Plasma echo in metals

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(Submitted April 24, 1974)
Zh. Eksp. Teor. Fiz. 67, 1843–1858 (November 1974)

A theory of spatial plasma echo (nonlinear penetration of a wave through a metal) is developed for transverse electromagnetic waves under infrared-optics conditions and for an arbitrary Fermi surface. In an anisotropic crystal, plasma echo arises even in the second-order perturbation theory in the wave amplitude. The effect is due to specular reflection of the electrons from the metal boundary. Corrections to the surface impedance, due to electron scattering by the metal surface, are calculated in the first order in the amplitude. This extends the results of Gurzhi, Azbel', and Hao [3] to include the case of an arbitrary specularity coefficient. A metal with an ellipsoidal Fermi surface is considered separately. The result in this case is simpler and can be subjected to experimental study.

1. INTRODUCTION

The question of plasma echo for transverse electromagnetic waves in an isotropic medium, such as a plasma in a pure metal with spherical Fermi surface, was considered in[1,2]. It is known that in the case of an isotropic medium the echo appears only in third order in the wave field amplitude. The main result of the present paper is that when electromagnetic waves pass through a metal, an effect of theplasma-echo type should appear already in second order in the field amplitude.

Assume that a transverse electromagnetic wave $\mathbf{E}_1$ of frequency ω_1 is incident from the left on the surface 1 of a metallic plate of thickness d (see Fig. 1). We apply from the right a field $\mathbf{E}_2$ of frequency ω_2 to the surface 2 of the plate. We assume satisfaction of rather stringent conditions, which can be expressed in the form of the following chain of inequalities:

$$v_0/\omega_0=\lambda_D\ll v_0/\omega_i\ll\delta_i\ll d\ll l,\quad i=1,\ 2 \qquad (1.1)$$

(see[2,3], Sec. 1).

Here $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the plasma frequency, m is the effective mass of the electron, $v_0 = hm^{-1}(3\pi^2 n)^{1/2}$ is the Fermi velocity[1)], λ_D is the Debye length, $\delta_i \sim c/\omega_0$ is the depth of penetration of the field $\mathbf{E}_1$ in the metal, and $l = v_0/\nu_{eff}$ is the electron mean free path in the metal.

The inequality $\delta_i \ll d$ means that the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ inside the metal are significant in the linear approximation only near the corresponding surfaces of the plate, and do not penetrate into the interior. In the next higher approximation, however, one can observe near the surface 1 a field $\mathbf{E}^e$ of frequency $\omega_1 - \omega_2$, the amplitude of which is proportional to E_1E_2 and has a sharp maximum at $\omega_2 = 2\omega_1$. The echo is produced by the electrons perturbed by the field $\mathbf{E}_1$ at the surface 1, and then reflected specularly from the surface 2. Of course, a similar effect can be produced at $\omega_2 = \omega_1/2$, near the surface 2, by the electrons specularly reflected from the surface 1.

We direct the x axis along the inward normal to the surface 1. In this case, if the Fermi surface of the metal is a sphere, then the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ (and also $\mathbf{H}_1$ and $\mathbf{H}_2$) are transverse, and the normal components E_{1x} and E_{2x} on the exterior and on the interior of the metal are equal to zero. Since it is impossible to make up a third transverse vector from two transverse vectors in an isotropic space, the echo is produced only in third order in the amplitude (see[2]). The situation is

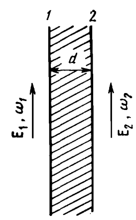

FIG. 1

different in a metal with arbitrary Fermi surface. Indeed, from the equation of the continuity of the normal component of the vector D on the surface of the metal it follows that

$$D_x=\hat{\varepsilon}_{xx}E_x+\hat{\varepsilon}_{xy}E_y+\hat{\varepsilon}_{xz}E_z=0 \quad \text{if} \quad x=0,\ d.$$

Consequently the longitudinal component of the field is given by the formula

$$E_x=-\hat{\varepsilon}_{xx}^{-1}(\hat{\varepsilon}_{xy}E_y+\hat{\varepsilon}_{xz}E_z)$$

and generally speaking is not equal to zero. An exception is the case when the x axis coincides with the principal axis of the tensor $\hat{\epsilon}_{\mu\nu}$. Since the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ inside the metal are no longer transverse, the plasma echo is produced in second order in the amplitude. Gurzhi et al.[3] have constructed a linear theory of the penetration of a high-frequency field into a metal occupying a half-space. It was assumed there that the inequalities (1.1) are satisfied[2)]. The problem was solved for an arbitrary electron dispersion $\epsilon = \epsilon(\mathbf{p})$. It was shown that the normal component of the $E_x(x)$ changes in the interior of the metal over a distance on the order v_0/ω, in contrast to the components $E_y(x)$ and $E_z(x)$, which attenuate monotonically over considerably larger distances of the order of $\delta \sim c/\omega_0$. However, it is impossible to use directly the results of[3] as initial equations for the solution of our problem, since the electron scattering by the surface was assumed in[3] to be diffuse. A plasma echo is the result of electrons reflected specularly from the surface, and the presence of these electrons must be taken into account by means of an appropriate boundary condition.

2. INITIAL EQUATIONS. LINEAR EQUATION IN THE WAVE FIELD AMPLITUDE

For the sake of convenience we change over to the dimensionless quantities

$$\boldsymbol{\rho}=\frac{\mathbf{p}}{mv_0},\quad \mathbf{u}=\frac{1}{v_0}\frac{\partial\varepsilon(\mathbf{p})}{\partial\mathbf{p}}.$$

The equation for the Fermi surface in ρ space takes the form

$$g(\rho)=1, \tag{2.1}$$

where $g(\rho) = \epsilon_F^{-1}\epsilon(mv_0\rho)$ and ϵ_F is the Fermi energy. Assuming $m = \hbar^2(3\pi^2 n)^{2/3}/2\epsilon_F$, we obtain the relation

$$\mathbf{u}={}^1/_2\partial g(\rho)/\partial\rho. \tag{2.2}$$

We consider first the distribution of the field near the surface 1. Just as in[3], we introduce the dimensionless coordinate $\xi = x(v_0/\omega_1)^{-1}$, express the electric-field intensity in the form $\mathbf{E}_1 = \mathbf{E}_1(\xi)\exp(i\omega_1 t)$, and express the continuous increment to the distribution function in the form

$$f-f_0=\frac{ev_0}{\omega_1}\psi_1(\xi,u)\frac{\partial f_0}{\partial\varepsilon}\exp(i\omega_1 t),$$

where

$$\frac{\partial f_0}{\partial\varepsilon}=-\delta(\varepsilon-\varepsilon_F)=-\frac{1}{\varepsilon_F}\delta[g(\rho)-1].$$

The current density is determined from the formula

$$j_\mu=-\frac{2e}{(2\pi\hbar)^3}\int v_\mu f d^3\mathbf{p}=\frac{\omega_1\eta_1}{4\pi}\frac{3}{4\pi}\oint\frac{\psi_1 u_\mu}{u}dS\exp(i\omega_1 t), \tag{2.3}$$
$$\mu=x,\ y,\ z;\quad \eta_1=\omega_0^2/\omega_1^2.$$

The intrgration occurs here over the surface (2.1), and u_μ/u is the μ-component of a unit vector directed along the normal to the surface. The system of initial equations consists of the kinetic equation, Maxwell's equations, and the boundary conditions.

