An acoustic heterodyne detector of gravitational radiation

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A type of heterodyne gravitational-radiation receiver based on the interaction between surface waves and deformations induced by the gravitational wave in a nonlinear medium is proposed. The receiver operates in the $\Omega \approx 10^4$–10^6 rad/sec frequency range, and may be 4–5 orders of magnitude more sensitive than the Weber-type detector. The changes in the structure of the surface wave are computed and the feasibility of the experimental realization of the detector is discussed. Estimates for detectable gravitational-radiation fluxes are given.

1. INTRODUCTION

The search for effective methods of detection of gravitational radiation has recently led to the investigation of detectors of the heterodyne type. Mechanical (M) and electromagnetic (EM) variants of such a detector have been considered in the literature[1-4]. The gravitational wave (GW) of frequency Ω interacts in such a detector with a mechanical or an electromagnetic system having a characteristic rotational frequency ω_0. Upon the fulfilment of the synchronism condition $\Omega = 2\omega_0$, there arise in the system perturbations that build up in time if the damping is sufficiently weak. In the M-variant[1,2] of the detector, the gravitational wave accelerates (decelerates) a rotating dumbbell, causing the phase to increase (decrease) quadratically in time. In the EM-variant[3,4], the gravitational wave changes the relative phase of an EM-wave packet in a ring wave guide, or leads, by interacting with a monochromatic wave, to the production of EM-wave harmonics.

Both types of detectors are resonant gravitational receivers, but are designed for the diametrically opposite ends of a GW-frequency range. Because of the low admissible rates of rotation, the M-variant can be realized for frequencies $\Omega \lesssim 5 \times 10^2$; in contrast, the EM-variant, owing to the high velocity of the electromagnetic wave, encompasses the super-high gravitational-wave frequencies $\Omega \gtrsim 10^8$. For the reception of gravitational radiation from outer space, both detectors are of little use: it is not yet sufficiently clear which extraterrestrial objects emit in the uhf region, and it is difficult to expect radiation of significant intensity from low-frequency gravitational-wave sources[5].

Below we propose an acoustic heterodyne detector of gravitational radiation in the interesting–from the astrophysical point of view–frequency range $\Omega \sim 10^4$–10^6, where we can expect the appearance of gravitational-radiation pulses of duration ~ 1 sec and energy density $\sim (1 - 10^3)$ erg/cm^2 [1,5]. The detector is based on the interaction between a high-frequency (tens-hundreds of MHz) elastic surface wave propagating along the outer surface of a solid or hollow cylinder (the wave front is parallel to the generatrix of the cylinder) and GW-induced deformations of the low-frequency quadrupole mode. The geometry of the cylinder is chosen such that the synchronism condition $2v_S/R = \Omega$, where v_S is the velocity of the surface wave and R is the radius of curvature of the surface of the cylinder, is satisfied. Since for solid bodies $v_S \sim (1.5 - 5) \times 10^5$ cm/sec, we obtain for reasonable dimensions $R \sim 1$–30 cm a GW frequency lying in the requisite region $\Omega \sim 10^4$–10^6. Furthermore, the GW frequency should coincide with the frequency of the low-frequency quadrupole mode of the transverse flexural vibrations of the cylinder.

A key feature guaranteeing the GW-surface wave interaction is the requirement of the acoustic nonlinearity of the material of cylinder. In a linear medium the deformations induced by the gravitational and elastic surface waves are independent, and the waves do not interact with each other[1)]. In a nonlinear medium the deformations induced by the GW alter the elastic parameters of the medium, thereby changing the conditions of propagation of the elastic surface wave. The choice of the type of surface wave (a Rayleigh, Love, Bluestein-Gulyaev, or Lamb wave) and of the quadrupole mode of the low-frequency vibrations is then determined by the existence of nonzero, nonlinear-coupling coefficients and the choice of the interaction with the maximum coefficients.

Let us, for concreteness, point out that the nonlinear coupling of the cylinder's tidal mode, which induces radial and tangential deformations, to a Rayleigh surface wave will be described by the same coefficients that are responsible for the generation of the second harmonic of the surface wave.

