Avalanche ionization induced in solid transparent dielectrics by strong laser pulses

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(Submitted April 11, 1974)
Zh. Eksp. Teor. Fiz. **67**, 1805–1817 (November 1974)

A consistent analysis of the kinetic equation in the quasicontinuous energy distribution approximation for conduction-band electrons is performed by taking into account intraband scattering by phonons at arbitrary dispersion laws. The choice of boundary conditions is analyzed. The breakdown criterion and the effect of spatial diffusion of the electrons are investigated. The temperature dependence of the critical field strength is studied. Concrete estimates are presented for the electron-avalanche development time.

1. INTRODUCTION

The development of an electron avalanche in the conduction band was first considered in the approximation of the Fokker-Planck equation in [1]. A qualitative examination of the influence of the energy losses in intraband scattering of electrons by phonons was carried out in [2]. In addition, certain features of the development of an electron avalanche under the influence of a very strong electromagnetic field were clarified in [3]. In this paper we propose a general method of solving the kinetic equations, which makes it possible to obtain concrete estimates of the avalanche-development constant and of the critical value of the electromagnetic field, to analyze the temperature dependence of the indicated parameters, and also to find the amount of energy transferred to the lattice by the conduction-band electrons in a strong electromagnetic field, as the result of which it is possible to formulate a physically verified criterion of optical damage.

The mechanism whereby a pure transparent dielectric is damaged can be visualized in the following manner: The multiphoton processes of ionization of the valence electrons in the conduction band gives rise to a certain initial number of electrons. These become accelerated in the field of the electromagnetic wave and reach an energy exceeding the ionization energy, and thus initiate the cascade process of multiplication. Damage takes place if the electron-number density reaches during the time of action of the pulse the critical value at which the energy absorbed by the electrons and transferred to the lattice is sufficient for the development of irreversible processes.

The acceleration of the electrons in the field of the light wave can proceed in different manners. Let ω be the frequency of the electromagnetic radiation, and τ_ϵ the characteristic relaxation time of the electrons along the energy axis (τ_ϵ is smaller by approximately two orders of magnitude than the relaxation time of the electron momentum over the directions in collisions with acoustic phonons). In the case when $\omega \gg 1/\tau_\epsilon$, the energy acquisition by the electron is either due to successive collisions with the photons and phonons, or to multiphoton absorption processes in the conduction band. The first case, neglecting the energy lost by the electrons in the collisions with the phonons, was considered in [1], while the second was considered in [2], where it was shown, in addition, from qualitative considerations, that if the kinetic-equation terms that describe the electron-phonon interaction are discarded beforehand, the threshold values of the damage-inducing field may be greatly underestimated. We shall show below that these considerations are valid, but the critical field values obtained from a consistent solution of the kinetic equation in the Fokker-Planck approximation are still much smaller than the critical fields indicated in [2] for multiphoton processes.

The general method of determining the critical field and the duration of the process of cascade ionization is described in Sec. 2. The only assumption in this case is $\hbar\omega \ll I$, where I is the ionization potential. In Sec. 3 are given concrete estimates of the critical field for the physical region of the spectrum, when it is possible to investigate analytically the general equations of Sec. 2.

In the other limiting case, $\omega \ll 1/\tau_\epsilon$, the processes in the conduction band do not differ in main outline from the processes in a constant electric field [4]. The region $\omega \sim 1/\tau_\epsilon$, just like the region $\hbar\omega \ll I$, requires further investigation when the Fokker-Planck approximation is not applicable. In Sec. 3 we analyze also the dependence of the critical field on the temperature.

In Sec. 4 is presented the solution of the kinetic equation in the region $\epsilon > I$ (ϵ is the electron energy); this solution is necessary to analyze the correctness of the boundary conditions formulated in Secs. 2 and 3. In Sec. 5 will be investigated the question of the absorption of the energy by the conduction-band electrons, and the critical density n_{cr} of the number of electrons will be determined; by the same token, a breakdown criterion will be formulated. Finally, in Sec. 6 the effectiveness of the spatial diffusion of the electrons and its influence on the cascade ionization process are estimated.

Since the solution of the kinetic equation obtained in Secs. 2 and 3 leads to a very peculiar—very strong—dependence of the duration of the development of the process on the intensity of the electromagnetic radiation, we specially discuss the questions connected with this dependence in Sec. 7.

2. SOLUTION OF KINETIC EQUATION IN THE REGION $\epsilon < I$

We shall henceforth consider electromagnetic-radiation pulses of duration from 10^{-10} to 10^{-7} sec, so that recombination processes, which are apparently characterized by times $\tau_p > 10^{-7}$, can be neglected.

