Similarity relations for strong fluctuations of the intensity of light propagating in a turbulent medium

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Fluctuations arising in a light wave propagating in an optically inhomogeneous turbulent medium are considered. Equations had been derived heretofore for coherence functions—statistical moments of arbitrary order n for a light field in a medium. However, in the case of strong intensity fluctuations, when the relative dispersion of intensity is of order of magnitude unity, no solution of these equations for $n \geq 4$ had been obtained. For a locally isotropic turbulent medium with a power spectrum of inhomogeneities of the dielectric constant, similarity relations are obtained for coherence functions of the fourth order. The characteristic longitudinal (along the direction of propagation) and tranverse scales of the problem of strong intensity fluctuations are determined. Results of measurements of relative dispersions, of the correlation coefficients, and of the frequency spectra of the intensity fluctuations of coherent radiation at wavelengths of 0.63 and 10.6 μm have confirmed the consequences of the derived similarity relations over sufficiently broad ranges of variation of the turbulence characteristics and of distances traversed by light in a turbulent medium.

1. Equations for the coherence functions of a wave in a turbulent medium have been obtained by Tatarskiĭ[1] under the additional assumption of the smallness of the longitudinal (along the direction of propagation) scale of inhomogeneities of the medium compared to other longitudinal dimensions of the problem. The applicability of these equations is not limited by the condition of smallness of intensity fluctuations[2,3]. However, it has been found possible to obtain an exact solution of the equation only for the coherence function of the second order Γ_2[4,5]. This solution agrees well with experiment[6,7] and, in particular, enables us to determine the range of coherence of the wave field and to calculate the average intensity in restricted beams.

In order to evaluate intensity fluctuations it is necessary to solve the equation for the coherence function of the fourth order Γ_4. Numerical solutions of this equation for three-dimensional inhomogeneities with a Gaussian correlation function[8] and two-dimensional inhomogeneities with a power law spectral density[9] agree qualitatively with experimental data[10]. A qualitative investigation of the fluctuations for a single-scale correlation function of the refraction index has been carried out by Shishov[11]. Considerable difficulties arise in the solution of the equation for Γ_4 in the case of greatest practical interest for a turbulent medium with three-dimensional locally isotropic inhomogeneities with a power law spectral density.

In connection with the fact that a general solution has not been obtained it appears to be useful to pose the following question: do some characteristic scales exist in the problem of strong intensity fluctuations which can be determined from the equation without solving it. Having determined such scales and using them to treat appropriate experimental data it is possible to establish similarity relations for strong intensity fluctuations. Experimental results treated on the basis of similarity will be, in addition to everything else, useful for evaluating the validity of approximate methods of solution.

2. For the coherence function of the fourth order

$$\Gamma_4(x, \rho_1, \rho_2, \rho_3, \rho_4) = \langle E(x, \rho_1)E^*(x, \rho_2)E(x, \rho_3)E^*(x, \rho_4)\rangle \quad (1)$$

($E(x, \rho)$ is a wave propagating in the x direction in a turbulent locally isotropic medium) Tatarskiĭ[1] has obtained an equation which in the special case of a plane wave and for $\rho_2 - \rho_1 = \rho_3 - \rho_4 = \mathbf{u}$, $\rho_4 - \rho_1 = \rho_3 - \rho_2 = \mathbf{v}$ reduces to the following form:

$$\frac{\partial \Gamma_4}{\partial x} = \frac{i}{k}\left(\frac{\partial^2}{\partial y_1 \partial y_2} + \frac{\partial^2}{\partial z_1 \partial z_2}\right)\Gamma_4 - \frac{\pi k^2}{4} F(\mathbf{u}, \mathbf{v})\Gamma_4,$$
$$F(\mathbf{u}, \mathbf{v}) = 4\iint_{-\infty}^{\infty} \Phi_\varepsilon(\varkappa)(1-\cos \varkappa\mathbf{u})(1-\cos \varkappa\mathbf{v})\, d^2\varkappa \quad (2)$$

with the boundary condition $E(0, \rho) = E_0 = \text{const}$, where $k = 2\pi/\lambda$, λ is the wavelength, the vectors ρ, $\mathbf{u} = \{y_1, z_1\}$, $\mathbf{v} = \{y_2, z_2\}$ lie in the plane $x = \text{const}$, $\Phi_\epsilon(\kappa)$ is the three-dimensional spectral density of fluctuations of the dielectric constant ϵ.

