Anomalous penetration of electromagnetic field into a metal near cyclotron resonance

A. S. Rozhavskiĭ and M. A. Lur'e

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
(Submitted April 15, 1974)
Zh. Eksp. Teor. Fiz. **67**, 1168–1175 (September 1974)

The distribution of an electromagnetic field in a metal is studied at external-electromagnetic-wave frequencies close to the cyclotron-resonance frequency. It is shown that if the deviation from resonance satisfies the inequalities obtained in the present paper, then the electromagnetic field at distances from the surface approximately equal to an integer number of characteristic cyclotron diameters should be the short cyclotron wave whose spectrum was previously derived by Kaner and Skobov. The physical cause of this phenomenon is cyclotron-wave generation in all the internal skin layers. Thus, under certain conditions, short cyclotron waves may effectively penetrate through samples of sufficiently large thickness. Conditions under which the waves can be detected experimentally are indicated.

The amplitude of an electromagnetic field in a metal attenuates rapidly with increasing distance from the surface, and differs noticeably from zero only in a small layer of thickness δ. The situation changes significantly when a metallic sample is placed in a strong constant and homogeneous magnetic field, when $\Omega\tau \gg 1$. Here $\Omega = |e|H/m^*c$ is the cyclotron frequency, e is the charge of the electron, H is the magnetic field intensity, c is the speed of light, m^* is the effective mass of the electron in the magnetic field, and τ is the mean free path time. In this case the electromagnetic field can penetrate into the interior of the sample to a depth greatly exceeding δ. This anomalous penetration of the electromagnetic field into the metal is of particular interest in the region of high frequencies ω of the electromagnetic wave, when the interaction of the wave with the conduction electrons is of resonant character—cyclotron resonance (CR).

Azbel'[1] has shown in 1960 that a system of spikes of electromagnetic field is produced in a metal near CR at depths that are multiples of the cyclotron diameters of the resonant electrons $D_e = 2cp_\perp(p_z^e)/|e|H$, where $p_\perp(p_z)$ is the radius of the intersection of the Fermi surface with the plane p_z = const, and p_z is the projection of the electron momentum on the direction of the magnetic field. In 1964, Kaner and Skobov[2] have shown that near the resonant frequencies there exist weakly-damped electromagnetic oscillations—cyclotron waves with wavelength much shorter than the cyclotron diameter. Later, in 1966, Walsh and Platzman[3] have experimentally observed the existence in the vicinity of CR of electromagnetic waves of a different character—with wavelength that is not small, but is to the contrary large in comparison with the cyclotron diameter. The amplitude of these waves is much smaller than that of the waves considered by Kaner and Skobov (by a factor D/δ), but the longer wavelength causes these waves to be more intense than the short cyclotron waves if a depth is appreciable.

Experimental observation of short-wave oscillations encounters significant difficulties. The reason is that although the short waves are weakly damped (the damping length l_0 is many times larger than δ), nevertheless l_0 is not large enough in real cases for the electromagnetic wave produced by the currents in the skin layer at the surface of the metal to pass without appreciable damping through a sample of thickness d which is in any case not less than D_e.[1)] Consequently, not one of the types of the short waves in metals considered in the literature (see, e.g., the review[4]) has a large enough penetrating power to pass through a sample and be observable. Therefore the observation of short waves passing through a sample would be of considerable physical interest.

It is shown in this paper that under certain conditions cyclotron waves can penetrate effectively through samples of sufficiently large thickness. The reason is the generation of an electromagnetic field in all the internal skin layers located at distances that are multiples of D_d from the surface. The field spikes in these layers generate waves that are damped over distances of order l_0, and consequently if the sample has a thickness equal to $pD_e + y (p = 1, 2, \ldots)$, while $y \lesssim l_0$, then the wave passes through the sample. The transmitted signal is in this case a nonmonotonic function of the magnetic field.