For the isotropic case, and for a large cylinder radius $R \gg \lambda$, the change in the phase velocity of the surface wave will be expressible in terms of the strain components for the surface of the cylinder in the following manner[6]:

$$v_s^2 = v_{s0}^2(1+\Gamma_1 u_t + \Gamma_2 u_r),$$

where v_{S0} is the velocity of the unperturbed surface wave; u_t and u_r are the tangential and radial strain components induced by the gravitational wave; Γ_1 and Γ_2 are the corresponding nonlinear parameters:

$$\Gamma_1 = \frac{2c_{155}}{c_{11}-c_{12}}, \quad \Gamma_2 = \frac{2c_{155}}{c_{11}-c_{12}}\left[\frac{1-(v_{s0}/v_l)^2}{1-(v_{s0}/v_t)^2}\right]^{1/4}.$$

Here c_{11} and c_{12} are the corresponding linear, and c_{155} the nonlinear, elastic moduli, while v_l and v_t are the velocities of the longitudinal and shear bulk waves.

2. COMPUTATION OF THE INTERACTION BETWEEN SURFACE AND GRAVITATIONAL WAVES

Let us carry out a simple analysis of the interaction between surface and gravitational waves, phenomenologically introducing in the process a nonlinearity parameter Γ for the medium. Let a plane GW of fixed frequency and polarization propagate at an angle θ to the symmetry axis of the cylinder. Let the frequency Ω of the GW coincide with that ω_m of the lowest-frequency mode of the mechanical flexural vibrations of the cylinder in the planes perpendicular to the axis of the cylinder (let us call this condition, i.e., the condition that $\Omega = \omega_m$, the "mechanical" resonance requirement, in contrast to the "synchronism" condition considered above). Then the GW-induced relative deformations of the surface of the cylinder will be determined by the expression

$$u_b = 0{,}125\, h_0 \Omega t (1+\cos^2\theta)\cos 2\varphi \sin(\Omega t + \Phi_b), \tag{1}$$

where φ is the polar angle, h_0 is the amplitude of the variation of the metric associated with the GW, $h = h_0 \cos(\Omega t + \Phi_b)$, and Φ_b is the initial phase of the GW. Here u_b may be taken as denoting both the radial and the tangential strain components (if it is remembered that they differ in their φ phases by $\pi/2$). It should be noted that the formula (1) is exact only in the case of an infinitely thin cylinder or for an arbitrary thickness l when $\theta = 0$ or $\pi/2$. We apply this formula in the general case approximately, assuming that $l \ll R$. The relation between the thickness of the cylinder and the wavelength of the surface wave does not then play a fundamental role, affecting not the conditions for the existence of the wave itself, but only its profile (the distribution of the deformations along the generatrix of the cylinder) and the dispersion law.

The formula (1) is valid in the absence of damping, or for times of action less than the mechanical-relaxation time τ_m^* for the mode in question. For a more prolonged action the time t in (1) should be replaced by τ_m^*, which will yield a vibration amplitude $(u_b)_{max} = 0.5 h_0 Q_m$, where Q_m is the Q of the cylinder with respect to the mechanical mode in question. Let us also emphasize that the assumption made in[1-4] that the GW propagates along the symmetry axis (i.e., that $\theta = 0$) is of no fundamental importance: the strains u_b exist at any value of θ, having at $\theta = \pi/2$ half the values they have at $\theta = 0$.

The displacements $u_s(\varphi, t)$ induced on the surface of an isotropic cylinder by an elastic surface wave should, at $\lambda \lesssim R$, satisfy the wave equation in which the velocity of the surface wave will be modulated by the deformations induced by the gravitational wave:

$$\frac{\partial^2 u_s}{\partial t^2} - \frac{v_s^2(1+\Gamma u_b)}{R^2}\frac{\partial^2 u_s}{\partial \varphi^2} = 0. \tag{2}$$

This equation can be derived rigorously in the framework of nonlinear three-constant acoustics for isotropic bodies. The exact calculation is, however, tedious, it requiring allowance for the boundary conditions in the determination of the dispersion law $v_s(\omega)$[7], the knowledge of which is not fundamental to the validity of the assertions made below. Therefore, we shall restrict ourselves to a phenomenological derivation of (2), as has been repeatedly done in the literature[6,8].

The smallness of the quantity Γu_b allows us to use the method of successive approximations. In using this method we shall neglect the solutions describing the action of the surface wave on itself (the generation of the harmonics of the surface wave). In the zeroth approximation the displacements on the surface are determined by the initial conditions and the conditions for the existence of a continuous surface wave of fixed frequency[9]:

$$\omega_k = k v_s / R, \quad k = 2\pi R/\lambda = 1, 2, 3\ldots$$

Then

$$u_s^{(0)}(\varphi, t) = u_0 \sin(\omega_k t - k\varphi + \Phi_s).$$

Here λ is the wavelength of the surface wave and Φ_s is its initial phase.