The parameter characterizing the properties of the turbulent medium is the structural characteristic C_ϵ^2 of the inhomogeneities of the dielectric constant which appears in the structural function D_ϵ:

$$D_\varepsilon(r) = \langle(\varepsilon(\mathbf{r}_0) - \varepsilon(\mathbf{r}+\mathbf{r}_0))^2\rangle = C_\varepsilon^2 r^{2/3}. \quad (3)$$

The structure function (3) corresponds to the three-dimensional spectral density

$$\Phi_\varepsilon(\varkappa) = AC_\varepsilon^2 \varkappa^{-11/3}, \quad A = \Gamma(8/3)\sin(\pi/3)(2\pi)^{-2}. \quad (4)$$

Formally in order that equation (2) should have an analytic solution it is necessary[1)] that

$$\left|\left(\frac{\partial^2}{\partial y_1 \partial y_2} + \frac{\partial^2}{\partial z_1 \partial z_2}\right) F(\mathbf{u}, \mathbf{v})\right|$$

should be bounded for all $\mathbf{u}$ and $\mathbf{v}$. This condition will be satisfied if for $\kappa \to \infty$ the spectral density $\Phi_\epsilon(\kappa)$ falls off faster than κ^{-4}, and correspondingly $D'_\epsilon(0) = 0$. Without adversely affecting the subsequent discussion it is sufficient to impose a more rigid but physically clear requirement, viz., the existence of an internal scale κ_m^{-1} for inhomogeneities of the dielectric constant. In this case the spectral density (4) remains unchanged for $\kappa \ll \kappa_m$, but for $\kappa \gg \kappa_m$ it falls off so rapidly that $D'_\epsilon(0) = 0$, $D''_\epsilon(0) \sim \kappa_m^{-2}$. It is quite obvious that if the internal scale κ_m^{-1} is much smaller than all the other dimensions, then it will not significantly affect the nature of the solution and the results obtained below should be regarded as a limiting case corresponding to $\kappa_m^{-1} \to 0$.

From the very method used by Tatarskiĭ[1] to the problem it follows in the derivation of equation (2) that there must exist at least two characteristic dimensions: the longitudinal one L_T and the transverse one l_T. We introduce a change of variables:

$$x=L_T\xi,\ u=l_T\eta,\ v=l_T\zeta. \qquad (5)$$

For the spectrum (4) we define the dimensions L_T and l_T in such a manner that in substituting (4) and (5) into (2) the latter should not contain any parameters. The dimensions L_T and l_T satisfying this condition are equal to:

$$l_T=(\pi AC_\varepsilon^2k^3)^{-3/11},\ L_T=(\pi AC_\varepsilon^2k^{7/6})^{-6/11},\ L_T=kl_T^2. \qquad (6)$$

In terms of the new variables (5) and (6) the equation for Γ_4:

$$\frac{\partial\Gamma_4}{\partial\xi}=i\nabla_\eta\nabla_\zeta\Gamma_4-f(\eta,\zeta)\Gamma_4,$$
$$f(\eta,\zeta)=\iint_{-\infty}^{\infty}|\varkappa|^{-11/3}(1-\cos\varkappa\eta)(1-\cos\varkappa\zeta)\,d^2\varkappa=\text{const}|\eta^2+\zeta^2|^{5/6}, \qquad (7)$$

no longer contains any parameters and its solution

$$\Gamma_4(\xi,\eta,\zeta)=\Gamma_4(x/L_T,\ u/l_T,\ v/l_T) \qquad (8)$$

will be a universal function of dimensionless variables.

The dimensions L_T and l_T have a well-defined physical meaning. Substituting $x = L_T$ into formula (9):

$$\beta_0^2(x)=\frac{\pi^3}{2}\frac{AC_\varepsilon^2x^{11/6}k^{7/6}}{\Gamma(^{17}/_6)\sin(5\pi/12)}\approx 0.31C_\varepsilon^2k^{7/6}x^{11/6}, \qquad (9)$$

for the evaluation of the relative dispersion of the intensity fluctuations $\beta_0^2(x)$ in the approximation of the method of smooth perturbations[12] we obtain the following expression:

$$\beta_0^2(L_T)={}^1/_2\pi^2[\sin(5\pi/12)\Gamma(^{17}/_6)]^{-1}\approx 3.$$

Consequently, L_T is that distance at which the calculation by means of perturbation theory produces an already considerable error.