Assuming $\hbar\omega \ll I$, following the usual procedure of changing from the kinetic equation to a diffusion equation of the Fokker-Planck type [5,6], we can obtain the following equation for the electron energy distribution function $f(\epsilon, t)$:

$$-\frac{\partial}{\partial\varepsilon}(\tilde{g}(\varepsilon)\tilde{S}(\varepsilon,t))=N_i\tilde{g}(\varepsilon)\frac{\partial\tilde{f}(\varepsilon,t)}{\partial t}+\tilde{R}(\varepsilon,t;\tilde{f}). \qquad (1)$$

Here

$$\tilde{S}(\varepsilon, t)=-\tilde{D}_1(\varepsilon)\frac{\partial \tilde{f}(\varepsilon, t)}{\partial \varepsilon}-\tilde{Q}^0(\varepsilon)\tilde{f}(\varepsilon, t),$$

$\tilde{g}(\epsilon)\tilde{S}(\epsilon, t)$ is the flux of the electrons through an equal-energy surface ϵ, $\tilde{D}_1(\epsilon)$ is the coefficient of diffusion along the energy axis, $\tilde{Q}^0(\epsilon)$, as shown in [6], is the power loss due to the spontaneous emission of the phonons, $\tilde{g}(\epsilon)$ is the density of the number of states, $2N_i/3$ is the total number of states with energies $\epsilon \leq I$, and $\tilde{R}(\epsilon, t; \tilde{f})$ is the term describing the outflow or inflow of electrons as a result of ionization. The diffusion coefficient $\tilde{D}_1(\epsilon)$ can be reduced to the form [2,3]

$$\tilde{D}_1(\varepsilon)=\tilde{D}_0(\varepsilon)\tilde{q}^0(\varepsilon)(1+\tilde{q}(\varepsilon))=\tilde{D}(\varepsilon)(1+q(\varepsilon)) \tag{2}$$

and analogously

$$\tilde{Q}^0(\varepsilon)=\tilde{Q}_0^0(\varepsilon)\tilde{q}^0(\varepsilon), \tag{2a}$$

where $\tilde{D}_0(\epsilon)$ and $\tilde{Q}_0^0(\epsilon)$ are the corresponding coefficients in the absence of an electromagnetic field,

$$\tilde{q}^0(\varepsilon)=\frac{4}{3}\int_0^1(1-\zeta^3)J_0^2(H\zeta)\,d\zeta, \tag{3}$$

$$\tilde{q}(\varepsilon)=\frac{1}{\tilde{q}^0(\varepsilon)}\sum_i\left(\frac{l\hbar\omega}{v_s p(\varepsilon)}\right)^2\int_0^1(1-\zeta)J_l^2(H\zeta)\,d\zeta, \tag{3a}$$

$$H=2eEp(\varepsilon)/\hbar m\omega^2,$$

J_l is a Bessel function, v_s is the speed of sound, $p(\epsilon)$ is the quasimomentum of an electron with energy ϵ, E is the effective value of the electric field acting on the electron, and e and m are respectively the charge and effective mass of the electron. In this expression, account is taken of the contribution of the multiphoton processes in the conduction band to the diffusion coefficient along the energy axis. Strictly speaking, relations (2) and (3) are valid only in the high-temperature approximation, $kT \gg v_s p$, and in the low-temperature approximation ($kT \ll v_s p$) the expression for $\tilde{q}(\epsilon)$ acquires an additional factor 5/4, which will henceforth be disregarded.

The form of Eq. (1) is significantly different in the regions $\epsilon < I$ and $\epsilon > I$, inasmuch as at $\epsilon > I$ the term $\tilde{R}(\epsilon)$ is significant (it is essential to taken into account the outflow of the electrons as a result of ionization), it is precisely this process which governs primarily the distribution function, whereas in the region $\epsilon < I$ it is important only in the vicinity of the point $\epsilon = 0$ and can be taken into account in the boundary conditions [4].

From the solution of Eq. (1) in the region $\epsilon \geq I$ we can determine the quantity

$$\sigma=\tilde{S}(I)/\tilde{f}(I)Q^0(I). \tag{4}$$

If the quantity σ is known, Eq. (4) is in fact the first boundary condition for the region $\epsilon \leq I$. We shall determine σ in Sec. 4.

It will be convenient subsequently to change over to the dimensionless variable $x = \epsilon/I$ and use all the functions of ϵ, introduced above, as functions of x without a tilde, so that the coefficients D(x) and $Q^0(x)$ will have the dimension sec^{-1} after division by the corresponding power of I.

We seek the solution of (1) in the form

$$f(x, t)=e^{\gamma t}f(x). \tag{5}$$

In the time interval of interest to us, the condition

$$\gamma \ll Q^0(x), D(x)$$

($Q^0(1) \gg 10^{12}$ sec^{-1}) is satisfied with large margin, with the possible exception of the very narrow region $x < [\gamma/Q^0(1)]^2$, which turns out to be even narrower than the region where the distribution function is governed mainly by the influx of electrons as a result of ionization. The width of the latter is x_0, and formulation of the boundary conditions connected with this width will be discussed later. It follows from the foregoing that a good approximation for the rate γ of the cascade-ionization) process can be obtained from the following successive-approximation procedure:

In Eq. (1), after substituting (5), we put $\gamma_0 = 0$ and obtain the zero-order approximation for the distribution function $f_0(x)$; with the aid of the boundary conditions we determine the first approximation for γ_1 and obtain $f_1(x)$, substituting $\gamma_1 f_0(x)$ in the right-hand side of (1), etc.

