface: here there is a characteristic mean displacement of the electrons perpendicular to the surface, and the general picture of the oscillations, including geometric resonance, can be a very rich one. It should be noted that the considered effects are physically related to the well-known phenomena of the current spikes inside a metallic sample, which are connected with oscillations in the conductivity (see the review in [5]). However, the latter are determined by the part of the distribution function of the electrons that is odd in the velocities, while the oscillations of u^∞ depend on the behavior of the deformational exciting force

$$F_i^d = -\frac{\partial \Sigma_{ik}}{\partial x_k}, \quad \Sigma_{ik} = \langle \lambda_{ik} f \rangle, \quad \langle Af \rangle = -\frac{2}{h^3} \int d\mathbf{p}\, A f \frac{\partial f_0}{\partial \varepsilon}, \tag{1}$$

which is connected with that part of the nonequilibrium contribution to the distribution function which is even in the velocities $f\partial f_0/\partial\epsilon$ (we recall that $\hat{\lambda}(\mathbf{p}) = \hat{\lambda}(-\mathbf{p})$).

We also consider the case of a normal field H_0, when the problem can be solved exactly for spherical Fermi surfaces and specular scattering of the electrons.

1. BASIC EQUATIONS

An electromagnetic wave $\sim\exp(-i\omega t)$ is incident on the half-space $y > 0$. We assume that sound waves of different polarizations are excited and propagate along the y axis independently (it is not difficult to carry out the generalization). Then the equation of motion of the medium is written down in the form[6]

$$\rho c_s^2 \left(\frac{d^2 u_i}{dy^2} + k_s^2 u_i\right) = \frac{d\Sigma_{iy}}{dy} - \frac{1}{c}[\mathbf{j}\mathbf{H}_0]_i, \tag{2}^*$$

where ρ is the density, c_s the sound velocity corresponding to the polarization, $k_s = \omega/c_s$, the second term on the right is the ponderomotive force (the force associated with the inertia of the electrons is not taken into account because of its small role in the problems considered below). As a consequence of the weakness of the electromechanical coupling, the problem of determination of the exciting force in (2) can be solved independently, i.e., we take into account the variations of f only under the action of the electromagnetic wave, and then find the sound amplitude. Here the electronic damping of the sound is not taken into consideration, but when needed it can be introduced phenomenologically. (Naturally, such an approach is suitable only at a large distance from the possible intersection of the electromagnetic spectrum and the sound-wave spectrum; this condition is assumed to be satisfied in what follows.) We seek a solution of (2) which satisfies the condition of balance of forces on the free surface $y = 0$:

$$\rho c_s^2 du_i/dy = \Sigma_{iy} \tag{3}$$

and has the form of an outgoing wave ($\sim\exp(ik_s y)$) as $y \to \infty$:

$$2\rho c_s^2 u_l(y)=e^{ik_s y}\left[\int_0^y dt\, e^{-ik_s t}\left(\Sigma_{ly}(t)+\frac{i}{k_s c}[\mathbf{jH}]\right)\right.$$
$$\left.-\int_0^\infty dt\, e^{ik_s t}\left(\Sigma_{ly}(t)-\frac{i}{k_s c}[\mathbf{jH}_0]_l\right)\right]-e^{-ik_s y}\int_y^\infty dt\, e^{ik_s t}\left(\Sigma_{ly}(t)-\frac{i}{k_s c}[\mathbf{jH}_0]_l\right). \quad (4)$$

We use the kinetic equation

$$v_y\frac{\partial f}{\partial y}+\Omega\frac{\partial f}{\partial \tau}+\hat{\nu} f=g\equiv e\mathbf{Ev}, \quad (5)$$

to find the distribution function. Here Ω is the cyclotron frequency and $\hat{\nu}$ the collision frequency. Since our principal interest is in the qualitative picture of the oscillations, we shall not take into account here the scattering of electrons on the surface, i.e., the boundary condition for f. (As is known, in related problems of anomalous field penetration, such an approximation is sufficiently good.) Then, using the usual method of even continuation of the field $g(-y)=g(y)$, we find the solution of (5) by means of a Fourier transformation:

$$f^k=\Omega^{-1}(1-G)^{-1}\int_{\tau-2\pi}^{\tau} d\tau'\, g^k(\tau,\tau')\exp[H(\tau')-H(\tau)]. \quad (6)$$