The transverse dimension l_T, as can be seen from (7), is smaller by a factor of $(2\pi)^{1/2}$ than the radius of the Fresnel zone for a distance L_T in free space. However, a calculation of the coherence function of the second order $\Gamma_2(x,\rho) = \langle E(x,\rho_0+\rho)\cdot E^*(x,\rho_0)\rangle$ in a turbulent medium leads in accordance with[4,5] to the result that $\Gamma_2(L_T, l_T)/E_0^2 \ll 1$. This means, in turn, that the concept of the Fresnel zone which retains its meaning for a randomly inhomogeneous medium for $x \ll L_T$, loses its meaning for $x \gtrsim L_T$.

From (1) it follows that in a plane wave the relative dispersion $\beta^2(x)$ and the correlation coefficient $b_I(x,\rho)$ of the fluctuations of intensity $I(x,\rho) = E(x,\rho)E^*(x,\rho)$ are expressed in terms of Γ_4 in the following manner:

$$\beta^2(x)=\frac{\Gamma_4(x,0,0)-[\Gamma_2(x,0)]^2}{[\Gamma_2(x,0)]^2},\quad b_I(x,\rho)=\frac{\Gamma_4(x,\rho,0)-[\Gamma_2(x,0)]^2}{\Gamma_4(x,0,0)-[\Gamma_2(x,0)]^2}. \qquad (10)$$

The one-dimensional spatial spectrum $U(x,\kappa)$ is determined in terms of the correlation coefficient by:

$$U(x,\varkappa)=\frac{\varkappa}{\pi}\int_0^\infty b_I(x,\rho)\cos\varkappa\rho\,d\rho. \qquad (11)$$

From (8), (10), (11) it follows that $\beta^2(x)$, $b_I(x,\rho)$, $U(x,\kappa)$ can be expressed in terms of certain universal functions f_β, f_b, f_U by:

$$\beta^2(x)=f_\beta\left(\frac{x}{L_T}\right),\quad b_I(x,\rho)=f_b\left(\frac{x}{L_T},\frac{\rho}{l_T}\right),$$
$$U(x,\varkappa)=f_U\left(\frac{x}{L_T},\varkappa l_T\right), \qquad (12)$$

which must be determined from experimental data.

Utilizing the equations for the coherence functions of order $2n$:

$$\Gamma_{2n}=\langle E(x,\rho_1)\ldots E(x,\rho_n)E^*(x,\rho_{n+1})\ldots E^*(x,\rho_{2n})\rangle,$$

obtained by Chernov[13] and Klyatskin[14] it can be shown that all the statistical moments of the intensity fluctuations $\langle(I(x))^n\rangle/\langle I(x)\rangle^n$ for a plane wave depend only on the parameter x/L_T. Consequently, the distribution of probabilities of the random quantity $I(x)/\langle I(\kappa)\rangle$ is determined only by this parameter.

It is not difficult to obtain similarity relations analogous to (12) in the case of a power law spectral density of the dielectric constant with the exponent ν in the range $-5 < \nu < -3$.

It is convenient to use the dimensions l_T and L_T in the case of strong fluctuations when it is not possible to utilize results obtained by the methods of perturbation theory. In the case of weak fluctuations $(x/L_T \ll 1)$ the characteristic transverse dimension for a turbulent medium with a spectral density of inhomogeneities (4) is the radius of the first Fresnel zone $(\lambda x)^{1/2}$[12]. The connection between l_T and $(\lambda x)^{1/2}$ is given by the following obvious formula: $l_T = (\lambda x)^{1/2}(2\pi x/L_T)^{-1/2}$. A characteristic longitudinal dimension in the case of weak fluctuations is the distance x. The connection between $\beta_0^2(x)$ and x/L_T is obtained from (9) and (6):

$$\beta_0^2(x)/\beta_0^2(L_T)=(x/L_T)^{11/6}. \qquad (13)$$

The similarity relations (12) derived above refer to the case of a plane wave incident on a medium containing inhomogeneities. In an experiment one always has to deal with beams of light which, for example, are modelled by the following boundary condition:

$$E(0,\rho)=E_0\exp[-\rho^2/2\alpha_0^2+ik\rho^2/2F_0],$$

where α_0 and F_0 are the width and the focal distance of the beam.