Consider a plate of thickness d placed in a magnetic field parallel to the surface. We choose the direction of the z axis along the magnetic field **H**, and direct the y axis along the inward normal to the surface. The spectrum, damping, and distribution of the field of the wave are determined by the Fourier components of the high-frequency conductivity tensor of the metal, the expression for which near the n-th harmonic of the CR in the case of a quadratic carrier dispersion law is[4]

$$\sigma_{ik}(k,\omega)=\frac{3Ne^2}{2m^*}\int_{-1}^{1}d\mu\,\frac{w_i(\mu)w_k^*(\mu)}{\nu+i(n\Omega-\omega)},\qquad(1)$$

where N is the concentration of the conduction electrons, $\nu = \tau^{-1}$ is the characteristic frequency of the electron collisions with the scatterers, and the dimensionless velocity components $w_i(\mu)$ are given by

$$w_x(\mu)=-i(1-\mu^2)^{1/2}dJ_n(\xi)/d\xi,\quad \xi=kR(1-\mu^2)^{1/2};\qquad(2)$$

$$w_y(\mu)=\frac{n}{kR}J_n(\xi),\quad w_z=\mu J_n(\xi).\qquad(3)$$

Here $\mu = p_z/p_0$, p_0 is the Fermi momentum of the carriers, $R = cp_0/eH$, k is the wave number, and $J_n(\xi)$ is a Bessel function of order n.

In the vicinity of the CR there exist two types of weakly-damped electromagnetic oscillations: an ordinary cyclotron wave excited by polarization of the electromagnetic field **E** = (0, 0, E), and an extraordinary wave excited at **E** = (E, 0, 0). The spectrum of each of the waves is determined by the diagonal components $\sigma_{zz}(k, \omega)$ and $\sigma_{xx}(k,\omega)$ of the conductivity tensor, respectively (as shown in[2], the off-diagonal components of the tensor σ_{ik} can be neglected in the expression for the spectrum of the short cyclotron waves).

To study short-wave oscillations it is necessary to obtain an asymptotic expression for $\sigma_{zz}(k, \omega)$ and $\sigma_{xx}(k, \omega)$ at $kR \gg 1$. Integrating with respect to μ in formula (1) by the stationary-phase method, with allowance of oscillating increments that are small in terms of the anomaly parameter kR, and determining the field spikes at distances that are multiples of the cyclotron diameter we have

$$\sigma_{zz}(k,\omega)=\sigma_0(k,\omega)\left\{1-\frac{(-1)^n}{2\sqrt{\pi}}\frac{\sin(2kR+\pi/4)}{(kR)^{1/2}}\right\}, \qquad (4)$$

$$\sigma_{xx}(k,\omega)=\sigma_0(k,\omega)\left\{1-\frac{2}{\sqrt{\pi}}(-1)^n\frac{\sin(2kR-\pi/4)}{(kR)^{3/2}}\right\}. \qquad (5)$$

Here

$$\sigma_0(k,\omega)=\frac{3Ne^2}{4\pi i m^*\omega kR}(\Delta-i\gamma)^{-1}, \quad \gamma=(\omega\tau)^{-1}, \qquad (6)$$

where $\Delta = H_{res} - H)/H_{res}$ is the detuning from resonance and $H_{res} = \omega m^*c/ne$.

The field distributions in the plate is determined by[4]

$$E_\alpha(y)=-2\pi^{-1}E_\alpha'(0)T_\alpha(y), \quad \alpha=(x,z), \qquad (7)$$

where

$$T_\alpha(y)=\int_0^\infty\frac{dk\cos ky}{k^2-4\pi i\omega c^{-2}\sigma_{\alpha\alpha}(k,\omega)}. \qquad (8)$$

At distances shorter than or of the order of one or several cyclotron diameters, the main contribution to the integral is made by large k, at which the expression for $\sigma_{\alpha\alpha}(k, \omega)$, as seen from (4) and (5) can be represented in the form

$$\sigma_{\alpha\alpha}(k,\omega)=\sigma_0(k,\omega)\left\{1+a\frac{\sin(2kR+\alpha)}{(kR)^s}\right\}. \qquad (9)$$

The simple expressions (8) and (9) are sufficient for the study of the qualitative picture of the distribution of the field in the metal, if the character of the reflection of the conduction electrons from the sample boundary is not close to specular[2].