We shall seek the correction of the first approximation under the assumption that the mechanical-resonance and synchronism conditions, $\Omega = \omega_m$ and $\Omega = \omega_2 = 2v_s/R$, are simultaneously satisfied. Technically, this is possible: e.g., for a hollow isotropic cylinder of radii R and r, we have

$$\omega_m \approx \pi \frac{R-r}{R^2}\sqrt{\frac{E}{\rho}}, \quad v_s = \xi\sqrt{\frac{E}{\rho}}, \quad \xi < 1$$

(E is Young's modulus)[9,10]; therefore, it is sufficient to choose the geometry such that $(R-r)/R = 2\xi/\pi$. The parameter ξ is (in the isotropic case) determined by the Poisson coefficient of the material.

It is not difficult to derive the equation for the first-order correction:

$$\frac{\partial^2 u_s^{(1)}}{\partial t^2} - \frac{v_s^2}{R^2}\frac{\partial^2 u_s^{(1)}}{\partial \varphi^2} = -\frac{u_0}{2}\Gamma a \omega_k^2 \Omega t \{\sin[(k+2)\psi - \varphi_1] + \sin[(k-2)\psi - \varphi_2]\}. \tag{3}$$

Here we have introduced the notation:

$$a = 0{,}125(1+\cos^2\theta) h_0, \quad \varphi_1 = \Phi_{rp} + \Phi_s,$$
$$\varphi_2 = \Phi_{rp} - \Phi_s, \quad \psi = \Omega t/2 - \varphi.$$

The analysis below is similar to the scheme investigated earlier for the EM variant[4]. The solution to Eq. (3) is:

$$u_s^{(1)} = 0{,}25 \Gamma a u_0 \omega_k \Omega t^2 \left\{\frac{k}{k+2}\cos[(k+2)\psi - \varphi_1] + \frac{k}{k-2}\cos[(k-2)\psi - \varphi_2]\right\}. \tag{4}$$

As in the case of (1), the solution (4) is valid for time intervals shorter than the relaxation times τ_m^* and τ_s^* (τ_s^* is the relaxation time for the surface wave). In the opposite case, it is necessary to replace t^2 by $\tau_m^* \tau_s^*$. It follows from (4) that there arise, as a result of the interaction between the gravitational and surface waves, harmonics of the surface waves at frequencies ω_{k+2} and ω_{k-2} not present in the initial high-frequency signal. At large k and for $\theta = 0$, the fraction of the energy transferable to a harmonic is

$$\frac{\Delta\varepsilon}{\varepsilon} \approx \frac{1}{256}\Gamma^2 h_0^2 \omega_k^2 \Omega^2 t^4 \sim \Gamma^2 h_0^2 Q_m^2 Q_s^2. \tag{5}$$

The last expression pertains to the steady-state regime; $Q_s = \omega_k / 2\alpha v_s$ is the equivalent Q-factor for the surface wave; α is the attenuation constant of the surface wave.

The case k = 1 corresponds to the parametric amplification of the fundamental wave:

$$u_s = u_0\left\{\cos(\psi + \Phi_s) + \frac{1}{16}\Gamma h_0 \Omega^2 t^2 \cos(\psi + \varphi_2)\right\} \tag{6}$$

a) for $\Phi_s = \varphi_2$:

$$u_s = u_0\left(1 + \frac{1}{16}\Gamma h_0 \Omega^2 t^2\right)\cos(\psi + \Phi_s),$$
$$\Delta\varepsilon/\varepsilon = h_0 \Omega^2 t^2/16 \sim \Gamma h_0 Q_m Q_s; \tag{7}$$

b) for $\varphi_2 = \Phi_s + \pi/2$:

$$u_s \approx u_0 \cos\left[\psi + \Phi_s + \frac{1}{32}\Gamma h_0 \Omega^2 t^2\right]. \tag{8}$$

Thus, upon the fulfilment of the phase relations in the case a), the amplitude of the wave with the fundamental frequency increases, the energy contribution being proportional to the first power of h_0, while in the case b) there is observed a shift in the phase of the fundamental wave.