The restricted size of the beam leads to the absence of local isotropy for $E(x,\rho)$ in the plane x = const and, as a consequence of this, to an equation for Γ_4 more complicated than (2). Correspondingly the number of parameters of the problem is increased. It is not difficult to show that the simplest characteristic—the value of $\beta^2(x)$ for the axis of a restricted beam—depends on three dimensionless parameters:

$$\beta^2(x)=f_b(x/L_T,\ k\alpha_0^2/x,\ x/F_0). \qquad (14)$$

It is not possible without solving the corresponding equation to predict theoretically the degree to which the restricted dimension of the beam is significant for the fluctuation of intensity in the region near its axis. As will be shown below, the experiments that have been carried out give some idea of the effect of the restricted dimension of the beams.

Conditions for carrying out an experimental investigation of the similarity of strong fluctuations in intensity can be satisfied, as is shown by the estimates of the values of C_ϵ^2 and κ_m, when laser beams propagate in the atmosphere. Realization of sufficiently clean experiments in the laboratory utilizing artificially created inhomogeneous media encounters certain difficulties, and the corresponding measurements have not been carried out.

Inhomogeneities of the dielectric constant in the atmosphere are described by the Kolmogorov structure function (3)[15]. Moreover, over quite a wide range of scales of inhomogeneities in the atmosphere the hypothesis is valid, with a high degree of accuracy, of the "frozen-in" turbulence[16] the meaning of which consists of the circumstance that the inhomogeneities are carried by the wind with an average velocity V without significant changes over a distance which considerably exceeds their dimensions. As a consequence of this in a plane wave the autocorrelation coefficient $\tilde{b}_I(\tau)$ and the frequency spectrum $\tilde{U}(\omega)$ corresponding to it are related to the spatial characteristics by the following formula:

$$\tilde{b}_I(x, \tau) = b_I(x, \rho = V_\perp \tau), \quad \tilde{U}(x, \omega) = U(x, \varkappa = \omega / V_\perp), \qquad (15)$$

where $V_\perp$ is the velocity of transfer transversely to the x direction.

The use of the "frozen-in" property (15) yields a considerable advantage in carrying out experiments, since from a single realization it turns out to be possible to determine the functions $\tilde{b}_I(x, \tau)$ or $\tilde{U}(x, \omega)$ over a wide range of values of their second argument, and not only the value of $b_I(x, \rho)$. The difference between the frequency spectrum of the intensity fluctuations along the axis of a restricted collimated beam for $k\alpha_0^2/x \gtrsim 1$ and the spectrum in a plane wave has been investigated by Time[17]; she has shown that the restricted nature of the beam leads to the suppression of high frequencies.

The possibility of the dropping of the spectrum in the high frequency region was taken into account by us in utilizing the data of measurements at a wavelength of 10.6 μm, where it is difficult in practice to satisfy simultaneously the two requirements: $k\alpha_0^2/x \gg 1$, $\beta_0^2 \gg 1$.

3. In order to check the similarity relations (12), (14), involving the utilization of (15) measurements of $\beta^2(x)$, $b_I(x, \rho)$, $\tilde{U}(x, \omega)$ were carried out using laser beams of wavelengths $\lambda = 0.63$ and 10.6 μm over path lengths from x = 0.25 to 16.3 km with different beam diameters. The lasers generated only axial modes. Measurements of β^2, b_I, $\tilde{U}$ were carried out over flat steppe areas or over a water surface, and this guaranteed the constancy of the parameters of the medium along the path of propagation of light. The structural characteristics C_ϵ^2 and the velocity $V_\perp$ were determined independently. The method of carrying out the measurements is described in[10,18-20].