In accord with the foregoing, we rewrite (1) in the form

$$\frac{\partial}{\partial x}\{g(x)[D(x)(1+q(x))f_0'(x)+Q^0(x)f(x)]\}=0, \tag{6}$$

so that

$$f_0'(x)+\frac{Q^0(x)}{D(x)(1+q(x))}f_0(x)=-\frac{S(1)g(1)}{g(x)D(x)(1+q(x))},$$

and, taking (4) into account, we obtain

$$f_0(x)=f(1)\exp\{-F(x)\}\left\{1-\sigma\int_1^x dy\exp\{F(y)\}\frac{g(1)Q^0(1)}{g(y)D(y)(1+q(y))}\right\}, \tag{7}$$

where

$$F(x)=\int_1^x\frac{Q^0(y)\,dy}{D(y)(1+q(y))}$$

We now consider the narrow region $x < x_0$. The exact form of the distribution function in this region does not play any role in the determination of γ, since $x_0 < 10^{-2} \ll 1$ (see (22)). For the flux $S_0 = gS$ we have

$$S(0, t)-S_0(x_0, t)=-2S_0(1, t). \tag{8}$$

In $S_0(0, t)$ it is possible to take formally into account the processes of nonradiative recombination [4]. At $S_0(0, t) = 0$ and $x_0 \to 0$, we obtain the usual "flux doubling" boundary condition [5,1]:

$$S_0(0)=2S_0(1). \tag{9}$$

Neglecting the recombination processes, the requirement $S_0(0) = 0$ is equivalent to the requirement that the function g(x)f(x) be finite at x = 0. This condition is obviously not satisfied by the boundary conditions (9), but again, owing to the smallness of x_0, the relative error turns out to be small—it is determined by the ratio

$$\int_0^{x_0}f(x)g(x)\,dx\Big/\int_{x_0}^1 f(x)g(x)\,dx,$$

which, as shown by concrete numerical calculations, does not exceed 5% at $x_0 < 10^{-2}$.

We next carry out the procedure of finding the first approximation for γ:

$$S_0(x_0)-S_0(1)=\gamma_1 N_i\int_{x_0}^1 g(x)f_0(x)\,dx, \quad x_0\to 0, \tag{10}$$

or, taking (8) and (4) into account,

$$\gamma_1=Q^0(1)\sigma\Big/\int_{x_0}^1 g(x)\frac{f_0(x)}{f_0(1)}\,dx, \quad x_0\to 0. \tag{11}$$

Relations (7) and (11) give the cascade-development constant in first-order approximation. If necessary, the next higher approximations are constructed analogously.

3. ESTIMATES OF THE CRITICAL FIELD IN THE VISIBLE PART OF THE SPECTRUM. DEPENDENCE OF THE CRITICAL FIELD ON THE TEMPERATURE

The formulas of Sec. 2 enable us to find, with the aid of numerical calculations, the dependence of the development constant of the cascade-ionization process on the value of the electric field in the electromagnetic wave in the entire range of frequencies satisfying the relation

$$h/\tau_e \ll \hbar\omega \ll I, \tag{12}$$

at arbitrary values of the dispersion. For concrete estimates, we shall dwell on the case

$$H \ll 1,$$

and then we obtain from (3a)

$$q^0 \approx 1,\ q \approx {}^1/_6 (eE/mv_s\omega)^2, \tag{3b}$$

which coincides with the analogous expression in [2]. Using the corresponding expressions for $\tilde{D}_0(\epsilon)$ and $\tilde{Q}_0^0(\epsilon)$ from [6] in the parabolic-band approximation, and also expressions (2) and (3b) of the present paper, we obtain in accordance with (7)

$$f_0(x) = f_0(1) \exp\left\{\frac{1-x}{\delta}\right\}\left(1 - \frac{\sigma}{\delta}\int_1^x \frac{1}{y^2}\exp\left\{\frac{y-1}{\delta}\right\}dy\right), \tag{13}$$

$$\delta = (1+q)\,kT/I \quad (kT \gg v_s p(\varepsilon));$$

$$f_0(x) = f_0(1)\exp\left\{2\frac{1-\sqrt{x}}{\delta_0}\right\}\left[1 - \frac{\sigma}{\delta_0}\int_1^x \frac{1}{y^{3/2}}\exp\left\{2\frac{\sqrt{y}-1}{\delta_0}\right\}dy\right], \tag{14}$$

$$\delta_0 = \frac{8mv_s}{5\sqrt{2mI}}(1+q) \quad (kT \ll v_s p).$$

The approximation (14), which corresponds to the "zero-point oscillations" of the lattice, can obviously not be used in the entire region $\epsilon < I$.