The case of double parametric amplification, which also allows us to produce an energy effect proportional to h_0, is possible. For this purpose, it is necessary to excite two surface waves u_1 and u_2 with frequencies ω_k and ω_{k+2}. Then the nascent harmonics will combine with the available signals of fundamental frequencies and be amplified, so that

$$\frac{\Delta\varepsilon}{\varepsilon}=\frac{1}{16}\Gamma\frac{u_1}{u_2}\Omega\omega_k t^2\sim\frac{u_1}{u_2}\Gamma h_0 Q_m Q_S. \tag{9}$$

The formulas (4)–(9) differ from the corresponding relations for the EM variant[4] by the presence of the nonlinear acoustic parameter Γ and by the quadratic increase in time of the amplitudes. The latter is a consequence of the simultaneous occurrence of the two resonances–the simple "mechanical" resonance and the "acoustic-gravitational synchronism." Notice also that the harmonics generated as a result of the action of the elastic surface wave on itself in the case when $k \gg 2$ can easily be filtered from the harmonics generated by the interaction between the surface and gravitational waves.

As in the EM variant[3, 4], the detection of GW in the acoustic case can be accomplished with the aid of surface-wave packets propagating along the free surface of the cylinder. For two packets with a spatial φ-phase shift of $\pi/2$, the relative phase shift with respect to the high-frequency population is determined by the relation

$$\Delta\varphi\approx 0.25\,\Gamma h_0\Omega\omega_k t^2, \tag{10}$$

this relation being valid in the coordinate system connected with the wave packets, i.e., in a coordinate system rotating with an angular velocity of v_S/R. To measure the phase shift under laboratory conditions, we must employ time strobing at the observation points, selecting the moments of transmission of its, i.e., of only the accelerated or only the decelerated, packet, as was implied in[3]. In such an experiment the requirement that $t < \tau_S^*$ is unimportant. For $t \gtrsim \tau_m^*$, instead of (10), the phase shift will be determined by the expression $\Delta\varphi = 0.5\Gamma h_0 Q_m\omega_k t$, i.e., the phase will increase in linear proportion to the time.

3. DISCUSSION OF THE RESULTS AND THE FEASIBILITY OF THE EXPERIMENT

To appraise the feasibility of the experimental realization of the above-described gravitational-radiation detector, and to estimate the attainable sensitivity, let us introduce into the formulas (4)–(10) the energy flux density I of the gravitational radiation, using the relation $h_0 = \Omega^{-1}[64\pi GI/c^3]^{1/2}$, where G is the gravitational constant and c is the velocity of light. Then from the expression (4) we have for the amplitude of the (k + 2)-th harmonic in the case when $k \gg 1$ the expression

$$u_s^{(1)}=u_0\Gamma\omega_k t^2(G\pi I/c^3)^{1/2}. \tag{11}$$

The propagation of elastic surface waves along an isotropic cylindrical surface has been considered by Viktorov[9, 11, 12]. The presently available methods of excitation and reception of surface waves are most efficient in the case of piezoelectric crystals[13], when, as the transducer, we can use an interdigital array of metallic electrodes. Thus, unidirectional interdigital transducers on lithium niobate enable us to reduce the losses due to conversion to 4–6 dB[13, 14], which significantly increases the dynamical range of the acoustic detector. Therefore, the practical realization of the considered method of reception of gravitational radiation is, apparently, possible if we use cylinders fabricated from piezoelectric single crystals (quartz, lithium niobate, bismuth germanium oxide). Such cylinders will also possess a high Q at the mechanical-resonance frequency.

The propagation of elastic surface waves along anisotropic crystalline surfaces is more complicated, but for $\lambda \ll R$ it is possible to select in certain crystals transversely-isotropic cylindrical sections, for which the propagation of surface waves will be almost the same as in the isotropic case, and in which case the piezoelectric method of wave excitation can be used. For example, it is possible to cut out from quartz a cylinder having the c axis (i.e., the threefold axis) of the crystal as its axis. In experiments performed at Moscow State University, an elastic surface wave of frequency 22 MHz was excited by interdigital transducers on the surface of a cylinder (R = 1 cm) cut out from quartz in the above-described manner. In this case 150–200 circulations of the ultrasonic pulse were observed. The total delay time was about 3500 μsec for a dynamical range of 100 dB and losses due to conversion of 40 dB. The effective Q for the surface waves at room temperature in this experiment was $Q_S \sim 10^5$ and the relaxation time for the surface wave was accordingly equal to $\tau_S^* \approx Q_S/\omega \sim 10^{-2}$ sec. The lowering of the temperature of the cylinder to the temperature of liquid helium and the vacuuming of the sample to reduce emission into the air enable us to increase the value of Q_S by roughly 2–3 orders of magnitude. This also allows the frequency of the surface wave to be increased by one-two orders of magnitude, which is useful in the phase-shift measurements. The multielectrode interdigital piezoelectric transducer is at the same time a good filter, allowing us to separate out the requisite-harmonic signal. Finally, it should be borne in mind that we can use dielectric single-crystal cylinders fabricated from nonpiezoelectric single crystals, using for the excitation of the surface waves thin piezoelectric films (e.g., AlN on sapphire[15]).