Here

$$f^k=\int_{-\infty}^{\infty} dy\, f(y)\exp(iky),\quad f(y)=(2\pi)^{-1}\int_{-\infty}^{\infty} dk\, f^k\exp(-iky),$$

and similarly for g^k. The following notation has been introduced:

$$H(\tau)=\int^{\tau}\Omega^{-1}\mathbf{kv}\,d\tau',\quad g^k(\tau,\tau')=g^k(\tau')\exp\int_{\tau}^{\tau'}\gamma\,d\tau'',$$
$$g^k(\tau)=e\mathbf{E}^k\mathbf{v}(\tau),\qquad \gamma=\hat{\nu}\Omega^{-1}, \quad (7)$$
$$G=\exp[-2\pi(\bar{\gamma}+i\Omega^{-1}\overline{\mathbf{kv}})],\quad \bar{a}=(2\pi)^{-1}\int_0^{2\pi} d\tau\, a(\tau).$$

In experiments with direct observation of sound generation, the sound amplitude u^∞ is measured far from the surface that is irradiated with the electromagnetic waves.[7-9] According to (4), u^∞ is determined by the coefficient of $e^{ik_s y}$ as $y\to\infty$.

Using the Fourier expansion for f in (4), and taking into account the symmetry relation $f^k(\mathbf{v})=-f^{-k}(-\mathbf{v})$, which follows from (5), we obtain an expression for the sound amplitude as $y\to\infty$ in the form

$$2\rho c_s^2 u_l^\infty=\frac{i}{k_s c}[\mathbf{j}^{k_s}\mathbf{H}_0]_l-\Sigma_{ly}^{k_s},$$
$$\mathbf{j}^{k_s}=e\langle \mathbf{v}f_{k_s}^+\rangle,\quad \hat{\Sigma}^k=\langle\hat{\lambda}f_k^-\rangle,\quad 2f_k^\pm=f^k\pm f^{-k}. \quad (8)$$

Thus the amplitude of the excited sound wave is expressed in terms of the Fourier components j and $\hat{\Sigma}$ for the value of $k=k_s$; these are determined by the parts of f^k that are even and odd, respectively, in k.

We further consider the case in which the characteristic dimensions that enter into the problem obey the following inequalities:

$$\delta\ll k_s^{-1}\ll r\ll l \quad (9)$$

(δ is the thickness of the skin layer, r the radius of the Larmor orbit, l the free path length). As a consequence of the left-hand inequality of (9), the Maxwell equations that serve to determine E^k reduce for $k=k_s$ to the relation

$$\frac{4\pi i\omega}{c^2}j_l^{k_s}=\frac{4\pi i\omega}{c^2}\sigma_{lj}^{k_s}E_j^{k_s}=-2\left.\frac{dE_l}{dy}\right|_{y=0},\quad l=x,z. \quad (10)$$

In short, the first term of (8), which is due to the ponderomotive force, is expressed in terms of the derivative of the electric field on the surface of the crystal, i.e., in terms of the value of the magnetic field of the incident wave, and does not depend on the kinetic characteristics of the metal. We therefore limit ourselves below to consideration of the second term of (8) only:

$$\hat{\Sigma}^k=\frac{2}{h^3}\int dp_H m^*\int_0^{2\pi} d\tau\,\hat{\lambda}f_k^- \quad (11)$$

(p_H is the projection of p on the direction of H_0 and m^* is the cyclotron mass). The integral over τ can be calculated here by the stionary-phase method because of the second inequality of (9) ($H(\tau)\gg 1$):

$$\int_0^{2\pi} d\tau\,\hat{\lambda}f_k=-\frac{i}{\Omega}\int_0^{2\pi} d\tau\frac{\hat{\lambda}g_k}{H'(\tau)}+\frac{\pi}{\Omega}\frac{1+G}{1-G}\sum_{i=\alpha,\beta}\frac{g_k(\tau_i)\hat{\lambda}(\tau_i)}{|H''(\tau_i)|}$$
$$+\frac{2\pi}{\Omega}\frac{\exp[i\pi(s_\beta-s_\alpha)/4]}{(1-G)|H''(\tau_\alpha)H''(\tau_\beta)|^{1/2}}\{\hat{\lambda}(\tau_\alpha)g^k(\tau_\alpha,\tau_\beta)G\exp i[H(\tau_\alpha) \quad (12)$$
$$-H(\tau_\beta)]+\hat{\lambda}(\tau_\beta)g^k(\tau_\beta,\tau_\alpha)\exp i[H(\tau_\alpha)-H(\tau_\beta)+\pi(s_\alpha-s_\beta)/2]\}.$$