Bearing (11) in mind and estimating the obtained integrals at $\delta \ll 1$, we get

$$\nu = \frac{Q^0(1)\sigma e^{-1/\delta}}{\delta^{3/2}\Gamma(3/2)(1+r\sigma)} \quad (kT \gg v_s p). \tag{15}$$

Here $r \sim 1$,

$$Q^0(1) = 2m^2\mathscr{E}_1^2\sqrt{2mI}/\pi\rho\hbar^4,$$

ρ is the density of the dielectric, and $\mathscr{E}_1$ is the constant of the deformation potential. The limiting case $\delta \gg 1$ leads to an expression for γ which coincides exactly with the corresponding expression of [1]. (In the boundary condition (8) it is possible to introduce in obvious fashion the probability of the electron "jumping over" the excitation region.)

Let us make a few remarks and consider the consequences of the foregoing results.

1. The boundary-value problem of the "flux doubling" type and with $f(1) = 0$ leads in the high-temperature approximation, for a parabolic band, when the asymptotic solutions at $\delta \ll 1$ are considered, to the following equation:

$$\frac{\delta^2 e^{1/\delta}}{\Gamma(a\gamma/Q^0(1)\delta^{1/2})} = 1 \quad (\gamma/Q^0(1)\delta^{1/2} \ll 1),$$

or

$$\delta^{3/2} e^{1/\delta}\gamma = Q^0(1)a^{-1},$$

with the estimates $1/8 \le a \le 1/2$ obtained for a. Thus, the first approximation obtained above for γ can be somewhat underestimated (but not overestimated!) in comparison with the exact solution of the boundary-value problem of the type described in [5], which unfortunately cannot be expressed in terms of known functions.

2. We assume the following values of the parameters: $T = 300^\circ K$, $\rho = 3.8$ g/cm^3, $v_s = 8 \times 10^5$ cm/sec, $\mathscr{E}_1 = 11$ eV, $I = 9$ eV [1], $\hbar\omega = 1.17$ eV, $\tau = 3 \times 10^{-9}$ sec, $m = m_e$, $r = 1$, and $\sigma \to \infty$. Using the "40 generations" criterion, we obtain from (15) $E \approx 1.3 \times 10^7$ V/cm. Under these conditions and at $\tau = 3 \times 10^{-8}$ sec, the estimates [2] for the multiphoton processes turn out to be much higher. The dependence of γ on E in the form $\gamma \propto e^{-1/\delta}$ was indicated from qualitative considerations in [2], but it was concluded that the critical fields exceed by one order of magnitude those indicated above. Unfortunately, these results cannot be compared with the available experimental data, since the effective value of the electric fields in the experiments remains unknown, owing to the influence of self-focusing. We indicate only that the limiting subcritical fields in pure sapphire crystals amount, according to the latest data, to 8×10^6 V/cm at $\tau = 3 \times 10^{-9}$ sec.

3. The possible order-of-magnitude agreement between the estimates of the present paper and the critical values typical of short pulses of strong constant electric fields is not more than an accident, since in the case of optical fields an important role is assumed by the symmetrical part of the distribution function which, as is well known, makes no contribution to the flux along the energy axis in the case of a constant electric field.

4. Under the influence of the electromagnetic field, the average energy of the electrons becomes $\sim (3/2)kT(1+q)$. In fact, integrating by parts the expression

$$A = \int_0^1 \left(\sqrt{x}\, e^{-x/\delta}\int_1^x y^{-2}e^{y/\delta}\,dy\right)dx,$$

we obtain

$$A^{-1}I\int_0^1 \left(x^{3/4}e^{-x/\delta}\int_1^x y^{-2}e^{y/\delta}\,dy\right)dx = \frac{3}{2}kT(1+q) + \frac{2\delta I}{A} \approx \frac{3}{2}kT(1+q),$$

since $A \gg 1$. In many cases it turns out that $kT(1+q) \approx \epsilon_0$, where ϵ_0 is determined from the equation

$$(1+q)kT = v_s p(\varepsilon_0), \tag{16}$$

and the use of the high-temperature approximation may turn out to be unwarranted. As already noticed, it is impossible to solve the equation for f(x) in the approximation $kT \ll v_s p(\epsilon)$ for the entire conduction band; performing the corresponding cutoff near $x = 0$, we obtain the following estimate for $\delta_0 \ll 1$:

$$\gamma_1^0 = \frac{2Q^0(1)\sigma\exp\{-2/\delta_0\}}{\delta_0^3(1+3r\sigma)}. \tag{17}$$

In a field $E = 1.1 \times 10^7$ V/cm, we have $\gamma \approx 2 \times 10^9$ sec^{-1} and $\gamma^0 \approx 3 \times 10^{10}$ sec^{-1}. At $\gamma^0 = 2 \times 10^9$ sec^{-1}, the critical field is $E_{cr} = 9 \times 10^6$ V/cm in the "zero-point oscillation" approximation. It follows therefore that the high-temperature approximation can be used if $E_{cr.high\text{-}temp} < E_{cr.low\text{-}temp}$. This corresponds approximately to the criterion

$$T \gg T_1;\ kT_1 = v_s p[kT_1(1+q)], \tag{18}$$

the meaning of which is clear from the foregoing. At $I = 9$ eV, $m = m_e$ (the mass of the free electron), and $v_s \approx 10^6$ cm/sec, the high-temperature approximation is valid at $T \gg 300^\circ K$ at the frequency $\omega \approx 10^{15}$ sec^{-1}.