The nonlinear acoustic parameter Γ determining the interaction between the modes can be calculated in terms of the linear and nonlinear elastic moduli of the crystal[5, 16], but this is a rather complicated problem in the case of surface waves, especially for a cylindrical surface. A more admissible way of estimating the quantity Γ is by experiment. Thus, for an X-cut quartz, in the plane-surface case experiments yield the value $\Gamma = 0.8$[17]. According to rough estimates, $\Gamma \sim 7$ for lithium niobate and sapphire. A stronger nonlinearity is the electronic nonlinearity, which is due to the interaction of the elastic wave with the conduction electrons. For Bluestein-Gulyaev waves in a GdS crystal, the nonlinearity parameter Γ was experimentally found to have the value $\Gamma \approx 690$, while theoretical estimates yield $\Gamma \approx 1200$[18]. In this case the magnitude of the nonlinearity parameter depends on the conductivity of the crystal. An even larger value of Γ can be expected in CdSe. However, a rather strong sound absorption, due again to the interaction with the conduction electrons, is observed at high frequencies in piezoelectric semiconductor crystals. As a compromise, we can propose a system consisting of a high-Q dielectric cylinder covered by a piezoelectric semiconductor film that will simultaneously serve as a transducer for surface-wave excitation and as a high-Γ medium.

The cited data allow us to estimate the limiting technical feasibility of the acoustic heterodyne method of reception of gravitational radiation. Let us choose the frequencies of the gravitational and surface waves to be $\Omega = 10^5$ and $\omega_k = 10^9$ respectively. We shall take the maximum value for the nonlinearity parameter: we shall take $\Gamma \sim 10^3$ when the electronic nonlinearity is used. In the narrow-band regime (to separate out the (k+2)-th harmonic, we need a relative receiver-band width $\Delta\omega/\omega \lesssim 10^{-4}$), it is not difficult to realize harmonic-signal reception in a dynamical range ~140 dB. Then the ratio of the amplitude of the (k + 2)-th harmonic to the amplitude of the signal having the fundamental frequency, as recorded by the receiving device, will be $u_s^{(1)}/u_0 \approx 10^{-7}$.

As was noted above, cooling to the temperature of liquid helium and vacuuming enable us to increase the mechanical and the equivalent surface Q factors, so that the corresponding relaxation times will be $\tau_m^*, \tau_s^* \gtrsim 1$ sec (it is also implied that the surface of the cylinder has been worked to a high degree of quality). Notice that, for a sensitivity $\sim 10^{-14}$ W($u_s^{(1)} \sim 1$ μW, $r \sim 100$ ohm), the amount of heat released in the acoustic resonator will not exceed 1 W and the rate of expenditure of helium will not be more than 2 liters per hour ($T \approx 2$°K). Using (11), we have for the limiting technical sensitivity the expression

$$I_{\text{tech}} = \left[\frac{u_\text{п}^{(1)}}{u_0} \Big/ \Gamma\omega_k t^2\right]^2 \frac{c^3}{\pi G} \approx 1\ \text{erg/cm}^2 \times \text{sec}. \qquad (12)$$

Thus, the instrumental limitations allow the detection of second-long GW pulses of energy density ~1 erg/cm².

From the point of view of the fluctuation-imposed limitations, I_{min} is determined by the thermal Brownian noise of the fundamental quadrupole mode of the cylinder. Under the condition $t \ll \tau_m^*$, we have

$$I_{min} \approx \frac{c^3}{8\pi G} \frac{\varkappa T}{m\Omega Q_m R^2 t} \eta. \qquad (13)$$

Here $\eta < 1$ is a factor determining the gain in the signal/noise ratio due to the fluctuating polarization of the Brownian vibrations of the cylinder (i.e., to the randomness of the angular orientation of the mode), as opposed to the fixed polarization of the deformations u_b.