Here $\tau_{\alpha,\beta}$ are the roots of the equation $H'(\tau)=0$, $s_i=\operatorname{sign}H''(\tau_i)$, and it is assumed that $H''(\tau_{\alpha,\beta})\neq 0$ (the prime denotes the derivative with respect to τ). The first term of (12) is the contribution of the region of integration which does not contain points of stationary phase (in the calculation of the conductivity tensor σ_{ij}^k the similar term vanishes). It determines the part of the amplitude

$$u_{lI}^\infty=\frac{ieE_j^{k_s}}{\rho c_s^2 h^3}\int dp_H m^*\int_0^{2\pi} d\tau\frac{\lambda_{ly}v_j}{k_s v}, \quad (13)$$

in which the dependence on the field H_0 enters only through the quantity E^k, calculated from (10) (the dependences of the $\hat{\sigma}^k$ on H_0 are well known; see, for example, [5]). At $H_0=0$, (13) is identical with the result obtained in [1]. The contributions of the second and third terms of (12) to (8) (we denote them by u_{II} and u_{III}) depend significantly on the value and orientation of H_0. We shall analyze this in specific examples.

2. OPEN ORBITS

In this case, the period T of the motion of the electron enters into (12) in the form $2\pi\Omega^{-1}$; it depends on the period P_0 of the dispersion law along the direction of openness. It follows from the equations of motion that, on the average over the time interval T, the electron moves perpendicular to the directions of openness and of the field H_0 through a distance $d=cP_0/eH_0$, which does not depend on the value of p_H. The quantity $2\pi\Omega^{-1}kv$ enters into expression (7) for G and is connected with the displacement d in the following fashion:

$$2\pi\Omega^{-1}k_s v=k_s\frac{P_0 c}{eH_0}\sin\theta\equiv k_s d_y \quad (14)$$

(θ is the angle between the direction of openness and the normal to the surface). At $k_s d_y>1$ (i.e., $\sin\theta>(k_s r)^{-1}$), the denominator $1-G$ in (12) causes resonant oscillations of u^∞ in the magnetic field. An estimate with the use of (12) gives for the resonant part of u^∞

$$u^\infty\sim u_I^\infty\frac{\sin k_s d_y}{\pi^2\gamma^2+\sin^2(k_s d_y/2)}. \quad (15)$$

The maximum and minimum of (15) are respectively achieved at the points $k_s d_y=2\pi(n\pm\gamma)$. The curve of

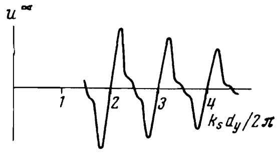

FIG. 1. Dependence of the amplitude of the excited sound on H_0^{-1} in the presence of open orbits.

the dependence of u^∞ on H_0^{-1} is antisymmetric relative to the points $k_s d_y = 2\pi n$ (see Fig. 1), in contrast to the resonant oscillations of the sound-absorption coefficient α on the open orbits.[4] For $k_s d_y < 1$ ($\theta < (k_s r)^{-1}$), the resonant oscillations disappear.

In an inclined field H_0 that forms an angle φ with the surface, the contribution to u^∞ from the open trajectories can change. It is not difficult to establish that for $\varphi < (k_s l)^{-1}$ we have Eq. (15) as before, and that for

$$(k_s r)^{-1} > \varphi > (k_s l)^{-1}$$

this contribution is comparable in magnitude with the result for closed trajectories considered below.

3. CLOSED ORBITS

In an inclined field and for closed orbits, the mean displacement of electrons in the direction of the normal is different from zero and is given by $d_y = v_H \Omega^{-1} \sin\varphi$, which enters into the expression for G. All the terms of (12) make contributions to u^∞; their values depend on the value of $2\pi k_s d_y$, i.e., on the angle φ. We first consider the nonresonant case $2\pi k_s d_y \ll 1$, i.e., $\varphi < (k_s r)^{-1}$. Here

$$1-G \sim (2\pi\gamma + ik_s d_y)^{-1},$$

and calculation shows that

$$u^\infty_{jII} = \frac{ieE_x^{k_s}}{\rho c_s^2 h^3} \int dp_H m^* \sum_{i=\alpha,\beta} \frac{\lambda_{jy}(\tau_i) v_x(\tau_i)}{|H''(\tau_i)|} \begin{cases} k_s \bar{v}_H \bar{v}^2 \sin\varphi \\ P(k_s \bar{v}_H \sin\varphi)^{-1} \end{cases}$$
$$\sim u^\infty_{jI} \begin{cases} k_s l \varphi \gamma^{-1}, & \varphi < (k_s l)^{-1}, \\ (k_s r \varphi)^{-1}, & \varphi > (k_s l)^{-1}. \end{cases} \qquad (16)$$

(P is the principal value of the integral; the x axis is perpendicular to the normal and to H_0).