5. Since (17) gives a value that does not depend on the temperature, and the high-temperature approximation leads to the relation

$$E_{cr} \sim 1/\sqrt{T}, \tag{19}$$

we can expect a weak dependence of the critical field on the temperature up to the value T_1 defined by the condition (18); at higher temperatures, Eq. (19) is valid. To obtain more exact results in particular cases it is reasonable to solve the equation in two regions: $p(\epsilon_0) \lesssim kT/v_s$, after which the solutions are matched at the point ϵ_0 determined from (16).

6. At sufficiently high electron energies, the spectrum of the phonons with which they interact can be cut off not at the value $\sim 2p(\epsilon)$, as assumed above (see, e.g., [3]), but at the limiting value of the phonon quasimomentum q_{Deb}. If umklapp processes are forbidden, this leads to the following expressions:

$$D_0(\varepsilon) = \begin{cases} \dfrac{16 v_s m^3 \varepsilon_{cr}^{5/2} \mathscr{E}_1^2}{5\pi\rho\hbar^4 \varepsilon^{1/2}}, & kT \ll v_s p(\varepsilon_{cr}) \\ kT Q_0^0(\varepsilon), & kT \gg v_s p(I) \end{cases}; \tag{20}$$

$$Q_0^0(\varepsilon) = \frac{2\sqrt{2}\, m^{3/2} \varepsilon_{cr}^2 \mathscr{E}_1^2}{\pi\rho\hbar^4 \varepsilon^{1/2}}, \quad \varepsilon > \varepsilon_{cr},$$

where $\epsilon_{cr} = (k\Theta)^2/8mv_s^2$ and Θ is the Debye temperature. The solution of Eq. (1) with the indicated quantities leads approximately to the same values of E_{cr} as before.

4. SOLUTION OF KINETIC EQUATION IN THE REGION $\epsilon > I$

Our task is now to find the previously introduced parameter σ. We consider in somewhat greater detail the low-temperature approximation, since in most cases of interest the condition $kT < v_s p(I)$ is satisfied.

If we assume for the ionization probability an expression of the type (D4) of [4], then Eq. (1), after suitable substitution of (5) and neglecting the term $\sim \gamma/Q^0(1)$, reduces to

$$\frac{\partial}{\partial x}(\delta_0 x^{5/2} f'(x) + x^2 f(x)) = Q\sqrt{x}(x-1) f(x), \tag{21}$$

where the dimensionless quantity $Q \sim 10^{16}/Q^0(1)$, and under the previously employed conditions $Q \sim 3 \times 10^3$. We use the parabolic-band approximation. The value of δ_0 is defined by (14) and $\delta_0 \approx 0.15$ in fields on the order of 10^7 V/cm. Making the substitution

$$f(x) = \exp(\alpha y)\, y^\nu u(y), \quad y = \sqrt{x},$$
$$\nu = -3/2 + 2\eta i, \quad \alpha = -[1 + (1+4Q\delta_0)^{1/2}]/\delta_0,$$
$$\eta = (Q/\delta_0 - 9/16)^{1/2}$$

($Q/\delta_0 > 10^4$ and $\eta \approx \sqrt{Q/\delta_0}$), we reduce (21) to the standard form of the confluent hypergeometric equation

$$yu'' + \left[(1+4\eta i) - 2y\frac{(1+4Q\delta_0)^{1/2}}{\delta_0}\right]u' - \frac{1}{\delta_0}[(1+4Q\delta_0)^{1/2}(1+4\eta i) - 4]u = 0.$$

The solution f(x) of the initial equation with the asymptotic form

$$f(x) \to 0 \text{ as } x \to \infty$$

takes the form

$$f(x) = C\left(\frac{2(1+4Q\delta_0)^{1/2}}{\delta_0}\right)^{2\eta i} (\sqrt{x})^{-3/2+2\eta i} \exp\left\{-\frac{1+(1+4Q\delta_0)^{1/2}}{\delta_0}\right\} \times \Psi\left(\frac{1+4\eta i}{2} - \frac{2}{(1+4Q\delta_0)^{1/2}},\ 1+4\eta i;\ \frac{2[(1+4Q\delta_0)x]^{1/2}}{\delta_0}\right). \tag{22}$$

We note that the region where f(x) differs significantly from zero has a width $\Delta x \sim (Q/\delta_0)^{1/2} \simeq 10^{-2}$, which justifies the assumption made above concerning the smallness of the quantity x_0 (see [8]).