The linearized kinetic equation takes the form

$$\frac{\partial}{\partial\xi}\psi_1+\frac{i}{u_x}\psi_1=\frac{u_\mu}{u_x}E_{1\mu}(x). \tag{2.4}$$

The tangential components of the field are described by a wave equation in which we neglect the displacement current:

$$\frac{d^2E_{1\alpha}}{d\xi^2}=\frac{4\pi}{\omega_1^2}\left(\frac{v_0}{c}\right)^2\frac{\partial j_\alpha}{\partial t},\quad E_{1\alpha}(0)=E_{1\alpha}^0,\quad \alpha=y,z. \tag{2.5}$$

The longitudinal component of the field is obtained from the condition

$$j_x(\xi)=0. \tag{2.6}$$

Let us formulate the boundary conditions for the distributuion function. As usual, we denote by ψ_{1+} and ψ_{1-} the values of the function ψ_1 for $u_x > 0$ and $u_x < 0$, respectively. Unlike in[3], we assume that part of the electrons is scattered from the surface 1 specularly. This means that when these electrons are reflected from the surface, no changes occur in their energy and in their tangential component of the quasimomentum ρ_T (see[5-7] and the review[8]). For simplicity, we assume a one-to-one correspondence between the state of the incident (ψ_+) and reflected (ψ_-) electron. If the straight line ρ_T = const is so directed that the period of the translational symmetry in this direction is equal to one of the principal periods of the reciprocal lattice, and if any such line crosses within the limits of the period the equal-energy surface at only two points, then the one-to-one correspondence indicated above does obviously indeed takes place. We note that the average surface density of the metal is in this case a crystal plane that can be chosen as the basal plane of the crystal.

The uniqueness of the transition from the given state to a specular state makes it possible to introduce in unique fashion a function that realizes this transition, i.e.,

$$u_\mu^*=W_\mu(\mathbf{u}). \tag{2.7}$$

Here the vector function $\mathbf{W}(\mathbf{u})$ realizes a transition from the velocity $\mathbf{u}$ in the given state to the velocity $\mathbf{u}^*$ in the mirror state. The function $\mathbf{W}(\mathbf{u})$ has the following obvious properties:

$$\operatorname{sign}W_x(\mathbf{u})=-\operatorname{sign}u_x, \tag{2.8}$$
$$W_\mu[\mathbf{W}(\mathbf{u})]=u_\mu, \tag{2.9}$$
$$W_\mu(-\mathbf{u})=-W_\mu(\mathbf{u}), \tag{2.10}$$

and the last relation follows from the central symmetry of the Fermi surface.

The formulas for the transition from ρ to ρ^* are

$$\rho_x^*=f(\rho),\quad \rho_y^*=\rho_y,\quad \rho_z^*=\rho_z. \tag{2.11}$$

Henceforth, when integrating over phase space, it will be necessary to change from the variables ρ_x, ρ_y, ρ_z to the variables ρ_x^*, ρ_y^*, ρ_z^*. It is easy to calculate the Jacobi determinant of this transformation. Using the invariance of $g(\rho)$, and also relations (2.11) and (2.2), we obtain

$$\frac{\partial(\rho_x,\rho_y,\rho_z)}{\partial(\rho_x^*,\rho_y^*,\rho_z^*)}=\frac{\partial\rho_x}{\partial\rho_x^*}=\frac{\partial g}{\partial\rho_x^*}\Big/\frac{\partial g}{\partial\rho_x}=\frac{u_x^*}{u_x}. \tag{2.12}$$

Consequently the transition from dS to dS* in the integrals over the surface (2.1) should be realized in the following manner:

$$dS_x=\frac{u_x dS}{u}=2u_x\frac{d\rho_x d\rho_y d\rho_z}{dg(\rho)}$$
$$=2u_x\left|\frac{\partial(\rho_x,\rho_y,\rho_z)}{\partial(\rho_x^*,\rho_y^*,\rho_z^*)}\right|\frac{d\rho_x^* d\rho_y^* d\rho_z^*}{dg(\rho^*)}=-\frac{u_x^* dS^*}{u^*}=-dS_x^*. \tag{2.13}$$

The relation $dS_x = -dS_x^*$ is obvious even from geometrical considerations.

We can now introduce the boundary conditions. On the surface 1, the Fuchs boundary conditions take the form

$$\psi_{1+}(0,\mathbf{u})=p_1\psi_{1-}(0,\mathbf{u}^*). \tag{2.14}$$

Here p_1 is the specularity coefficient for the reflection from surface 1. The second boundary condition, as usual, is

$$\psi_{1-}(+\infty,\mathbf{u})=0. \tag{2.15}$$

From (2.12), (2.14), (2.16), and (2.3) we get

$$\psi_{1-}(\xi,\mathbf{u})=-\frac{1}{u_x}\int_\xi^{+\infty}d\xi'\mathbf{E}_1(\xi')\mathbf{u}\exp\left[\frac{i}{u_x}(\xi'-\xi)\right],$$
$$\psi_{1+}(\xi,\mathbf{u})=\frac{1}{u}\int_0^\xi d\xi'\mathbf{E}_1(\xi')\mathbf{u}\exp\left[\frac{i}{u_x}(\xi'-\xi)\right]$$
$$-p_1\frac{1}{u_x^*}\int_0^{+\infty}d\xi'\,\mathbf{E}_1(\xi')\mathbf{u}^*\exp\left[i\left(\frac{\xi'}{u_x^*}-\frac{\xi}{u_x}\right)\right], \tag{2.16}$$

$$j_\mu(\xi)=\frac{\omega_1\eta_1}{4\pi}\int_0^\infty d\xi'[L_{\mu\nu}(|\xi-\xi'|)-p_1G_{\mu\nu}(\xi,\xi')]E_\nu(\xi'),$$
$$L_{\mu\nu}(\xi)=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\mu u_\nu}{u_x}\exp\left(-i\frac{\xi}{u_x}\right)\frac{dS}{u}, \tag{2.17}$$
$$G_{\mu\nu}(\xi,\xi')=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\mu u_\nu^*}{u_x^*}\exp\left[-i\left(\frac{\xi}{u_x}-\frac{\xi'}{u_x^*}\right)\right]\frac{dS}{u},$$
$$\mu,\ \nu=x,\ y,\ z.$$

In analogy with[3], we obtain from (2.6)

$$E_{1x}(\xi)=iE_\alpha(\xi)\left[\chi_{1\alpha}(\xi)+i\frac{K_{\alpha x}(0)}{K_{xx}(0)}\right],\quad \alpha=y,z\,(E_\alpha\approx\text{const});\tag{2.18}$$

$$\int_0^\infty d\xi'[L_{xx}(|\xi-\xi'|)-p_1G_{xx}(\xi,\xi')]\chi_{1\alpha}(\xi')=-K_{x\alpha}(\xi)+\frac{K_{\alpha x}(0)}{K_{xx}(0)}K_{xx}(\xi)$$

$$-p_1\left[-D_{x\alpha}(\xi)+\frac{K_{\alpha x}(0)}{K_{xx}(0)}D_{xx}(\xi)\right];\tag{2.19}$$

$$K_{\mu\nu}(\xi)=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\mu u_\nu}{u}\exp\left(-i\frac{\xi}{u_x}\right)dS,$$

$$D_{\mu\nu}(\xi)=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\mu u_\nu^*}{u}\exp\left(-i\frac{\xi}{u_x}\right)dS.$$