Substituting expressions (9) in (8) and expanding in the small parameter $(kR)^{-s}$ we obtain, neglecting the rapidly oscillating terms under the integral sign, the function $T_\alpha(y)$ in the form

$$T_\alpha(y)=T_\alpha^{(0)}(y)+T_\alpha^{(1)}(y)+\ldots+T_\alpha^{(m)}(y)+\ldots; \qquad (10)$$

$$T_\alpha^{(m)}(y)=(4\pi i\omega c^{-2}a)^m\int_{k_1}^\infty\frac{dk\,\varphi_m(k)}{(kR)^{sm}[k^2-4\pi i\omega c^{-2}\sigma_0(k,\omega)]^{m+1}},$$

$$\varphi_m(k)=\frac{(-1)^{m/2}}{2^m}\begin{cases}\cos[k(2mR-y)+m\alpha], & m=2f\\ -i\sin[k(2mR-y)+m\alpha], & m=2f+1\end{cases} \qquad (11)$$

($f = 0, 1, 2, \ldots$), where k_1 satisfies the inequality $R^{-1} \ll k_1 \ll k_0$, and k_0 is a solution of the dispersion equation and determines the short cyclotron waves[2]

$$k^2=4\pi i\omega c^{-2}\sigma_0(k,\omega). \qquad (12)$$

It is obvious that the existence of a weakly-damped wave is possible only under the condition $\mathrm{Im}\,\sigma_0(k, \omega) \gg \mathrm{Re}\,\sigma_0(k, \omega)$, which is valid if the inequality

$$\Delta\gg\gamma \qquad (13)$$

is satisfied. When this inequality is taken into account, the solution of the dispersion equation (12) becomes

$$k_0=\frac{1}{R}\left(\frac{\sqrt{3}}{2}\frac{\omega_0R}{c}\right)^{2/3}\Delta^{-1/3}\left(1+i\frac{\gamma}{3\Delta}\right). \qquad (14)$$

Here $\omega_0^2 = 4\pi Ne^2/m^*$ is the plasma frequency. The quantity

$$\delta^{-1}=|k_0|\cong\mathrm{Re}\,k_0=\frac{1}{R}\left(\frac{\sqrt{3}}{2}\frac{\omega_0R}{c}\right)^{2/3}\Delta^{-1/3}$$

determines the thickness of the skin layer.

It is seen from (14) that the cyclotron wave (ordinary or extraordinary) generated by the skin layer, is damped over distances on the order of

$$l_0\cong\delta\Delta/\gamma. \qquad (15)$$

The damping length l_0 is comparable with or exceeds the cyclotron diameter when the relation $\Delta\omega\tau \gtrsim R/\delta$ is satisfied; this is attainable only in unprecedentedly pure samples. In the opposite limiting case

$$\Delta\ll\gamma R/\delta \qquad (16)$$

the amplitude $T_\alpha^{(0)}(y)$ is exponentially small at $|y - 2R| \lesssim l_0$, and at these distances the field is determined by the second term of expression (10). (This condition leads simultaneously to the absence of interference between the waves generated by the various skin layers).

An analysis of the expression for $T_\alpha^{(1)}(y)$ shows that the distribution of the field near the spike has a rather peculiar character. It is easily seen that the field amplitude at $|y - 2R| \lesssim \delta$ is smaller by a factor $(|k_0|R)^s$ than its value at the surface of the metal. At large distances, when $\delta \ll |2R - y| \leq l_0$, the value of the integral $T_\alpha^{(1)}$ is determined by the residue of the integrand at the point $k = k_0$, the evaluation of which yields

$$T_\alpha^{(1)}(y)\cong\frac{\pi a i}{18k_0}(k_0R)^{-s}|k_0|(2R-y)\exp\{ik_0|2R-y|+i\alpha\,\mathrm{sgn}(2R-y)\}. \qquad (17)$$

It follows from this formula that up to distances $y \lesssim 2R + l_0$ the field amplitude increases, after which it decreases exponentially with increasing $|2R - y|$. Naturally, the maximum field amplitude is smaller in the region of the spike than at the surface of the metal, and this leads to the inequality