For the quantity σ we have

$$\sigma = -(\delta_0 f' + f)/f|_{x=1},$$

with

$$\Psi'(a,c;z) = \Psi(a,c;z) - \Psi(a,c+1;z).$$

To estimate σ under conditions when $2/(1 + 4Q\delta_0)^{1/2} \ll 1/2$ or $Q\delta_0 \gg 4$, we write [7]

$$f(x) = C \exp\left(-\frac{\sqrt{x}}{\delta_0}\right) x^{-1/4} K_{2\eta i}\left(\left(\frac{1+4Q\delta_0}{\delta_0^2}x\right)^{1/2}\right). \tag{22a}$$

It is impossible to put $(1 + 4Q\delta_0)^{1/2} \approx 2(Q\delta_0)^{1/2}$ directly in the argument; we have neglected in fact only the quantity $1/Q$ compared with unity, since the first parameter of the function Ψ contains a pure imaginary quantity with large absolute value η, and the ratio of the neglected quantity to the modulus of the remaining quantity is $1/Q \ll 1/Q\delta_0$. We can use next the asymptotic form of the Bessel function of imaginary argument and pure imaginary order $K_{ip}(z)$ as $p \to \infty$, $z \to \infty$ and at $z > p$ (in our case $z^2 - p^2 \sim \delta_0^{-2}$). From the expansion of the hypergeometric function in Bessel functions it follows also that to change over from (22) to (22a) it is necessary to satisfy the condition $\delta_0 \gg (2Q)^{-1/2}$. Under the indicated assumptions, which are usually valid at the parameter values of interest to us, we obtain the estimates

$$\sigma \sim Q\delta_0^{2-1/2}, \tag{23}$$

and in most cases it can be assumed that $\sigma \gg 1$.

In the high-temperature approximation we can obtain in analogy with the foregoing

$$f(x) = C \exp\left(\frac{-1-(1+4Q\delta)^{1/2}}{2\delta}\right) \Psi\left(-\frac{Q+1}{(1+4Q\delta)^{1/2}} + 1,\ 2;\ \frac{(1+4Q\delta)^{1/2}}{\delta}x\right)$$

(the ionization probability is written in the form $\sim (x-1)x^{-1/2}$). Under the conditions $\delta \ll 1$, $Q \gg 1$, and $Q\delta \gg 1$, we can use the asymptotic form of the confluent hypergeometric function $\Psi(a, c, z)$ as $a \to \infty$, $z \to \infty$ (see [7], formulas (6), (13), and (25)). For σ we obtain

$$\sigma \approx \frac{1}{2}\sqrt{Q\delta}\left[1 - \frac{K_{1/4}(\xi_2)}{K_{1/4}(\xi_1)}\left(\frac{\xi_2}{\xi_1}\right)^{1/4} - \frac{1}{\sqrt{Q\delta}}\right],$$

where

$$\xi_1 = \frac{(1-4\delta)^{3/2}}{12Q\delta^2}\left(1 + O\left(\frac{1}{Q\delta}\right)\right),$$

$$\xi_2 = \xi_1\left(\frac{1-4\delta+4\delta(1+4Q\delta)^{1/2}}{1-4\delta}\right)^{3/2}.$$

At $Q = 3.5 \times 10^3$ and $\delta = 0.08$ we get $\sigma \approx 3$.

We note also that the "no-threshold ionization" discussed in [2], can play an important role in the process under consideration. First, it can lead to an increase of the parameter σ. Second, it decreases, as it were, the ionization potential and one can speak of an "effective ionization potential"

$$I_{eff} = I - l\hbar\omega,$$

where the quantity l characterizes the order of the multiquantum processes that lead to an appreciable ionization probability in the given fields. For fields $\sim 10^7$ V/cm we have $l = 1$–2.

5. ENERGY ABSORPTION AND LATTICE HEATING. BREAKDOWN CRITERION

The amount of absorbed energy per unit time, as seen from (1), can be expressed in the following form (high-

temperature approximation, parabolic band):

$$\frac{\partial}{\partial t}\left(IN_i\int x^{1/2}f(x,t)\,dx\right)_+ = N_iQ^0(1)I\delta\int x\frac{\partial}{\partial x}\left(x^2\frac{\partial f}{\partial x}\right)dx \qquad (24)$$
$$=2N_iQ^0(1)I\delta\int xf(x)\,dx\approx 2NQ^0(1)I\delta^{1/2}.$$

We have integrated twice by parts, using the relation $N = N_i\int\sqrt{x}\,f(x)dx$ and the properties of the integrals (13). It was assumed here that $f(x) \to 0$ and $\partial f/\partial x \to 0$ at $x \to \infty$, i.e., the matching at the point $x = 1$ was effected in the correct manner, and we have then neglected the integrals in the interval $x \geq 1$. The + subscript in (24) indicates that it is necessary to take the term responsible for the energy absorption from the left-hand side of (1). The fraction of this energy is of the order $\gamma/Q^0(1)$ and goes to increase the number of particles in the system, while the remainder, which is practically equal to the expression written above, goes to heating of the phonon field (and to acoustic phenomena of all kinds).