The constant term in (2.18) is chosen such that $\chi_{1\alpha}(\xi)$ vanishes at infinity. The right-hand side of (2.19) vanishes at $\xi=0$. Indeed,

$$D_{x\nu}(\xi)=\frac{3}{4\pi}\int_{u_x<0}\frac{u_\nu^* u_x}{u}dS\exp\left(i\frac{\xi}{u_x}\right)=-\frac{3}{4\pi}\int_{u_x<0}\frac{u_\nu^* u_x^*}{u^*}dS^*\exp\left(i\frac{\xi}{u_x}\right)$$

$$=-\frac{3}{4\pi}\int_{u_x>0}\frac{u_\nu u_x}{u}dS\exp\left(i\frac{\xi}{u_x^*}\right).\tag{2.20}$$

Therefore $D_{x\nu}(0)=-K_{x\nu}(0)$. In exactly the same way, using the central symmetry of the Fermi surface and relation (2.13), we easily get

$$G_{\mu\nu}(\xi,\xi')=C_{\nu\mu}(\xi',\xi).$$

Thus, the kernel in (2.19) is symmetrical.

In the derivation of Eq. (2.19) we took into account the fact that the tangential components of the field, as can be seen from (2.5) change significantly only at $\xi\sim\xi_1=c\omega_1/v_0\omega_0\gg 1$. From (2.17) we get

$$j_\alpha=-2i\frac{\omega_1\eta_1}{4\pi}K_{\alpha\mu}(0)E_{1\mu}(\xi),\quad \xi\gg 1;\ \alpha=y,z.$$

Hence, taking (2.18) into account, we obtain Eqs. (2.5) in the form

$$\frac{d^2E_{1\alpha}}{d\xi^2}=2\eta_1\left(\frac{v_0}{c}\right)^2B_{\alpha\beta}E_{1\beta},\quad E_{1\alpha}(0)=E_{1\alpha}^0,$$
$$B_{\alpha\beta}=K_{\alpha\beta}(0)-\frac{K_{\alpha x}(0)K_{\beta x}(0)}{K_{xx}(0)};\quad \alpha,\beta=y,z.\tag{2.21}$$

Solving Eqs. (2.21), we can find the surface-impedance tensor in the zeroth approximation $1/\xi_1$ (see, e.g.[9], formulas (2) and (17)). In the next approximation in $1/\xi_1$ it is easier to find the correction to the reciprocal of the surface impedance, by using formulas (2.17), (2.18), (2.19) and the relation

$$(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta}E_\beta=\frac{v_0}{\omega_1}\int_0^\infty[j_\alpha(\xi)-j_\alpha(\infty)]d\xi.$$

The expression obtained in this case takes the form

$$(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta}=\frac{v_0\eta_1}{4\pi}\left\{Q_{\alpha\beta}-\frac{K_{\alpha x}(0)Q_{\beta x}+K_{\beta x}(0)Q_{\alpha x}}{K_{xx}(0)}+\frac{K_{\alpha x}(0)K_{\beta x}(0)Q_{xx}}{K_{xx}^2(0)}\right.$$
$$+\int_0^\infty\int_0^\infty L_{xx}(|\xi-\xi'|)\chi_{1\alpha}(\xi')\chi_{1\beta}(\xi)d\xi'd\xi-p_1\left[R_{\alpha\beta}-\frac{K_{\alpha x}(0)R_{\beta x}+K_{\beta x}(0)R_{\alpha x}}{K_{xx}(0)}\right.$$
$$\left.\left.+\frac{K_{\alpha x}(0)K_{\beta x}(0)R_{xx}}{K_{xx}^2(0)}+\int_0^\infty\int_0^\infty G_{xx}(\xi,\xi')\chi_{1\alpha}(\xi')\chi_{1\beta}(\xi)d\xi'd\xi\right]\right\},$$

$$Q_{\alpha\beta}=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\alpha u_\beta u_x}{u}dS,\quad R_{\alpha\beta}=R_{\beta\alpha}=\frac{3}{4\pi}\int_{u_x>0}\frac{u_\alpha u_\beta^* u_x}{u}dS.\tag{2.22}$$

The tensor $(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta}$, as expected, is symmetrical. In the derivation we used the relation (2.20). At $p_1=0$ formula (2.22) coincides with the expression obtained in[3] for $(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta}$ in the case of diffuse scattering of electrons by a surface. The answer obtained in[3] for the case of specular reflection is incorrect.

3. SECOND-ORDER APPROXIMATION. PLASMA ECHO

Substituting (2.18) in (2.16) we obtain as $\xi\to+\infty$

$$\psi_{1+}(\xi,\mathbf{u})=iE_{1\alpha}^0[w_\alpha+\theta_{1\alpha}(u_x)-p_1w_\alpha^*-p_1\theta_{1\alpha}(u_x^*)]\exp\left(-i\frac{\xi}{u_x}\right),\quad \xi\gg\xi_1;$$

$$\theta_{1\alpha}(u_x)=\int_0^\infty\chi_{1\alpha}(\xi)\exp\left(i\frac{\xi}{u}\right),\quad w_\alpha=u_\alpha-\frac{K_{x\alpha}(0)}{K_{xx}(0)}u_x.$$

At $\xi\gg\xi_1$, the function

$$f_+^{(1)}(\xi,\mathbf{u},t)=\frac{ev_0}{\omega_1}\psi_{1+}(\xi,\mathbf{u})\frac{\partial f_0}{\partial\varepsilon}\exp(i\omega_1 t),$$

which describes electrons moving towards surface 2 is the solution of the kinetic equation for free electrons:

$$\frac{\partial f^{(1)}}{\partial t}+v_0u_x\frac{\partial f^{(1)}}{\partial x}=0.\tag{3.1}$$

The boundary condition on the surface 2 takes the form

$$f_-^{(1)}\left(\frac{\omega_1 d}{v_0},\mathbf{u},t\right)=p_2f_+^{(1)}\left(\frac{\omega_1 d}{v_0},\mathbf{u}^*,t\right).$$

From this we obtain, for the electrons moving away from the surface 2, the solution of Eq. (3.1):

$$f_-^{(1)}(\xi,\mathbf{u},t)=i\frac{ev_0}{\omega_1}p_2E_{1\alpha}^0[w_\alpha^*-p_1w_\alpha+\theta_{1\alpha}(u_x^*)$$
$$-p_1\theta_{1\alpha}(u_x)]\frac{\partial f_0}{\partial\varepsilon}\exp\left[-\frac{i\xi}{u_x}+\frac{i\omega_1 d}{v_0}\left(\frac{1}{u_x}-\frac{1}{u_x^*}\right)+i\omega_1 t\right].$$