Figure 1 presents the results of measurements of $\beta(x)$ at a wavelength of $\lambda = 0.63$ μm as a function of $\beta_0 = 1.72\,(x/L_T)^{11/12}$ for wide collimated ($F_0 = \infty$) beams obtained at distances of x = 0.25; 1.75 and 8.5 km. The use of the parameter β_0 in place of x/L_T facilitates comparison with numerous experimental data obtained earlier (review[21]) in the case of weak fluctuations. For small $\beta_0 < 1$ the experiment agrees well with results obtained in accordance with perturbation theory. For values of $\beta_0 \approx 1.5$ the dependence of β on β_0 has a weakly pronounced maximum and for large β_0 it falls off slowly. The presence of a more pronounced maximum in the dependence of $\sigma(\beta_0) = \langle(\ln I - \langle \ln I \rangle)^2\rangle^{1/2}$ on β_0 was noted earlier in[22,23]. The difference in the behavior of experimentally determined functions $\beta(\beta_0)$

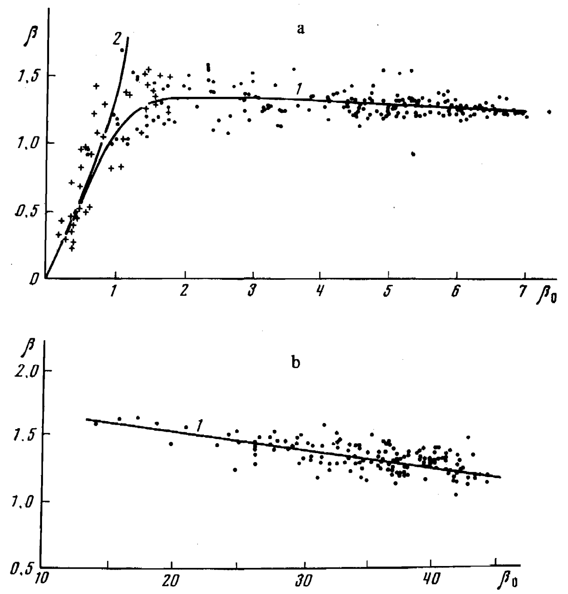

FIG. 1. Dependence of β on β_0 for wide collimated beams at a wavelength of 0.63 μm: a) $k\alpha_0^2/x = 130$, crosses—x = 0.25 km, dots—x = 1.75 km: b) $k\alpha_0^2/x = 27$, x = 8.5 km. Curves: 1-averaged over experimental data, 2-calculation by the method of smooth perturbations ($\beta^2 = \exp(\beta_0^2) - 1$).

and $\sigma(\beta_0)$ in the neighborhood of the maximum, as has been shown earlier[19], is explained by the fact that in this region the distribution of the probabilities of fluctuations of lnI differs to the greatest degree from the normal law by the existence of more pronounced regions of low values.

In the measurements at greater β_0 shown in Fig. 1 obtained at a distance of x = 8.5 km the same tendency is exhibited towards a certain diminution of the relative fluctuations with increasing β_0, although the average taken over all the measurements of Fig. 1b is somewhat greater than the values of β in Fig. 1a. In comparing the data of Figs. 1a and 1b it is necessary, however, to take into account the fact that measurements at distances of x = 1.75 and 8.5 km were carried out using an identical half-meter mirror collimator, and therefore in accordance with (14) the dependence on the wave parameter $k\alpha_0^2/x$ is manifested in them although to a small degree since both beams are still sufficiently wide.

A visual impression of the dependence of the fluctuations of intensity on the parameter $k\alpha_0^2/x$ is given by Fig. 2. It shows averaged results of measurements of β along the axis of the beams at a wavelength of $\lambda = 0.63$ μm at distances from 1.75 to 16.3 km for values of $\beta_0 > 2$. In this range the dependence of β on β_0 is very weak all the other conditions remaining the same. The table shows the parameters of the beams for which the measurements shown in Fig. 2 were carried out. The dependence of β on the parameter $k\alpha_0^2/x$ exhibits an easily noted maximum at $k\alpha_0^2/x \sim 1$. Fluctuations in a spherical wave ($k\alpha_0^2/x \ll 1$) are somewhat greater than in wide beams ($k\alpha_0^2/x \gg 1$). The asymptotic values of β for a plane wave and $\beta_0 \gg 1$[25,26] generally agree with experimental data. We note that when the beam is decollimated the fluctuations along its axis approach a value corresponding to a spherical wave: in wide beams for $k\alpha_0^2/x \gg 1$ they increase, while in narrow beams, for $k\alpha_0^2/x \sim 1$ they diminish. The existence of a maximum in β in beams narrowest