Let us estimate the number of electrons needed for an appreciable heating of the crystal. Neglecting the diffusion of the heat from the interaction region, we have

$$\frac{dT}{dt}=\frac{2Q^0(1)In}{C\rho}\left(\frac{\delta}{T_0}\right)^{1/2}T^{1/2},\quad \frac{dn}{dt}=\gamma n. \qquad (25)$$

Here C is the specific heat of the lattice, T_0 is the initial temperature, the value of δ was calculated at the temperature T_0, so that the temperature does not enter in practice in the round brackets).

The solution of the system of equations (25), which determines the dependence of the temperature on the time, has a vertical asymptote ($T \to \infty$ as $t \to t_1$). It is reasonable to assume that the pulse duration τ necessary for the damage is $\tau = t_1 - t_0$ (t_0 corresponds to the start of the action of the pulse). We then obtain the following equation for the determination of the cascade-development constant γ (and, in fact, the correct damage criterion):

$$\gamma\tau=\ln\left(1+\frac{\gamma C\rho}{Q^0(1)I(\delta/T_0)^{1/2}T_0^{1/2}}\frac{1}{n_0}\right). \qquad (26)$$

Here n_0 is the initial concentration of the electrons, and γ as a function of the field is determined by the relation (11).

At $T_0 = 300°K$ and $Q^0 = 7 \times 10^{12}$ sec^{-1} (under the conditions described in Sec. 3 above) we have $\delta = 0.08$, which corresponds at the indicated temperature to a field $E \approx 10^7$ V/cm, $C = 0.18$ cal/g-deg (sapphire), we have $\gamma \approx 10^9$ sec^{-1} and $\gamma\tau \approx \ln(4\times 10^{18}/n_0)$.

The occasionally employed "$\gamma\tau = 40$" criterion corresponds, as we see, to an initial electron-number density $n_0 \approx 20$ cm^{-3}, and the "40 generation" criterion ($2^{40} \approx 10^{12}$) corresponds to $n_0 \approx 10^6$ cm^{-3}. In real crystals we can expect values 10^{11}–10^{13} cm^{-3} and even higher, owing to the single-photon ionization of the impurities or to heating of the absorbing defects.

6. SPATIAL DIFFUSION OF THE ELECTRONS

The coefficient of spatial diffusion of the electrons turns out to be relatively small, even if we disregard the small mobility of the holes:

$$D=\frac{1}{3}l_{ae}v_{ep}\sim l_{ae}[(1+q)kT/m]^{1/2}. \qquad (27)$$

Here l_{ae} is the average electron mean free path. At $T = 300°K$, $E = 10^7$ V/cm, and $l_{ae} \approx 10^{-6}$ cm we obtain $D \approx 30$ cm^2/sec. However, since the time of development of the cascade-ionization process depends very strongly on the values of the field E,

$$\gamma^{-1}\sim 10^{-15}\exp(G/E^2)\ \text{sec},$$
$$G=\begin{cases}\dfrac{6I}{kT}\left(\dfrac{mv_s\omega}{e}\right)^2, & kT\gg v_sp(\varepsilon)\\[2ex] \dfrac{15\sqrt{2mI}}{2mv_s}\left(\dfrac{mv_s\omega}{e}\right)^2, & kT\ll v_sp(\varepsilon)\end{cases}, \qquad (28)$$

it follows that at any spatially non-uniform distribution of the density of the electromagnetic radiation there can occur, as the cascade develops, initial electron-density gradients $\partial n/\partial r$ (r is the spatial coordinate), and this gives rise to more or less strong diffusion fluxes.

We consider by way of estimate a cylindrically-symmetrical problem. The equation for the electron-multiplication process can be written in the form

$$\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}D\right)+r\gamma(r)n(r)=\frac{\partial}{\partial t}(rn(r,t)). \qquad (29)$$

Let the distribution of the field in the beam be given in the form

$$E^2=E_0^2\exp(-4r^2/d^2),$$

and then

$$q=q_0\exp(-4r^2/d^2)$$

(d is the diameter of the light spot), and let the process develop within the limits $R < d/2$. Integrating (29) term by term we obtain

$$\left(r\frac{\partial n}{\partial r}D\right)_{r=R}+\int_0^R r\gamma(r)n(r,t)\,dr=\frac{\partial}{\partial t}\int_0^R rn(r,t)\,dr. \qquad (30)$$