The second order correction, which is proportional to the field $\mathbf{E}_2(x,t)=\mathbf{E}_2(x)\exp(-i\omega_2 t)$, is obtained from the equation

$$\frac{\partial f_\pm^{(2)}}{\partial t}\pm v_0|u_x|\frac{\partial f_\pm^{(2)}}{\partial x}-\frac{eE_{2\nu}(x)}{mv_0}\exp(-i\omega_2 t)\frac{\partial f_\pm^{(1)}}{\partial\rho_\nu}=0.\tag{3.2}$$

Neglect of the magnetic field in (3.2) means neglect of terms of higher orders in the parameter $1/\xi_2=v_0\omega_0/c\omega_2$. Putting $f_\pm^{(2)}=\varphi_\pm^{(2)}(x,\rho)\exp[i(\omega_1-\omega_2)t]$, we arrive at the equation

$$i(\omega_1-\omega_2)\varphi_\pm^{(2)}\pm v_0|u_x|\frac{\partial\varphi_\pm^{(2)}}{\partial x}-\frac{eE_{2\nu}(x)}{mv_0}\frac{\partial\varphi_\pm^{(1)}(x,u)}{\partial\rho_\nu}=0,\tag{3.3}$$

in which $\varphi_\pm^{(1)}(x,\mathbf{u})=f_\pm^{(1)}(x\omega_1/v_0,\mathbf{u},0)$. In (3.3), the velocity $\mathbf{u}$, according to relation (2.2), is regarded as a function of $\boldsymbol{\rho}$. Using the boundary conditions

$$\varphi_-^{(2)}(d,\rho)=p_2\varphi_+^{(2)}(d,\rho^*),\quad \varphi_+^{(2)}(-\infty,\rho)=0.\tag{3.4}$$

we obtain the solution of (3.3):

$$\varphi_+^{(2)}(x,\rho)=\frac{e}{mv_0^2u_x}\int_{-\infty}^x E_{2\nu}(x')\frac{\partial\varphi_+^{(1)}(x',\mathbf{u})}{\partial\rho_\nu}\exp\left[\frac{i(\omega_1-\omega_2)(x'-x)}{v_0u_x}\right]dx',$$
$$u_x>0;\tag{3.5}$$

$$\varphi_-^{(2)}(x,\rho)=\frac{e}{mv_0^2u_x}\int_d^x E_{2\nu}(x')\frac{\partial\varphi_-^{(1)}(x',\mathbf{u})}{\partial\rho_\nu}\exp\left[\frac{i(\omega_1-\omega_2)(x'-x)}{v_0u_x}\right]dx'$$
$$+p_2\frac{e}{mv_0^2u_x^*}\int_{-\infty}^d E_{2\nu}(x')\frac{\partial\varphi_+^{(1)}(x',\mathbf{u}^*)}{\partial\rho_\nu^*}$$
$$\times\exp\left[\frac{i(\omega_1-\omega_2)}{v_0}\left(\frac{d-x}{u_x}+\frac{x'-d}{u_x^*}\right)\right]dx',\tag{3.6}$$
$$u_x<0.$$

When integrating in formulas (3.5) and (3.6) it is obviously necessary to use the following approximations for the components of the field $E_2(x)$:

$$E_{2\alpha}(x)\approx E_{2\alpha}(0)=E_{2\alpha}^0,$$
$$E_{2x}(x)=-iE_{2\alpha}^0\left[\chi_{2\alpha}\left(\frac{d-x}{v_0}\omega_2\right)-i\frac{K_{x\alpha}(0)}{K_{xx}(0)}\right],$$

where the functions $\chi_{2\alpha}$ are solutions of equations obtained from (2.19) by making the substitutions $p_1 \to p_2$ and $i \to -i$.

Let us calculated the contribution to the current density:

$$j_\alpha^e(x) = \frac{-2e}{(2\pi\hbar)^3} \int_{u_x<0} \varphi^{(2)}(x,\boldsymbol{\rho}) u_\alpha d^3\boldsymbol{\rho}. \tag{3.7}$$

A nonzero correction to the current is made by electrons that can move towards the surface 1, i.e., described by the function $\varphi_-^{(2)}$. We substitute expression (3.6) in formula (3.7). We first calculate the contribution made to the current by the first integral in (3.6). In the calculation it is necessary to interchange the order of integration with respect to dx′ and $d^3\rho$, and then evaluate the integral with respect to $d^3\rho$ by parts. Only the rapidly varying exponential $\exp[i(\omega_1 - \omega_2)(x' - x)/v_0u_x]$ need by differentiated with respect to ρ_ν. Recognizing that the current is significantly different from zero only near the surface 1, i.e., at $x \ll d$, we obtain

$$j_\alpha' = i\frac{3p_2ne^3d(\omega_1-\omega_2)}{4\pi m^2v_0^2\omega_1\omega_2}E_{1\gamma}^0E_{2\beta}^0 \int_{u_x<0} [w_\gamma^* - p_1w_\gamma + \theta_{1\gamma}(u_x^*) - p_1\theta_{1\gamma}(u_x)]$$
$$\cdot\left[u_x\frac{\partial w_\beta}{\partial\rho_x} - \theta_{2\beta}(u_x)\frac{\partial u_x}{\partial\rho_x}\right]\exp(i\Delta)\frac{u_\alpha dS}{uu_x^3}, \tag{3.8}$$

$$\theta_{2\alpha}(u_x) = \int_0^\infty \chi_{2\alpha}(\xi)\exp\left(i\frac{\xi}{u_x}\right)d\xi,\quad \Delta = -\frac{\omega_1-\omega_2}{v_0u_x}x + \frac{d}{v_0}\left(\frac{\omega_1}{u_x} - \frac{\omega_1}{u_x^*} - \frac{\omega_2}{u_x}\right),$$
$$\alpha, \beta, \gamma = y, z.$$

In the integration of the second term in (3.6) it is necessary to go over in (3.7) from the integration variables ρ_ν to the variables ρ_ν^*, using the Jacobi determinant (2.12)

$$j_\alpha'' = i\frac{3p_2ne^3d(\omega_1-\omega_2)}{4\pi m^2v_0^2\omega_1\omega_2}E_{1\gamma}^0E_{2\beta}^0 \int_{u_x<0} [w_\gamma^* - p_1w_\gamma + \theta_{1\gamma}(u_x^*) - p_1\theta_{1\gamma}(u_x)]\cdot$$
$$\times\left[u_x^*\frac{\partial w_\beta^*}{\partial\rho_x} - \theta_{2\beta}(u_x^*)\frac{\partial u_x^*}{\partial\rho_x}\right]\exp(i\Delta)\frac{u_\alpha dS^*}{u_x^3u^*}. \tag{3.9}$$

It follows from (3.8) and (3.9) that inasmuch as the integral contains an exponential that varies rapidly with u_x and u_x^*, the currents j_α' and j_α'' are generally speaking negligibly small for any x in comparison with the dimensional factor taken outside of the integral sign. An exception is the case in which u_x^*/u_x = const, i.e., $W_x(u) = Cu_x$. In this case, at definite frequencies, the exponentials that oscillate rapidly in velocity may cancel out. Using theproperties (2.8), (2.9), and (2.10) of the function W(u), we can see that the only possible value of the constant is $C = -1$.