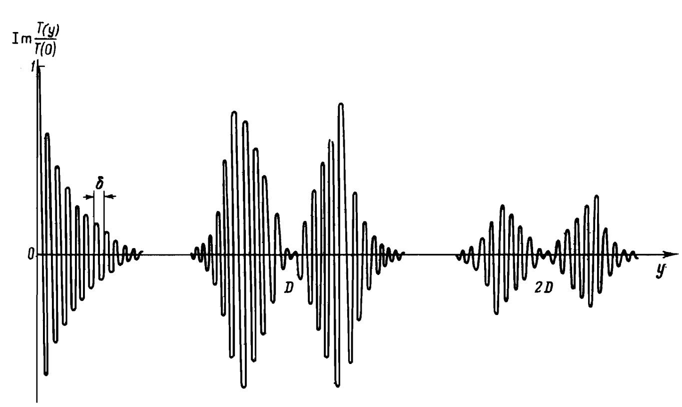
$$\frac{l_0}{\delta}(|k_0|R)^{-s}=\frac{\Delta}{\gamma}(|k_0|R)^{-s}<1, \qquad (18)$$

which was implicitly used in the calculation of the field distribution. It is easily seen that only in this case it is correct to represent the field in the form of the expansion (10).

Quite analogously, analyzing the integral (11), we arrive at the conclusion that the function $T_\alpha^{(m)}(y)$, which determines the electromagnetic field at distances on the order of $\delta \ll |y - 2mR| \lesssim l_0$, is a cyclotron wave that propagates from the m-th spike with an amplitude proportional to the quantity

$$(k_0R)^{-sm}[|k_0|(2mR-y)]^m.$$

The wave field distribution is shown schematically in the figure.



Substituting in (17) the values of a, s, and α determined by formulas (4) and (5), we obtain the field distribution for the propagation of the extraordinary and ordinary waves:

$$T_e^{(1)}(y) \cong \frac{(-1)^{n+1}\sqrt{\pi}i}{9k_0}(k_0R)^{-1/2}|k_0|(2R-y)\exp\left\{ik_0|2R-y| - i\frac{\pi}{4}\mathrm{sgn}(2R-y)\right\}, \quad (19)$$

$$T_0^{(1)}(y) \cong \frac{(-1)^{n+1}\sqrt{\pi}i}{36k_0}(k_0R)^{-3/2}|k_0|(2R-y)\exp\left\{ik_0|2R-y| + i\frac{\pi}{4}\mathrm{sgn}(2R-y)\right\}. \quad (19')$$

It is interesting to note that even in the case of an isotropic dispersion there arises an appreciable difference between the field distributions at different polarizations. The amplitude of the field of the ordinary wave at $|y-2R| \lesssim l_0$ is smaller by a factor $(|k_0|R)^{-1}$ than the amplitude of the field of the extraordinary wave.

In addition, the detuning limitations at which the discussed effect takes place are different for the ordinary and extraordinary waves. The condition (18) under which the ordinary waves is generated by the spikes is weaker than the condition (16) for the absence of interference effects. The situation is reversed for the extraordinary wave. Recognizing also that the propagation of cyclotron waves is possible only in the vicinity of the cyclotron resonance, we write down the final inequalities under which the considered field distribution in the metal is produced.

In the case of the ordinary wave

$$\gamma \ll \Delta \ll \min(1, \gamma R/\delta). \quad (20)$$

In the case of the extraordinary wave

$$\gamma \ll \Delta \ll 1 \quad \text{at} \quad \gamma\sqrt{R/\delta} > 1,$$
$$\gamma \ll \Delta < \gamma\sqrt{R/\delta} \quad \text{at} \quad \gamma\sqrt{R/\delta} < 1. \quad (20')$$

We call attention to the fact that in the latter case the amplitude of the cyclotron wave in the region of the first few spikes is comparable with the amplitude of the field at the surface of the metal.