In the oscillating part of u^∞, which is connected with the third term of (12), those orbits which correspond to the central cross section of the Fermi surface perpendicular to H_0 make a contribution at $\varphi < (k_s l)^{-1}$. After simple computations, we obtain the same result as for $\varphi = 0$:[2]

$$u^\infty_{lIII} = -\frac{2is_\alpha^0 eE_x^{k_s}}{\rho c_s^2 h^3 \bar{\gamma} k_s} \left|\frac{\partial^2\varepsilon}{\partial p_y^2}\right|^{-1} \left|\frac{\partial^2\varepsilon}{\partial p_z^2}\right|^{-1/2} \left|\frac{\pi e H_0}{k_s v_x c}\right|^{1/2}$$
$$\times v_x(\tau_\alpha^0) \lambda_{ly}(\tau_\alpha^0) \cos(k_s D_x^0 - \pi s_z/4). \qquad (17)$$

Here $D_x^0 = [p_x(\tau_\alpha^0) - p_x(\tau_\beta^0)]c/eH_0$ is the extremal diameter of the orbit, $\tau^0_{\alpha,\beta}$ are the points on the central cross section at which $v_y = 0$ (see Fig. 2), $v_x(\tau_\alpha^0) = -v_x(\tau_\beta^0)$, $\hat{\lambda}(\tau_\alpha^0) = \lambda(\tau_\beta^0)$, $s_\alpha^0 = -s_\beta^0$ as a consequence of the central symmetry of the Fermi surface; the values of all the quantities are taken at these points, except $\hat{\lambda}$, which is averaged over the orbit corresponding to the central cross section; $s_z = \mathrm{sign}(v_x^{-1}\partial^2\epsilon/\partial p_z^2)$ at $\tau = \tau_\alpha^0$. Equation (17) refers to the case in which $\lambda_{ly}(\tau_{\alpha,\beta}) \neq 0$ (the analysis for $\lambda_{ly}(\tau_{\alpha,\beta}) = 0$ will be given below for the example of a spherical Fermi surface). Expression (17) describes oscillations of u^∞ analogous to geometric resonance.[3] We note that similar terms in the conductivity and in the sound-absorption coefficient α have opposing phases and relative values $\sim(k_s r)^{-1/2} \ll 1$.[5,3] In our case, the ratio of (17) and (13) is $\gamma^{-1}(k_s r)^{-1/2}$ in order of magnitude, i.e., it can be considerable. For larger angles of inclination $\varphi > (k_s l)^{-1}$, the quantity u_{III} depends on which of the functions p_H is sharper—$(1-G)$ or $\exp[\pm i(H(\tau_\beta) - H(\tau_\alpha))]$. It is not difficult to establish that for

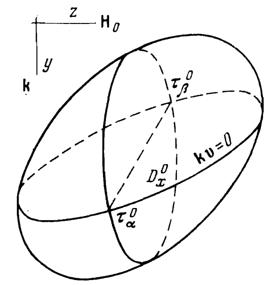

FIG. 2

$$(k_s l)^{-1} < \varphi < (\gamma/k_s l)^{1/2}$$

u_{III} is expressed by Eq. (17), and that for

$$(k_s r)^{-1} > \varphi > (\gamma/k_s l)^{1/2}$$

$$u^\infty_{jIII} = -\frac{2is_\alpha^0 eE_x^{k_s} \Omega \lambda_{jy}(\tau_\alpha^0) \cos(k_s D_x^0)}{\rho c_s^2 h^3 k_s^2 \varphi} \left|\frac{\partial^2\varepsilon}{\partial p_\eta^2}\right|^{-1} \left|\frac{\partial v_H}{\partial p_H}\right|^{-1} \qquad (18)$$
$$\sim u^\infty_{jI} (k_s r \varphi)^{-1}$$

(p_η is the projection of p on the direction perpendicular to x and H_0).