The spectrum leading to such a function W(u) is given by

$$g(\boldsymbol{\rho}) = F[\alpha(\rho_\tau)\rho_x^2 + \beta(\rho_\tau)\rho_x, \rho_\tau];\quad \alpha(\rho_\tau) \neq 0. \tag{3.10}$$

In particular, $W_x(u) = -u_x$ for a quadratic dispersion law. From the uniqueness of the transition to the mirror state it follows that the function $F(h, \rho_\tau) = F_{\rho_\tau}(h)$ should be monotonic in h at each value of ρ_τ. In addition, obviously, the following identity should be satisfied:

$$F[\alpha(\rho_\tau)\rho_x^2 + \beta(\rho_\tau)\rho_x, \rho_\tau] = F[\alpha(-\rho_\tau)\rho_x^2 - \beta(-\rho_\tau)\rho_x, -\rho_\tau].$$

We put $u_x^* = -u_x$ in (3.8) and (3.9), and also change over in (3.9) to integration with respect to dS. We obtain

$$j_\alpha = j_\alpha' + j_\alpha'' = i\frac{3p_2ne^3d(\omega_1-\omega_2)}{4\pi m^2v_0^2\omega_1\omega_2}E_{1\gamma}^0E_{2\beta}^0 \int_{u_x<0} [w_\gamma^* - p_1w_\gamma + \theta_{1\gamma}(-u_x)$$
$$- p_1\theta_{1\gamma}(u_x)]\left\{u_x\frac{\partial(w_\beta - w_\beta^*)}{\partial\rho_x} + [\theta_{2\beta}(-u_x) - \theta_{2\beta}(u_x)]\frac{\partial u_x}{\partial\rho_x}\right\} \tag{3.11}$$
$$\times\exp\left[-\frac{i(\omega_1-\omega_2)x}{v_0u_x} + \frac{id(2\omega_1-\omega_2)}{v_0u_x}\right]\frac{u_\alpha dS}{u_x^3u}.$$

This, however, is still not the final answer; it is necessary to take into account, in the same approximation in the field, those electrons which collide with the surface 1 and travel towards the surface 2. We denote the distribution function of these electrons by $\varphi_-^{(2)\prime}\exp[i(\omega_1 - \omega_2)t]$, where $\varphi_-^{(2)\prime}$ is the solution of the kinetic equation for the free electrons (3.1) at $u_x > 0$ in conjunction with the boundary condition

$$\varphi_-^{(2)\prime}(0,\boldsymbol{\rho}) = p_1\varphi_-^{(2)\prime}(0,\boldsymbol{\rho}^*).$$

Calculating the corresponding contribution of these electrons to the current density, we obtain ultimately

$$j_\alpha^e(x) = i\frac{3p_2ne^3d(\omega_1-\omega_2)}{4\pi m^2v_0^2\omega_1\omega_2}E_{1\gamma}^0E_{2\beta}^0I_{\alpha\beta\gamma}\left[\frac{(\omega_1-\omega_2)x}{v_0}\right],$$
$$I_{\alpha\beta\gamma}(\varkappa) = \int_{u_x<0}[w_\gamma^* - p_1w_\gamma + \theta_{1\gamma}(-u_x) - p_1\theta_{1\gamma}(u_x)]$$
$$\times\left\{u_x\frac{\partial(w_\beta - w_\beta^*)}{\partial\rho_x} + [\theta_{2\beta}(-u_x) - \theta_{2\beta}(u_x)]\frac{\partial u_x}{\partial\rho_x}\right\} \tag{3.12}$$
$$\times\exp\left[\frac{id(2\omega_1-\omega_2)}{v_0u_x}\right]\left[u_\alpha\exp\left(-\frac{i\varkappa}{u_x}\right) - p_1u_\alpha^*\exp\left(i\frac{\varkappa}{u_x}\right)\right]\frac{dS}{u_x^3u};$$
$$\alpha, \beta, \gamma = y, z.$$

We see from this that the amplitude of the echo differs from zero near the surface 1 if $|2\omega_1 - \omega_2| \lesssim v_0/d$. The current density then vanishes already at a distance $x \gtrsim v_0/|\omega_1 - \omega_2|$ from the surface.

Let us calculate the corresponding correction to the surface impedance:

$$(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta} = \frac{2p_2ne^3dE_{2\gamma}^0}{4\pi m^2v_0\omega_1\omega_2}J_{\alpha\beta\gamma};$$
$$J_{\alpha\beta\gamma} = \int_{u_x<0}[w_\beta^* - p_1w_\beta + \theta_{1\beta}(-u_x) - p_1\theta_{1\beta}(u_x)]$$
$$\times\left\{u_x\frac{\partial(w_\gamma - w_\gamma^*)}{\partial\rho_x} + [\theta_{2\gamma}(-u_x) - \theta_{2\gamma}(u_x)]\frac{\partial u_x}{\partial\rho_x}\right\} \tag{3.13}$$
$$\times\exp\left[\frac{id(2\omega_1-\omega_2)}{v_0u_x}\right]\frac{u_\alpha - p_1u_\alpha^*}{u\cdot u_x^2}dS;$$
$$\alpha, \beta, \gamma = y, z.$$

To obtain the radiation outside the sample, it is necessary to substitute (3.12) in the complete expression

$$\frac{d^2E_\alpha^e}{dx^2} + \frac{(\omega_2-\omega_1)^2}{c^2}E_\alpha^e = -\frac{4\pi i(\omega_2-\omega_1)}{c^2}j_\alpha^e,\quad x<0, \tag{3.14}$$

from which we obtain

$$E_\alpha^e(x) = \frac{-3p_2ne^3dE_{1\beta}^0E_{2\gamma}^0}{2cm^2v_0\omega_1\omega_2}J_{\alpha\beta\gamma}\exp\left[-i\frac{\omega_2-\omega_1}{c}x\right],\quad x<0. \tag{3.15}$$

The distribution of the echo field inside the plate can be obtained by substituting the components of the vector $-[4\pi iv_0^2/c^2(\omega_2 - \omega_1)]j_\alpha^e$ in the right-hand sides of the equations of the system (2.21) (in this case η_1 should be replaced by $\omega_0^2(\omega_2 - \omega_1)^{-2}$). The obtained system can be easily solved by reducing the tensor $B_{\alpha\beta}$ to its principal axes. The field $\mathbf{E}^e$ inside the metal is damped exponentially at distances on the order of $\delta \sim c/\omega_0$.

From (3.12), (3.13), and (3.15) we see the obvious need for the existence of electrons specularly reflected from the surface 2; the specularity coefficient p_2 enters in the formulas in the form of a factor. The scattering by the surface 1, for example, can be completely diffuse.

4. ELLIPSOIDAL FERMI SURFACE

The quadratic dispersion law is a particular case of the dispersion law (3.10). However, the results obtained in the preceding section for metals with a quadratic dispersion law are incorrect. The point is that in this case the calculation by means of formula (3.12) yields zero.

Let

$$g(\rho)=a_{\mu\nu}\rho_\mu\rho_\nu, \quad a_{\mu\nu}=a_{\nu\mu}. \tag{4.1}$$

Then

$$u_\mu=a_{\mu\nu}\rho_\nu; \quad \mu, \nu=x, y, z. \tag{4.2}$$

We assume the quadratic form (4.1) to be positive definite, so that the coefficients $a_{\mu\nu}$ satisfy all the criteria needed for this purpose.