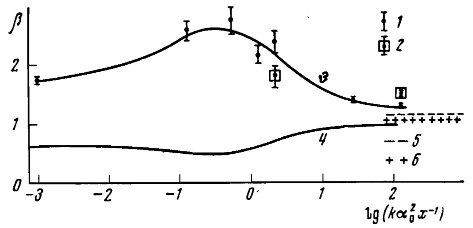

FIG. 2. Dependence of β on $k\alpha_0^2/x$ for $\beta_0 > 2$. Experimental data: 1-collimated beam, 2-divergent beam, 3-empirical dependence; vertical bar—mean square error of the average value of β. Calculations: 4-β/β_0 for $\beta_0 \ll 1$; asymptotic ($\beta_0 \to \infty$) values for a plane wave ($k\alpha_0^2/x \to \infty$); 5-according to [25], 6-according to [26].

$k\alpha_0^2/x$	Number of measurements	β_0	x, km	Place of measurements	Beam Geometry
130	72	2÷7	1.75	Tsimlyansk	Collimated
26	52	10÷40	8.5	Tsimlyansk	»
2.2	21	4÷7	16.3	Sevan	»
1.35	5	16÷38	11.6	Odessa*	»
0.56	6	4÷7	16.3	Sevan	»
0.14	42	4÷7	16.3	Sevan	»
0.001	31	4÷7	16.3	Sevan	»
130	61	2÷7	1.75	Tsimlyansk	Divergent
2.2	5	4÷7	16.3	Sevan	»

*Data in this line from [24].

at the receiving end and the decrease in β when such beams are decollimated can not ascribed, as will be shown later in the discussion of measurements of correlation, only to the effect of an accidental displacement of the beam. It is of interest to note that the dependence of β on $k\alpha_0^2/x$ in the case of strong fluctuations is qualitatively opposite to the calculation by means of perturbation theory[27]. It should be noted that the values of β remain practically constant within the limits of the half width of the beam smeared out by an inhomogeneous medium.

Measurements of the spatial correlation functions $b_I(\rho)$ the results of which have been partially published earlier[19] provide evidence that for $\beta_0 < 1$ experiment agrees with calculations by the method of smooth perturbations[12]. Correlation of fluctuations in intensity in the portion of wide collimated beams adjacent to the axis ($k\alpha_0^2/x \gg 1$) decreases smoothly to zero when the observation points are separated by a distance $\rho \sim (\lambda x)^{1/2}$, while as ρ is increased further a weakly pronounced minimum is observed in which $b_I(\rho) < 0$. For $\beta_0 \gtrsim 1$ the correlation falls off rapidly as the points are separated by a distance $\rho \sim l_T$, which is considerably smaller than $(\lambda x)^{1/2}$: $l_T = 1.35\beta_0^{-6/11}(\lambda x)^{1/2}$. As ρ is increased further the correlation $b_I(\rho)$ falls off slowly both in collimated and in divergent beams. For measured correlation coefficients for sufficiently large β_0 no significant negative values of $b_I(\rho)$ are observed as the observation points are maximally separated right up to the half-width of the narrowest beams. This provides evidence of the fact that a displacement of the beam as a whole has a small effect on the magnitude of the intensity fluctuations in the region near the axis of the beam. Owing to the slow falling off of the correlation coefficient and the low accuracy of measurement of small values of $b_I(\rho)$ at large ρ, no success has been achieved in verifying for $\beta_0 > 1$, on the basis of experimental data in wide beams, that the condition

$$\int_0^\infty b_I(\rho)\rho\, d\rho = 0, \qquad (16)$$

which is a consequence of the law of conservation of energy for a plane wave, is satisfied. However, for $\beta_0 < 1$ in wide beams condition (16) is satisfied with sufficient accuracy.