We now return to consideration of the case of sufficiently large angles of inclination $\varphi > (k_s r)^{-1}$, when $k_s d_y > 1$. Here resonant oscillations of u similar to those considered in Sec. 2 for open trajectories may appear. The difference is that in the given case, d_y depends on p_H; therefore, the resonance, as in the analogous case for the conductivity [5] and the sound absorption,[4] appears only in the vicinity of the elliptical limiting point and in the presence of an extremum in $d_y(p_H)$ outside the central cross section.

In the analysis of resonant oscillations in the neighborhood of the elliptical limiting point corresponding to the limiting value of the momentum p_H^0, we represent the dispersion law $\epsilon(\mathbf{p})$ in the form of an expansion in $\Delta p_i = p_i - p_i^0$ with accuracy to within quadratic terms and, using the equations of motion of the electron, we find the dependence $H(\tau)$ in the explicit form (7). From the condition of the presence of the effective points $\tau_{\alpha,\beta}$ on the orbits in the vicinity of a limiting point determined by the width of the resonance denominator $1-G$, we get the limits on the angle of inclination φ, which ensure the presence of resonant oscillations:

$$(k_s r)^{-1} < \varphi < (k_s l)^{-1/2}.$$

Here the resonant point of u^∞ equals

$$u^\infty_{jII} = -\frac{i\pi e E_m^{k_s} \lambda^0_{jy} v_m^0}{\rho c_s^2 h^3 k_s K_0^{1/2} n^{1/2}} s_H M(-\Delta) \left|\frac{\partial^2\varepsilon}{\partial p_H^2} \frac{\partial^2\varepsilon}{\partial p_\eta^2}\right|_0^{-1/4}, \qquad (19)$$

where

$$M(\Delta) = \gamma(\gamma^2+\Delta^2)^{-1/2}[(\gamma^2+\Delta^2)^{1/2} - s_H\Delta]^{-1/2},$$
$$\Delta = k_s d_y - n = k_s \varphi \Omega^{-1} v_H^0 - n \ll 1,$$

and n is the number of the resonance.

The values taken at the limiting point are marked with zeroes in (19), and K_0 is the Gaussian curvature,

$$s_H = \text{sign}\left(\frac{k_s\varphi}{\Omega}\frac{\partial^2\varepsilon}{\partial p_H^2}\right)_0.$$

The maximum of the function $M(-\Delta)$ is equal to $3^{3/4}/2\gamma^{1/2}$ and is reached at $\Delta = \gamma s_H/3^{1/2}$ (in the analogous increments to $\hat{\sigma}^k$ and α, the maximum occurs at opposite values of Δ).

If the Fermi surface is not convex and there are extrema of $d_y(p_H)$ on it, resonant oscillations of the type (19) are also possible at these points (for angles of inclination $\varphi > (k_s r)^{-1}$).

4. SPHERICAL FERMI SURFACE

The case in which $\lambda l_y = 0$ at $v_y = 0$ is not included in the foregoing results. We analyze it for a spherical Fermi surface, for which one can use the following form of the deformation potential:

$$\lambda_{ij} = \frac{\lambda}{v^2} v_i v_j, \tag{20}$$

the parameter $\lambda \sim \epsilon_F$ (for a gas of free electrons, $\lambda = -mv^2$ [10]).

a) H_0 parallel to the surface. In this case, $v_x = v_\perp \cos\tau$ $v_y = v_\perp \sin\tau$, $v_z = v\cos\theta$, $v_\perp = v\sin\theta$, θ is the azimuthal angle. The integrals over $d\tau$ in (6) can be calculated conveniently by using an expansion of $\exp(ikr\sin\theta\cos\tau)$ in a series in Bessel functions. Simple calculations lead to the following results.

The amplitude of the transverse sound wave for $k_s r \gg 1$ is equal to

$$u_j^\infty = -\frac{i}{2}E_j^{k_s}\frac{\lambda}{\varepsilon_F}\frac{ne}{k_s\rho c_s^2}\begin{cases}1+(k_s r)^{-3/2}(2\pi)^{-1/2}\sin(2k_s r-\pi/4), & j=x,\\ 1, & j=z\end{cases} \tag{21}$$

The coefficient in front of the curly bracket in (21) has the same order of magnitude as u_I^∞ in (13). We note that the oscillating contributions not associated with E^{k_s} (i.e., with the oscillations of $\hat{\sigma}^k$) exist only for an exciting field $E_x \perp H_0$, and their relative value is $\sim(k_s r)^{-3/2} \ll 1$.