We proceed now to consider short cyclotron waves in metals with anisotropic carrier dispersion. In this case the cyclotron frequency Ω is different for different values of the electron-momentum projection on the direction of the magnetic field. Small selected electron groups take part in the resonance, namely electrons with extremal effective mass, the extremum of which always is reached on the central section of the Fermi surface, electrons with extremal cyclotron diameter, and electrons near a limiting point of the Fermi surface.

In the case of cyclotron resonance with electrons with extremal diameter, it is a small increment to the impedance that is resonant. The dissipative part of the principal term in the conductivity is of the same order as the imaginary part, and consequently the propagation of electromagnetic waves near these resonant frequencies is generally speaking impossible.

Near a limiting point of the Fermi surface, there is no preferred cyclotron diameter, so that this method of "dragging through" an ordinary cyclotron wave to an appreciable depth into the metal is excluded.

We consider the propagation of an extraordinary wave excited at resonance with electrons of the central section, in the case when $m^*(p_z)$ has a minimum at $p_z = 0$[2]. To simplify the calculations we assume that the sections of the Fermi surface at $p_z \simeq 0$ are circles. This makes it possible to use formula (1) and (2) in the calculation of the conductivity $\sigma_{xx}(k, \omega)$, but it is necessary at the same time to take into account the dependence of Ω on μ. Replacing in (2) the derivative $dJ_n(\xi)/d\xi$ by its asymptotic expression for $\xi \gg 1$ and substituting in (1), we get

$$\sigma_{xx}(k,\omega) = \frac{3Ne^2}{2\pi m^* i\omega kR}\left\{\int_{-1}^{1}\frac{d\mu}{\varkappa\mu^2+\Delta-i\gamma} - (-1)^n\int_{-1}^{1}d\mu\frac{\sin[2kR(1-\mu^2)^{1/2}]}{\varkappa\mu^2+\Delta-i\gamma}\right\},$$
$$\varkappa = \frac{1}{2m^*}\frac{\partial^2 m^*}{\partial\mu^2}\bigg|_{\mu=0} > 0. \quad (21)$$

The calculation of the first integral is elementary. The value of the second depends on the relative "sharpness" of the numerator and denominator. The characteristic scales of variation of the numerator and denominator are $\delta\mu_1 \cong \sqrt{\delta/R}$ and $\delta\mu_2 \cong \max(\sqrt{\Delta}, \sqrt{\gamma})$, respectively.

We consider first the case

$$\gamma \gg \delta/R. \quad (22)$$

The numerator at all values of Δ is then a sharper function and the resonant denominator can be taken outside the integral sign. Integration by the stationary-phase method then yields for the conductivity $\sigma_{xx}(k, \omega)$ the expression

$$\sigma_{xx}(k,\omega) = \frac{3Ne^2}{2m^* i\omega kR\sqrt{\varkappa}}\frac{1}{\sqrt{\Delta-i\gamma}}\left\{1-(-1)^n\sqrt{\frac{\varkappa}{\pi kR}}\frac{\sin(2kR-\pi/4)}{\sqrt{\Delta-i\gamma}}\right\}. \quad (23)$$

This expression for σ_{xx} and the dispersion equation (12) determine the wavelength and the damping of the extraordinary cyclotron waves. At $\Delta \gg \gamma$ we have

$$k_0^a = \left(\frac{3}{2\sqrt{\varkappa}}\right)^{1/3}\left(\frac{\omega_0 R}{c}\right)^{2/3}\frac{1}{R}\Delta^{-1/6}\left(1+\frac{i\gamma}{6\Delta}\right). \quad (24)$$

If the inequality (22) is satisfied, the damping length is $l_0 \cong |k_0^a|^{-1}\Delta\omega\tau \ll R$, and the field at the distances $|y-2R| \le l_0$ satisfies the expression (17). Substituting

$$a = (-1)^{n+1}\sqrt{\varkappa/\pi\Delta}, \quad s = 1/2, \quad \alpha = -\pi/4,$$

we obtain

$$T_e^{(1)}(y) \cong \frac{(-1)^{n+1}\sqrt{\pi\varkappa}\,i}{18k_0^a\sqrt{\Delta}}(k_0^aR)^{-1/2}|k_0^a|(2R-y)\exp\left\{ik_0^a|2R-y| - i\frac{\pi}{4}\mathrm{sgn}(2R-y)\right\}. \quad (25)$$

From expressions (11) at m = 1 and from (23) it follows that formula (25) is valid provided the inequality

$$\Delta < \gamma^2 R/\delta \quad (26)$$

is satisfied. This requirement is analogous to the inequality (18), which pertains to a quadratic dispersion law. The condition for the compatibility of the inequalities (13) and (26) leads to the already employed inequality (22).