The role of the low-frequency mechanical Q factor can be seen from (13). Guaranteeing a sufficiently high Q_m (while maintaining a large Γ) is the primary experimental difficulty in the process of lowering I_{min}. Orientating ourselves to the attained level, let us set $\eta/Q_m = 10^{-9}$ [19]; as reasonable—for laboratory conditions—values of the remaining parameters, let us take: R = 15 cm, $T \approx 2$°K, and $m = 10^4$ g. Then we obtain from (13) for an observation time of 1 sec the value: $I_{min} \approx 10$ erg/sec × cm², which is an order of magnitude higher than the instrumental limit I_{tech} given by (12). The lowering of the temperature to 0.2°K equalizes the two values; furthermore, there are no fundamental laws prohibiting the growth of Q_m.

Notice that the detector acts as an amplitude converter in the regime in which it measures the amplitude of the surface wave, converting the amplitude Δx of the GW-induced displacement into a surface-strain amplitude Δu. For $I \sim 1$ erg/sec × cm², the corresponding $\Delta x \sim 10^{-17}$ cm. The direct detection of such displacements with the aid of, for example, a piezoelectric pickup or a pickup of the parametric type[20] has thus far not been carried out (in[20] the displacements measured with the aid of a capacitive pickup were $\sim 10^{-14}$ cm). The present detector effects a transformation to a measurable quantity Δu according to the law $\Delta u/u_0 \approx \Gamma\omega_k t \Delta x/R$ with a very substantial conversion factor $\Gamma\omega_k t \sim 10^{10}$ for the parameters indicated in the text.

Experimentally, this regime can be realized in a continuous excitation of a surface wave of frequency ω_k by an external stable source. A typical relative line width of quartz generators is $\Delta f_g/f_g \sim 10^{-10}$, which is considerably narrower than the signal-frequency shift $\Omega/\omega_k \sim 10^{-4}$. To draw off the combination harmonic, it is sufficient to set up a filter with $\Delta f_f/f_f \lesssim 10^{-5}$. Owing to the generator fluctuations, the root-mean-square deviation at the exit of the filter will be

$$(\overline{\Delta u_{fl}^2})^{1/2} \approx \frac{u_0}{\Omega}\left(\frac{D}{2\pi}\Delta f_f\right)^{1/2}.$$

For quartz standards, the dispersion of the generator $D \approx 10^{-10}$ rad⁻¹, whence $(\overline{\Delta u_{fl}^2})^{1/2}/u_0 \lesssim 5 \times 10^{-8}$, which is smaller than the expected signal level $u_s^{(1)}/u_0 \approx 10^{-7}$. As in the experiment[20], in the search for GW from cosmic objects, the measurement process here amounts to the continuous tracking of the variations in the amplitude of the combination harmonic within one-second observation intervals.

The second operating regime of the detector is the version of the experiment in which surface-wave packets are used. An estimate of the phase shift from the formula (10) with the same parameter values used earlier yields for t = 1 sec and I = 1 erg/sec × cm² the phase-gain value $\Delta\varphi \approx 10^{-6}$, which is easily measurable. In fact, the thermal fluctuations produce a root-mean-square drift of $(\overline{\Delta\varphi_{fl}^2})^{1/2} \approx (\overline{\Delta u_{fl}^2})^{1/2}/u_0$, where

$$(u_{fl}^2)^{1/2} \approx [kT/\pi^2\rho v_s^2 l_p]^{1/2}/R,$$

ρ is the density of the detector material and $l_p - v_s\tau_p$ is the wave-packet dimension, which we take here to be equal to 0.1R. At a temperature $T \approx 2$°K and for $R \approx 10$ cm, we obtain $(\overline{\Delta u_{fl}^2})^{1/2} \sim 10^{-4}$, whence $(\overline{\Delta\varphi_{fl}^2})^{1/2} \lesssim 10^{-6}$ if $u_0 \gtrsim 10^{-9}$. The pump generator in this experiment should operate in the pulse regime, exciting simultaneously at two points at intervals of ~1 sec high-frequency surface-wave pulses of length $\tau_s < 10^{-5}$ sec. The two-channel signal-detection system should be gated with the circulation frequency of the wave packets and should have the phase detector at its exit.

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[1)] We omit here and below the direct action of the GW (but not the deformations induced by it!) on the surface wave, an action which produces effects considerably weaker than those of the nonlinear interaction between the deformations. Indeed, the direct acceleration of the surface wave in the field of the gravitational wave under the conditions of synchronism leads, for example, to a phase gain $\Delta\varphi \sim \Omega^2 h_0 t^2$, which is $\Gamma(\omega_k/\Omega)$ ($\sim 10^7$ for the parameters used in the text) times smaller than the corresponding phase shift (10) due to the nonlinear acoustic effect.

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