We calculate the factors $K_{\mu\nu}(0)$:

$$K_{\mu\nu}(0)=\frac{3}{4\pi}\int\frac{u_\mu u_\nu}{u}dS=\frac{3}{8\pi}\oint_{g(\rho)=1}u_\nu dS_\mu=\frac{3}{8\pi}a_{\mu\nu}V_g.$$

In the calculation we used the Gauss theorem; V_g is the volume enclosed by the surface $g(\rho)=1$.

It follows therefore that

$$w_\alpha=u_\alpha-\frac{a_{\alpha x}}{a_{xx}}u_x=b_{\alpha\beta}\rho_\beta, \quad \alpha, \beta=y, z, \tag{4.3}$$

where $b_{\alpha\beta}=a_{\alpha\beta}-a_{\alpha x}a_{\beta x}/a_{xx}$. This means that the w_α do not change on going to the mirror state[3]:

$$w_\alpha=w_\alpha^*. \tag{4.4}$$

In addition, by equating the quadratic form (4.1) to unity, and taking (4.2) into account, we easily obtain

$$\rho_x=-\frac{a_{x\alpha}}{a_{xx}}\rho_\alpha\pm\left[\frac{1-b_{\alpha\beta}\rho_\alpha\rho_\beta}{a_{xx}}\right]^{1/2}, \quad u_x=\pm\left[a_{xx}(1-b_{\alpha\beta}\rho_\alpha\rho_\beta)\right]^{1/2}. \tag{4.5}$$

It can be shown with the aid of (4.3) and (4.5) that the functions $\chi_{1\alpha}(\xi)$ in (2.18) should be equal to zero, i.e., the connection between the normal and tangential components

$$E_{1x}(\xi)=-\frac{a_{\alpha x}}{a_{xx}}E_{1\alpha}(\xi) \tag{4.6}$$

makes (2.6) an identity. Indeed, substituting (4.6) in (2.17) and taking (4.12) into account, we obtain

$$j_x=\frac{\omega_1\eta_1}{4\pi}\int_0^\infty d\xi'\left[H_{x\alpha}(|\xi-\xi'|)+p_1H_{x\alpha}(\xi+\xi')\right]E_{1\alpha}(\xi'),$$

$$H_{x\alpha}(\xi)=\frac{3}{4\pi}\int w_\alpha\frac{\exp(-i\xi/u_x)}{u_x}dS_x. \tag{4.7}$$

In (4.7), the integration is carried out in the plane $\rho_x=0$ over the area contained inside the ellipse $b_{\alpha\beta}\rho_\alpha\rho_\beta=1$. This integral is equal to zero since w_α is an odd function of the vector ρ_T while u_x is an even function, and consequently $j_x\equiv 0$.

Although we have dealt so far with a quadratic dispersion of the electrons, it is easily seen that to satisfy relation (4.6) it suffices to require only that the Fermi surface be ellipsoidal. Moreover, any dispersion law in the form

$$g(\rho)=F[a_{xx}\rho_x^2+2a_{x\alpha}\rho_\alpha\rho_x+\varphi(\rho_\tau)],$$

[$\varphi(\rho_T)$ is an arbitrary even function of the vector ρ_T] leads to relation (4.6). In this section we consider for simplicity only a dispersion law in the form (4.1), bearing in mind the fact that all the results can be easily generalized in the case of an ellipsoidal Fermi surface.

Substituting $\theta_{1\alpha}=\theta_{2\alpha}\equiv 0$ in (3.12), we obtain a zero result after taking (4.4) into account. We note that according to (4.6) the normal component of the field should experience on the metal surface a jump from zero to $E_{1x}(0)=-a_{\alpha x}E^0_{1\alpha}/a_{xx}$. This means in fact that the component E_{1x} changes near the surface over distances that are much smaller than v_0/ω_1, namely, over distances on the order of the Debye length λ_D.

Let us calculate the correction to the reciprocal of the surface impedance. Formula (2.22), as can be easily verified by direct substitution, assumes the form

$$(\hat{Z}^{-1}_{\text{sur}})_{\alpha\beta}=\frac{v_0\eta_1}{4\pi}(1-p_1)\frac{3}{4\pi}b_{\alpha\gamma}b_{\beta\gamma'}\int_{b_{\alpha\beta}\rho_\alpha\rho_\beta<1}\rho_\gamma\rho_{\gamma'}d\rho_y d\rho_z; \quad \alpha, \beta, \gamma, \gamma'=y, z. \tag{4.8}$$

Reducing the quadratic form $b_{\alpha\beta}\rho_\alpha\rho_\beta$ to a sum of squares, we calculate the integral (4.2), and recognizing that $\det|b_{\alpha\beta}|=a_{xx}^{-1}\det|a_{\mu\nu}|$, we obtain ultimately

$$(\hat{Z}^{-1}_{\text{sur}})_{\alpha\beta}=\frac{3v_0\eta_1}{64\pi}(1-p_1)\left(\frac{a_{xx}}{\det a}\right)^{1/2}b_{\alpha\beta}.$$

This formula at $p_1=0$ could be obtained by using the results of[9]. Indeed, it follows directly from (19) of[9] that the correction to the reciprocal impedance takes the form (4.8) at $p_1=0$.

We proceed to the calculation of the second-order corrections in the amplitude of the field. In the zeroth approximation in the small parameter $1/\xi_1$ there is no plasma echo. To find the corrections of first order in the parameter $1/\xi_1$ it is necessary to take into account in the kinetic equation (3.2) the magnetic field, and then we obtain in place of (3.3) the equation

$$i(\omega_1-\omega_2)\varphi_\pm^{(2)}\pm v_0|u_x|\frac{\partial\varphi_\pm^{(2)}}{\partial x}-\frac{e^2v_0}{m\omega_1 c}[\mathbf{u}\times\mathbf{H}_2]_\mu\frac{\partial}{\partial\rho_\mu}\left(\psi_{1\pm}\frac{\partial f_0}{\partial\varepsilon}\right)=0, \tag{4.9}$$

in which

$$\psi_{1+}(x,\rho)=iE_{1\alpha}(1-p_1)w_\alpha\exp\left(-\frac{ix\omega_1}{v_0|u_x|}\right),$$

$$\psi_{1-}(x,\rho)=iE_{1\alpha}p_2(1-p_1)w_\alpha\exp\left[\frac{i\omega_1(x-2d)}{v_0|u_x|}\right].$$

We note that

$$[\mathbf{u}\times\mathbf{H}_2]_\mu\frac{\partial}{\partial\rho_\mu}\left(\psi_{1\pm}\frac{\partial f_0}{\partial\varepsilon}\right)=[\mathbf{u}\times\mathbf{H}_2]_\mu\frac{\partial\psi_{1\pm}}{\partial\rho_\mu}\frac{\partial f_0}{\partial\varepsilon}$$

After differentiation we obtain

$$\frac{\partial\psi_{1+}}{\partial\rho_\mu}\approx-a_{x\mu}\frac{\omega_1}{v_0}\frac{x}{u_x^2}E_{1\alpha}^0(1-p_1)w_\alpha\exp\left(-\frac{ix\omega_1}{v_0|x_x|}\right), \tag{4.10}$$

$$\frac{\partial\psi_{1-}}{\partial\rho_\mu}\approx a_{x\mu}\frac{\omega_1}{v_0}\frac{2d-x}{u_x^2}E_{1\alpha}^0p_2(1-p_1)w_\alpha\exp\left[-\frac{i\omega_1(2d-x)}{v_0|u_x|}\right]. \tag{4.11}$$