We turn our attention to the following circumstance. If the field in the incident electromagnetic wave is parallel to the x axis, a Hall field E_y is generated in the sample, the Fourier component of which turns out to be equal to

$$E_y^k = E_x^k \frac{1}{2\pi\gamma}\left(\frac{\pi}{kr}\right)^{1/2}\cos\left(2kr - \frac{\pi}{4}\right). \tag{22}$$

The Hall field makes a negligible contribution to the excitation of the transverse sound, which is omitted in (21) (a similar situation also existed above for $\lambda l_y \neq 0$ at $v_y = 0$). However, for excitation of the longitudinal sound in our case with deformation potention (20), the Hall field plays a decisive role and gives the oscillating contribution

$$u_y^\infty = -\frac{i}{2}\frac{ne\lambda}{\rho c_s^2 \varepsilon_F k_s} E_y^k, \tag{23}$$

which is $(k_s r)^{1/2}$ times greater than the monotonic contribution.

b) $H_0 \parallel k_s \parallel y$—normal field. The paper by Kaner and Fal'ko[11] was devoted to analysis of this situation. The authors [11] considered not u^∞ but the quantity u at y = 0. In accord with (4), the latter has the form

$$2\rho c_s^2 u_l(0) = -\sum\nolimits_{ly}^{k_s} + \frac{i}{ck_s}[\mathbf{j}^{k_s}\mathbf{H}_0]_l$$

$$+\frac{1}{\pi i}\mathrm{P}\int_{-\infty}^{\infty}\frac{dp}{p-k_s}\left[-\sum\nolimits_{ly}^{p} + \frac{i}{ck_s}[\mathbf{j}^p\mathbf{H}_0]_l\right].$$

(the Fourier expansion has been used for f). Only the integral part of u(0) was calculated in [11]—the part connected with the features of the functions Σ^p and j^p that determine the penetration of the electromagnetic wave, i.e., the value actually found is for the deformation at y = 0, which is further damped in the skin layer, together with the electromagnetic field. Since experimentors usually measure u^∞, we now carry out a calculation for this quantity.

In the given case, $v_y = v\cos\theta$, $v_x = v_\perp\cos\tau$, and $v_z = v_\perp \sin\tau$. This case is of interest, in particular for the fact that the distribution function (6)

$$f^k = \Omega^{-1}\int_{-\infty}^{\tau} d\tau' g_k(\tau')\exp[(\tau'-\tau)(\gamma + i\Omega^{-1}kv_y)],$$
$$g_k = e[E_x^k v_x + E_z^k v_z] \tag{6'}$$

is, as can easily be shown, the exact solution of the kinetic equation for specular scattering of electrons on the surface.

Let the electric field of the incident wave be parallel to the x axis. We give the results of the calculations for the transverse waves u_x and u_z excited in this case:

$$u_x^\infty = u_0\lambda_1 K_1 \frac{dE_x}{dy}\bigg|_{y=0},$$
$$u_z^\infty = u_0[(k_s r)^{-1} + \lambda_1 K_2]\frac{dE_x}{dy}\bigg|_{y=0}, \tag{24}$$

where

$$u_0 = \frac{c^2 p_F}{4\pi\omega e\rho c_s^2}, \quad \lambda_1 = \frac{\lambda}{2\varepsilon_F}, \quad K_1 = \frac{4}{3}\,\mathrm{Im}\,Y^{-1} - (k_s l)^{-1},$$
$$K_2 = (k_s r)^{-1} - \frac{4}{3}\,\mathrm{Re}\,Y^{-1}, \quad Y = 2\Gamma + (1-\Gamma^2)\ln\frac{\Gamma+1}{\Gamma-1}, \quad \Gamma = \frac{1+i\gamma}{k_s r}.$$

The formulas (24) were obtained in the longwave approximation $k_s\delta \ll 1$, using the expressions for E^{k_s} that follow from (10). We note that u_x^∞ is expressed directly in terms of the absorption coefficient of transverse sound α, which is equal in our notation to

$$\alpha = \frac{\lambda_1^2 n p_F \omega}{2\rho c_s^2} K_1. \tag{25}$$

The contribution from the ponderomotive force (the term without λ_1 in u_z) was also included in (24); it is linearly dependent on H_0. The contributions from the deformation force (the terms with λ_1) are quite nonlinear. Figure 3 shows the dependences of $K_{1,2}$ on $(k_s r)^{-1} \sim H_0$ for different values of $k_s l$. The behavior of the function K_2, which varies nonmonotonically, is of interest: at $k_s r \sim 1$, it reaches a maximum, which is sharper the greater the value of $k_s l$, and then falls off (at large H_0, as H_0^{-1}). Thus, monotonic decay of u_x from the maximum value corresponding to $H_0 = 0$ should take place with increasing H_0; u_x falls off asymptotically as H_0^{-2} in strong fields. The character of the asymptote for u_z in large fields is clear: $u_z \sim H_0$, approaching