In the Opposite limiting case $\gamma \ll \delta/R$, the requirement on the detuning at which cyclotron waves can propagate in a metal is more stringent than $\Delta \gg \gamma$, namely

$$\Delta \gg \delta/R. \quad (27)$$

Although in this case the wave generated by the principal skin layer attenuates over distances larger than 2R, the satisfaction of the inequality

$$\gamma \ll \delta/R \ll \Delta \ll 1$$

is impossible in experiment at present (this calls for samples whose quality must ensure satisfaction of the relation $\omega\tau \cong 10^3$). At $\Delta \ll \delta/R$, a significant transformation of the wave spectrum takes place. The frequency ω corresponds to an entire series of waves with lengths satisfying the equation $q\lambda/2 = 2R$ (q = 1, 2, . . .) The only interference with physical meaning between these waves is the one leading to the existence in the metal of sharp slowly-damped field spikes[1].

We suggest that it is most convenient to study this effect by observation of the cutoff of the cyclotron resonance. Let H_0 be the cutoff field, i.e., $2R|_{H=H_0} = d$, and let H_n be the smallest resonant value of the magnetic field at which the corresponding cyclotron diameter is still less than the thickness of the conductor. By using a frequency-modulation method (see, e.g.,[7]), it is possible to decrease somewhat the frequency of the resonator in such a way that in the vicinity of the n-th resonance the inequality $D_n - d \lesssim l_0$ is satisfied. Then the transparency peaks become quite distinct. The measured signal, which is proportional to $T^{(1)}(y)$, as follows from expressions (19), (19′), and (25), is periodic in $\Delta^{-1/3}$ in the case of quadratic dispersion and in $\Delta^{-1/6}$ in the case of anisotropic dispersion of the carriers. All this, in our opinion, makes it possible to identify the considered effect uniquely.

The authors thank G. E. Zil'berman and É. A. Kaner for valuable advice and remarks.

1)At $D_e > d$, the character of the CR is appreciably altered [5,6] and the question of the existence of cyclotron waves in this case calls for a special analysis.

2)In the case of almost specular reflection, electrons that collide many times with the boundary participate in the production of the near-surface skin layer. The resonant electrons "record" the information on the skin layer at a considerable depth from the surface, and this can lead to a change in the picture of the field distribution in the metal. An analysis of this question is timely for all the presently known types of anomalous penetration of a field into a metal, and in this sense the effect studied here is no exception. No solution of this problem has been obtained so far, however, owing to its great complexity. It should nevertheless be noted that the very existence of the indicated effects is not subject to doubt even in this case.

1M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **39**, 400 (1960) [Sov. Phys.-JETP **12**, 283 (1961)].
2É. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela **6**, 1104 (1964) [Sov. Phys.-Solid State **6**, 851 (1964)].
3W. M. Walsh and P. M. Platzman, Phys. Rev. Lett., **15**, 784 (1965).
4E. A. Kaner and V. G. Skobov, Advances in Physics, **17**, 605 (1968).
5M. A. Lur'e and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. **66**, 240 (1974) [Sov. Phys.-JETP **39**, 114 (1974)].
6A. P. Volodin, V. S. Édel'man, and M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. **65**, 2105 (1973) [Sov. Phys.-JETP **38**, 1052 (1974)].
7M. S. Khaĭkin and V. S. Édel'man, Zh. Eksp. Teor. Fiz. **47**, 878 (1964) [Sov. Phys.-JETP **20**, 587 (1965)].

Translated by J. G. Adashko.
129