To simplify the subsequent calculations it is convenient to direct the axes y and z along the principal axes of the ellipsoid $u_x(\rho_T)=0$. Then

$$b_{yz}=b_{zy}=0, \quad a_{xx}a_{yz}=a_{xz}a_{xy}.$$

The system (2.21) does not contain crossing terms in this case. For the field components $\mathbf{E}_2(x)$, the analogous system of equations takes the form

$$d^2E_{2\alpha}/dx^2=E_\alpha/\delta_\alpha^2, \quad E_{2\alpha}(d)=E_{2\alpha}^0; \quad \alpha=y, z. \tag{4.12}$$

The δ_α are equal to

$$\delta_\alpha=\frac{c}{\omega_0}\left(\frac{\sqrt{\det a}}{b_{\alpha\alpha}}\right)^{1/2}.$$

From (4.12) we obtain

$$E_{2\alpha}(x)=E_{2\alpha}^0\exp[(x-d)/\delta_\alpha].$$

Using these expressions, we get

$$a_{x\mu}[\mathbf{u}\times\mathbf{H}_2]_\mu=(\mathbf{H}_2[\mathbf{a}\times\mathbf{u}])=-i\frac{c}{\omega_2}a_{xx}w_\alpha(E_2/\delta)_\alpha,$$

$$(E_2/\delta)_\alpha=E_{2\alpha}/\delta_\alpha. \tag{4.13}$$

Here **a** is a vector with components $a_{x\mu}$.

Substituting (4.10) and (4.11) in (4.9) we obtain, taking (4.13) into account, the following equation for $\varphi_\pm^{(2)}$:

$$i(\omega_1-\omega_2)\varphi_\pm^{(2)}\pm v_0|u_x|\frac{\partial\varphi_\pm^{(2)}}{\partial x}\ldots$$

$$-\left[\begin{matrix}1\\-p_2\end{matrix}\right] ia_{xx}\frac{e^2}{m\omega_2}E_{1\alpha}{}^0\left(\frac{E_2}{\delta}\right)_\beta \frac{w_\alpha w_\beta}{u_x^2}\left[\begin{matrix}x\\2d-x\end{matrix}\right](1-p_2) \times\frac{\partial f_0}{\partial\varepsilon}\exp\left\{-\frac{i\omega_1}{v_0|u_x|}\left[\begin{matrix}x\\2d-x\end{matrix}\right]\right\}=0. \qquad (4.14)$$

The symbol $\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right]$ defines a function that assumes a value unity at $u_x > 0$ and a value 2 at $u_x < 0$. We solve Eq. (4.14), using the boundary conditions (3.4). We obtain at $x \ll d$

$$\varphi_-^{(3)}=-p_2(1-p_1)a_{xx}\frac{2e^2d}{m\omega_2^2}\cdot \times\frac{w_\alpha w_\beta}{u_x^2}E_{1\alpha}{}^0(E_2{}^0/\delta)_\beta\frac{\partial f_0}{\partial\varepsilon}\exp\left\{\frac{i}{v_0|u_x|}[x(\omega_1-\omega_2)-d(2\omega_1-\omega_2)]\right\}.$$

Hence

$$j_\alpha{}^e=-\frac{3ne^3d\omega_0}{2m^2v_0\omega_2{}^2c}p_2(1-p_1)\cdot \times\frac{a_{\alpha x}}{\sqrt{a_{xx}}}\frac{b_{yy}^{3/2}E_{1y}{}^0E_{2y}{}^0+b_{zz}^{3/2}E_{1z}{}^0E_{2z}{}^0}{(\det a)^{3/4}}[p_1\Phi_1(\lambda_1)-\Phi_1(\lambda_2)], \qquad (4.15)$$

$$\lambda_{1,2}=\frac{-d(2\omega_1-\omega_2)\mp x(\omega_1-\omega_2)}{v_0\sqrt{a_{xx}}}, \qquad (4.16)$$

$$\Phi_1(\lambda)=\int_1^\infty\left(\frac{1}{y}-\frac{1}{y^3}\right)\exp(i\lambda y)\,dy=-\operatorname{Ei}(i\lambda)\left(1+\frac{\lambda^2}{2}\right)-e^{i\lambda}\left(\frac{1}{2}+\frac{i\lambda}{2}\right).$$

$\operatorname{Ei}(i\lambda) = \operatorname{Ci}(\lambda) + i\operatorname{si}(\lambda)$ is the integral exponential function of imaginary argument. Calculating the total current, we obtain the corrections to the reciprocal surface impedance:

$$(\hat{Z}_{\text{sur}}^{-1})_{\alpha\beta}{}^e=-i\frac{3ne^3d\omega_0}{2m^2\omega_2{}^2(\omega_2-\omega_1)c}\cdot p_2(1-p_1)a_{\alpha x}(b_{\beta\beta})^{3/2}(\det a)^{-3/4}E_{2\beta}{}^0[p_1\Phi_2(\lambda_3)+\Phi_2(-\lambda_3)],$$

$$\lambda_3=d(\omega_2-2\omega_1)/v_0\sqrt{a_{xx}}, \qquad (4.17)$$

$$\Phi_2(\lambda)=\int_1^\infty\left(\frac{1}{y^2}-\frac{1}{y^4}\right)\exp(i\lambda y)\,dy=-\operatorname{Ei}(i\lambda)\left(i\lambda+\frac{i\lambda^3}{6}\right)+e^{i\lambda}\left(\frac{2}{3}+\frac{\lambda^2}{6}-\frac{i\lambda}{6}\right),\quad \Phi_2(0)=\frac{2}{3}.$$

Plots of the functions Φ_1 and Φ_2 are shown in Figs. 2 and 3. Both functions tend asymptotically to $-2e^{i\lambda}/\lambda^2$ at $\lambda \gg 1$.

The function $\Phi_1(\lambda)$ is infinite at zero, and therefore at $\omega_2 = 2\omega_1$ the current density j_y^e becomes infinite near the surface 1. In fact, of course, the current density near the surface is determined near the electron collisions. Indeed, the condition under which an electron traveling from the surface 1 to the surface 2 and back experiences no collisions is

$$|u_x| \gg d/l. \qquad (4.18)$$

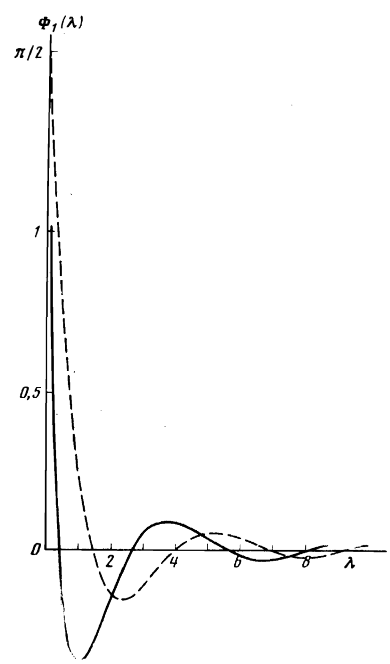

FIG. 2. Plot of the function $\Phi_1(\lambda)$: solid curve—plot of the function Re $\Phi_1(\lambda)$, dashed—plot of Im $\Phi_1(\lambda)$.