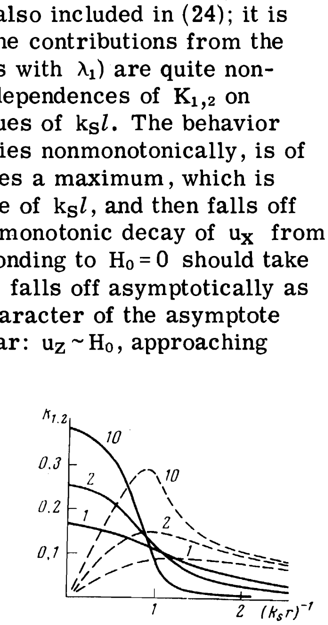

FIG. 3. Dependence of $K_{1,2}$ on $(k_s r)^{-1} \sim H_0$. K_1 is given by the solid lines, K_2 by the dashed ones. The numbers on the curves correspond to the values of the parameter $k_s l$.

this dependence from above for $\lambda_1 > 0$ and from below for $\lambda_1 < 0$.

5. DISCUSSION

As we have already observed, we have not considered cases of coincidence of the lengths of the sound waves and the weakly damped electromagnetic waves, i.e., the excitation of coupled waves. We shall touch on the possibility of such an approach for the case of dopplerons. For open orbits, as is well known,[12] the first dopplerons are obtained only upon satisfaction of special requirements as to the structure of the Fermi surface (the presence of additional electron groups with parameters of the necessary value); effective dopplerons of large number are possible at $\omega \gg \nu$. We shall assume that these requirements are not satisfied. For an inclined field, it suffices to assume that the frequencies satisfy a condition that is opposite to the condition for realization of the dopplerons:[13]

$$\omega \leqslant \varphi^{-2}\omega_L \sim \varphi^{-2}\Omega H_0^2/4\pi n\varepsilon_F \quad (\varphi \ll 1).$$

In a normal field, we have considered only the Fermi sphere, for which the interval of fields in which dopplerons are realized is extremely small. Resonant interaction of sound with other types of weakly damped electromagnetic waves was considered by Skobov and Kaner;[14] we assume that the corresponding conditions of interaction of the spectra are not satisfied in our cases either.

We now proceed to the analysis of the obtained results.

Formally, the oscillations are introduced by two cofactors into the expressions for u^∞. The first is connected with the part of the distribution function $f^k(v)/E^k$ that is even in the electron velocities; the character of its changes was analyzed in the text. The second is connected with the Fourier components of the conductivity, which enter through the values of E_k in accord with relations (10). As was shown in Secs. 2 and 3, the oscillating parts of u^∞/E^{k_s} depend in the case of closed orbits on the values of a number of parameters of the electron spectrum at definite points on the Fermi surface, and, in particular, in accord with (17)–(19), on the values of the deformation potential λ_{jy}. The oscillations of the conductivity make contributions to u_{osc} that depend on the averaged values of $\hat{\lambda}$. Therefore, to estimate $\hat{\lambda}$ at the different points of the Fermi surface, it is necessary to use cases in which the role of the oscillations of $\hat{\sigma}$ in the general pattern of variation of u^∞ is unimportant. It is not difficult to show that for an arbitrary Fermi surface (with $\lambda_{iy} \neq 0$ at $v_y = 0$—as assumed in Secs. 2 and 3), one such case is that of small angles of inclination of the field $\varphi < (k_S l)^{-1}$. Here the monotonic part is $\sigma^k \sim \sigma_0 (kr)^{-1} \sim \gamma_{av}^{-1}$ (to distinguish it from $\bar{\gamma}$ in (17) we denote the average over the Fermi surface by γ_{av}). In short, expression (17) depends only on the values of the parameters of the Fermi surface at the point τ_α^0 (or τ_β^0), with the exception of the factor $\gamma_{av}/\bar{\gamma}$. Using the independence of γ_{av}/γ of the rotation around the field H_0, given samples with different orientations, we can in principle obtain a comparative estimate of the values of λ at different points of each central cross section of the Fermi surface from the value of the oscillations of u^∞. In the case of larger angles of inclination $\varphi > (k_S l)^{-1}$, the relative contribution from $\hat{\sigma}^k_{osc}$ is of the same order as that described by formulas (18) and (19). The general picture of the oscillations of u^∞, which are considerable in magnitude, is complicated here, which makes it difficult to obtain estimates for $\hat{\lambda}$. In the case of very considerable resonance oscillations (15) due to electrons on open orbits, we obtain a result similar to the foregoing: for $\varphi < (k_S l)^{-1}$ the contribution of σ^k_{osc} is insignificant.