The quantity

$$\frac{\partial}{\partial t}2\pi l\int_0^R rn(r,t)\,dr$$

describes, obviously, the change in the total number of particles in a cylinder of radius R and length l. If the diffusion process is "turned off," then at the instant of time t we have $n = n_0\exp\{\gamma(r)t\}$, and the first term in (30) becomes

$$\left(r\frac{\partial n}{\partial r}D\right)_{r=R}\approx RD\left(\frac{\partial n}{\partial\gamma}\frac{\partial\gamma}{\partial q}\frac{\partial q}{\partial r}\right)_{r=R}\approx -\frac{8R^2DtGn(r)\gamma(r)}{d^2E^2}.$$

We apply to the second integral the mean-value theorem

$$\int_0^R r\gamma(r)n(r,t)=\frac{1}{2}n(R\xi)\gamma(R\xi)R^2.$$

Here $0 < \xi < 1$. It is clear that the diffusion processes are significant when both terms of the left-hand side of (30) become approximately equal to each other within the time of action of the pulse $t = \tau$, i.e., when

$$\frac{16DG\tau}{E^2d^2}\frac{n(R)\gamma(R)}{n(R\xi)\gamma(R\xi)}\approx 1. \qquad (31)$$

We introduce the quantity d_{cr} from relation (31) at $\xi = 1$:

$$d_{cr}^2=16DG\tau/E^2.$$

Since we always have $n(R)\gamma(R)/n(R\xi)\gamma(R\xi) < 1$, it follows that at $d > d_{cr}$ the condition (31) cannot be satisfied, and the diffusion processes are insignificant. Then $n(0) \gg n(d/2)$ and $R \ll d/2$. To the contrary, at $d \ll d_{cr}$, the spatial diffusion leads to a slowing down of the electron multiplication process, the process development time becomes dependent on the diameter of the light spot, $n(r) \sim n(0)$ in the entire region $r \leq d/2$, and experiment should reveal a complete cutoff of the momentum of the transmitted radiation as a result of the strong absorption of

the electromagnetic energy by the electrons in the entire volume of the caustic.

Under the conditions described in Sec. 3, $d_{cr} \approx 2 \times 10^{-3}$ cm and we can expect damage to occur in a region with radius on the order of several microns.

7. DISCUSSION OF RESULTS

We note that under the conditions of Sec. 3, at a field value 7.5×10^6 V/cm, the cascade-development constant is approximately 3×10^7 sec^{-1}, and in a field 9×10^6 V/cm we have $\gamma \approx 3 \times 10^9$ sec^{-1}, whereas at 1.2×10^7 V/cm we have $\gamma \approx 3 \times 10^{11}$ sec^{-1}. We have used (17), i.e., the low-temperature approximation. We see that small changes in the effective field in the light wave give rise to appreciable changes in the duration of the process. Resorting to the usual terminology, but which we see to be quite inaccurate in the present situation, we can state that the critical field is practically independent of the pulse duration in a rather wide range.

Let us examine in greater detail the process of the occurrence and development of the cascade in a real experiment.

With increasing field in the pulse—the leading front—multiphoton ionization takes place (in real crystals, in addition, single-photon ionization of the impurities takes place), and E reaches a value E_{cr} such that the process terminates within a time on the order of the entire duration of the pulse; then, after a very short time interval $\Delta t_1 \ll \tau$, we get $E > E_{cr}$ and the cascade-development process, after abrupt acceleration, terminates within a time $\Delta t \ll \tau$—the pulse of the transmitted radiation is cut off. The time of formation of the "cutoff" will depend mainly on the slope of the pulse front. Thus, cascade ionization is characterized by cutoff on the leading front of the electromagnetic-radiation pulse, and its duration depends on the rise time. This situation is observed in laser damage of pure samples and is apparently connected with the indicated feature of the development of the electron cascade.

The high-temperature approximation leads to analogous results.

As we have seen in Sec. 3, at sufficiently high temperatures we can expect the square of the critical field to be inversely proportional to the temperature. We note that it follows from the results of Sec. 5 that a similar dependence appears also in the case when the damage to the dielectric is due to purely thermal phenomena that are connected with the appearance of a large number of electrons in the conduction band (breakdown of a dielectric containing many impurities in an optical field).

From the results of Sec. 3 we can draw no conclusion concerning the character of the dependence of the critical field on the frequency. The analysis carried out there is valid only for a narrow region of frequencies, bounded by the conditions

$$2eEp/m\omega \ll \hbar\omega \ll I,$$

the first of which is not sufficiently well satisfied even in the case of a neodymium laser. Preliminary calculations in accordance with the general formulas of Sec. 2, as well as an analysis of expressions (3), shows that one can expect a rather weak dependence on the frequency in the region $\omega < 10^5$ sec^{-1}. On the other hand, the region $\hbar\omega \lesssim I$, as indicated in the Introduction, calls for special investigations. It can be assumed, however, that the possibility of ionization by an electron after absorption of only two or three light quanta leads approximately to the same thresholds as in the considered frequency band.

In conclusion, the author is sincerely grateful to academician A. M. Prokhorov for a valuable discussion and for interest in the work, and also A. A. Manenkov and Yu. K. Danileĭko for constant help during its preparation.

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Translated by J. G. Adashko
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