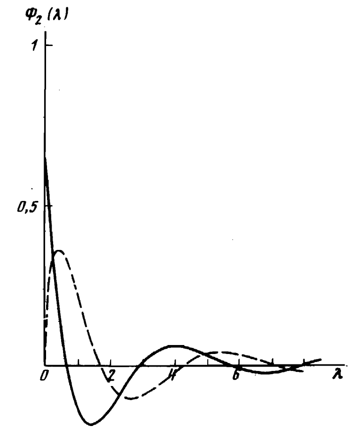

FIG. 3. Plot of the function $\Phi_2(\lambda)$: solid curve—plot of the function Re $\Phi_2(\lambda)$, dashed—plot of Im $\Phi_2(\lambda)$.

Therefore in fact, when integrating over the Fermi surface, the region of integration should be not $u_x > 0$ but $u_x > u_x^0 \sim d/l$. In addition, owing to the collisions, λ_1 and λ_2 differ from expression (4.16) by an amount on the order of id/l. Noting that

$$\Phi_1(\lambda)\approx-\ln|\lambda| \quad \text{at} \quad |\lambda|\ll 1,$$

we obtain

$$j_\alpha{}^e(0)=\frac{3ne^3d\omega_0}{2m^2c\omega_2{}^2v_0}\frac{p_2(1-p_1)^2}{\sqrt{a_{xx}}} \times a_{\alpha x}\frac{b_{yy}^{3/2}E_{1y}{}^0E_{2y}{}^0+b_{zz}^{3/2}E_{1z}{}^0E_{2z}{}^0}{(\det a)^{3/4}} \times\ln\left(\frac{l}{d}\right),\quad |\omega_2-2\omega_1|\leqslant\nu_{ef}. \qquad (4.19)$$

The specularity coefficients p_1 and p_2 should be attributed to the non-glancing electrons, since the glancing electrons do not take part in the echo production, by virtue of the condition (4.18). Substituting the current density (4.15) in Eq. (3.14), we obtain the field of the radiated wave

$$E_\alpha{}^e(x)=i\frac{3\pi ne^3\omega_0 d}{m^2\omega_2{}^2c^2(\omega_2-\omega_1)} \times p_2(1-p_1)a_{\alpha x} \times\frac{b_{yy}^{3/2}E_{1y}{}^0E_{2y}{}^0+b_{zz}^{3/2}E_{1z}{}^0E_{2z}{}^0}{(\det a)^{3/4}} \times[p_1\Phi_2(\lambda_3)+\Phi_2(-\lambda_3)] \times\exp\left[\frac{-i(\omega_2-\omega_1)x}{c}\right],\quad x<0. \qquad (4.20)$$

Inside the metal, the field distribution takes the form

$$E_\alpha{}^e(x)=\frac{3\pi ne^3d}{m^2\omega_2{}^2c^2}p_2(1-p_1)a_{\alpha x}\frac{b_{yy}^{3/2}E_{1y}{}^0E_{2y}{}^0+b_{zz}^{3/2}E_{1z}{}^0E_{2z}{}^0}{(b_{\alpha\alpha}\det a)^{1/2}}\cdot \times[p_1\Phi_2(\lambda_3)+\Phi_2(-\lambda_3)]\exp\left(-\frac{x}{\delta_\alpha}\right),\quad x>0.$$

From these formulas we can see clearly the anisotropy of the dispersion law; the coefficient $a_{\alpha x}$ enters in the form of a factor.

5. CONCLUSION

In concluding, a few words concerning the possible experimental verification. In this case, a convenient material for the investigation is, as expected, singlecrystal bismuth, in which the inequalities (1.1) can be attained at $\omega_i \sim 10^{10} - 10^{11}$ sec^{-1}. In addition, a particularly important fact is that we can apply to bismuth directly the results of the preceding section, since each of the valleys of the Fermi surface of bismuth can be regarded with good accuracy as an ellipsoid, in spite of the fact that the dispersion law differs strongly from quadratic (see the reviews[10,11]). Let the axes C_1, C_2, and C_3 of the crystal lattice of bismuth be respectively the axes y, z, and x in our problem. With such an orientation, the hole ellipsoid takes no part in the echo production. Neglecting the intervalley transitions, we can calculate independently the contribution of each of the three electron ellipsoids. The major axis of the electron ellipsoid is directed at an angle 6°20′ to the basal plane (y, x). Using the parameters of bismuth[10,11], we obtain the form of the function g(**ρ**) for an ellipsoid having one of the principal axes aligned with C_2:

$$g(\boldsymbol{\rho}) \sim 0{,}1(1{,}83\rho_y^2 + 86\rho_x^2 + 2\cdot 9{,}6\rho_x\rho_y + 195\rho_z^2).$$

Calculations by means of formula (4.20) yield (in cgs esu)

$$E_y^e(0) \sim i\cdot 10^4 dp_2(1-p_1^2)\cdot (0{,}021E_{1y}^0E_{2y}^0 + 86E_{1z}^0E_{2z}^0),$$
$$E_z^e = 0 \quad \text{at} \quad \omega_2 = 2\omega_1 \sim v_0\omega_0/c.$$

Taking into account the contribution made to the echo by the two other electron ellipsoids, we obtain ultimately (in cgs esu)

$$E_y^e(0) \sim i\cdot 10^5 dp_2(1-p_1^2)(E_{z1}^0E_{z2}^0 - E_{y1}^0E_{y2}^0); \tag{5.1}$$

$$E_z^e(0) \sim i\cdot 10^5 dp_2(1-p_1^2)(E_{y1}^0E_{z2}^0 + E_{z1}^0E_{y2}^0). \tag{5.2}$$

In a real experiment, the radiation takes place not in vacuum, but in the volume of the resonator located to the left of the plate. In this case we are dealing with excitation of oscillations induced in the resonator by the current $\mathbf{j}^e$. But since the frequency of this current is equal to the natural frequency ω_1 of the resonator[4)], the excitation has a resonant character[12]. The corresponding estimate differs from (5.1) and (5.2) by a factor equal to the Q of the resonant circuit, which includes the resonator and the bismuth plate.

The authors are grateful to L. P. Pitaevskiĭ, M. I. Kaganov, B. E. Meĭerovich, A. F. Andreev, V. Ya. Kravchenko, and G. I. Babkin for useful discussions.

[1)]In this paper we disregard, for simplicity, Fermi-liquid effects. In spite of the fact that in infrared metal optics the Fermi-liquid interaction is appreciable (see, e.g. [3], Sec. 4 and [4], Secs. 40 and 47), it appears that the picture of the phenomena of interest to us will not differ qualitatively from that described with the aid of the gas model.

[2)]With the exception, of course, of the last one, inasmuch as $l \ll d = \infty$ for a half-space.

[3)]Relation (4.4) determines the connection between **u*** and **u**, in analogy with Eq. (12) of [6] or (6) of [7].

[4)]For the same reason, the signal $\mathbf{E}_2$ must be modulated in order to distinguish $\mathbf{E}^e$ from $\mathbf{E}_1$.

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Translated by J. G. Adashko
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