Thus, measurements of the amplitude of u^∞ and its dependence on H_0 make it possible in principle to estimate the values of $\hat{\lambda}(p)$ in a number of cases; moreover, we can obtain the same information on the dimensions and shape of the Fermi surfaces from the magnitude of the periods of the oscillations as is obtained by means of the radiofrequency size effect for the case of ultrasound absorption.

We now review the results for a spherical Fermi surface. According to the data of Sec. 4a), the oscillations of u^∞ are determined by the current spikes, i.e., the contribution $\sigma_{osc} \sim \sigma_{mon}(k_S r)^{-1/2}$, and are comparatively small for transverse sound. Under these conditions, the excitation of longitudinal sound, in which the u_{osc} are considerable and, according to (23) and (22), are opposed in phase to the oscillations of transverse sound, is of interest. In a normal field, the weak-field behavior of u_Z^∞, which depends strongly on the values of $k_S l$ and $\lambda_1 = \lambda/2\epsilon_F$, deserves attention. Thus, at $k_S l > 1$ in the range of fields where the function K_2 increases (see Fig. 3), and at values of $|\lambda_1| \sim 3-5$, the quantity $|\lambda_1| K_2$ in u_Z^∞ numerically exceeds $(k_S r)^{-1}$. If these values of λ_1 are negative, then the value of u_Z in the given region of fields has a different sign than in strong fields, i.e., the opposite phase. At a certain value of H_0 (in the region $k_S r \sim 1$), the amplitude of u_Z^∞ gives to zero in the considered case and then begins to increase, approaching its asymptotic value from below. For $|u_Z^\infty|$, we get a maximum in this case at $k_S r > 1$ and a minimum at $k_S r \sim 1$. This behavior of u_Z^∞ was observed experimentally by Wallace, Gaerttner and Maxfield [8] for potassium.

To reconcile (24) with the experimental values of u_X^∞, values of $\lambda_1 \sim 7$ are required, but they lead to a strong difference between the independently measured coefficient of sound absorption α and Eq. (25). Evidently, the divergences of theory and experiment in this problem are of interest in principle, because the exact solution of the problem for specular scattering was used in Sec. 4b).

Account of diffuseness can scarcely correct the situation. Let us estimate the role of diffuseness of scattering for $H_0 = 0$. Using the distribution function for completely diffuse scattering of electrons on the boundary

$$f(v_y>0) = \int_0^y dy' Q(y', y), \quad f(v_y<0) = \int_\infty^y dy' Q(y', y),$$

$$Q(y', y) = \frac{e\mathbf{E}(y')\mathbf{v}}{v_y} \exp\frac{(y'-y)\nu}{v_y}$$

in Eq. (4), we obtain

$$u_x^\infty = \frac{\lambda_1 k_s em}{i\rho c_s^2}\left\{\left\langle \frac{v_x^2 v_y^2}{k_s^2 v_y^2 + \nu^2}\right\rangle \int_0^\infty dy E_x(y) \cos k_s y - \left\langle \frac{v_x^2 v_y^2}{k_s^2 v_y^2 + \nu^2}\right\rangle_{v_y>0} \int_0^\infty dy\, E_x(y) \exp\left(-\frac{y\nu}{v_y}\right)\right\}. \tag{26}$$

The first term of (26) coincides with (24) at $H_0 = 0$; the second term, which is lacking in the case of specular scattering, only reduces the general result: for $k_S\delta \ll 1$ and the maximum anomalous skin effect, i.e., as $\nu \to 0$, it is equal to ½ of the first term. Diffuseness does, of course, lead to a certain increase in E, but this growth is unimportant; thus, the surface impedance increases by no more than 9/8 times.

We shall not discuss the experiments of [9], in which oscillations of u^{∞} were observed, because its data are insufficient for comparison with our results.

*$[jH_0] \equiv j \times H_0$.

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