The most detailed data on similarity in a plane perpendicular to the direction of propagation are obtained from measurements of frequency spectra of intensity fluctuations. Measurements of frequency spectra were carried out at two wavelengths: $\lambda = 0.63\ \mu$m and $\lambda = 10.6\ \mu$m. Figure 3 presents selected examples from the total number of 107 experimentally measured spectra $\tilde{U}(x, \omega)$. The examples presented give us an idea of the possible forms of spectral dependence and of frequencies corresponding to maxima of the spectra as the wavelength, the distance and the characteristics of the medium are varied. It should be emphasized that the positions of the maxima in the scale of frequencies ω differ by more than three orders of magnitude.

Verification using individual measurements has shown that for $\beta_0 > 1$ and $k\alpha_0^2/x \gtrsim 1$ one does not observe any influence of the geometry of the beams on the spectrum within the utilized frequency range in agreement with estimates made on the basis of the results of Time[17]. When experiments are carried out in the atmosphere the possibility of specifying the values of β_0 in advance is excluded. For a more compact presentation of the experimental data we have to divide the results of the measurements into groups with close values of β_0 and to utilize for subsequent discussion averages over these groups. Therefore all the spectra obtained for $k\alpha_0^2/x > 1$ were grouped in accordance with the values of the parameter β_0 after going over to the dimensionless wave numbers $\Omega_T = \omega l_T/V_\perp$. Figure 4 presents the spectra $U(\Omega_T, \beta_0)$ averaged over the groups as functions of Ω_T and β_0.

Comparison of Figs. 4 and 3 shows that after transition to dimensionless frequencies all the spectra become close to one another and are positioned in a regular manner depending on the parameter β_0. From Fig. 4 it may be seen that the greatest similarity of the spectra for different values of β_0 occurs in the domain of high frequencies and at the position of the maxima. This provides evidence of the fact that the greatest contribution to the dispersion of the intensity fluctuations for $\beta_0 > 1$ is made by inhomogeneities of the intensity of the field of dimensions of the order of l_T, and the scale l_T is in this sense a characteristic transverse scale. As Tatarskiĭ has shown in[25], where this scale has been obtained as a result of an approximate solution of Eq. (2), it characterizes the dimensions of random bursts of intensity.

It is not excluded that a small difference in the spectra measured at wavelengths of 10.6 and 0.63 μm for close β_0 is due to a difference in the ratios of the value of l_T corresponding to these wavelengths to the internal scale of turbulence κ_m^{-1}, since the index of the power in the structure function (3) can differ somewhat from $2/3$ as the internal scale is approached. Therefore a further measurement of the spectra of intensity fluctuations at a wavelength of 10.6 μm, but for correspondingly greater β_0 than have been attained in the present work is of indubitable interest.

An explicit dependence of the spectra on the parameter β_0 is observed in the low frequency region. As β_0 increases the low frequency (large scale) components of the fluctuations of the intensity field increase. At the

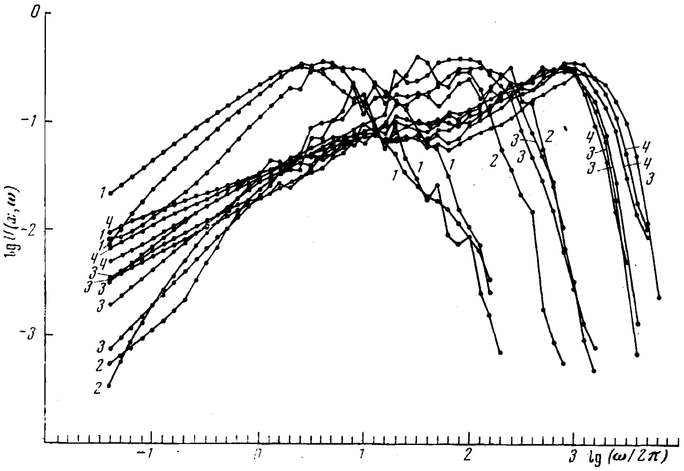

FIG. 3. Examples of frequency spectra of intensity fluctuations: 1) $\lambda = 10.6\ \mu m$, $x = 16.3$ km; 2) $\lambda = 0.63\ \mu m$, $x = 16.3$ km; 2) $\lambda = 0.63\ \mu m$, $x = 16.3$ km; 3) $\lambda = 0.63\ \mu m$, $x = 1.75$ km; 4) $\lambda = 0.63\ \mu m$, $x = 8.5$ km.

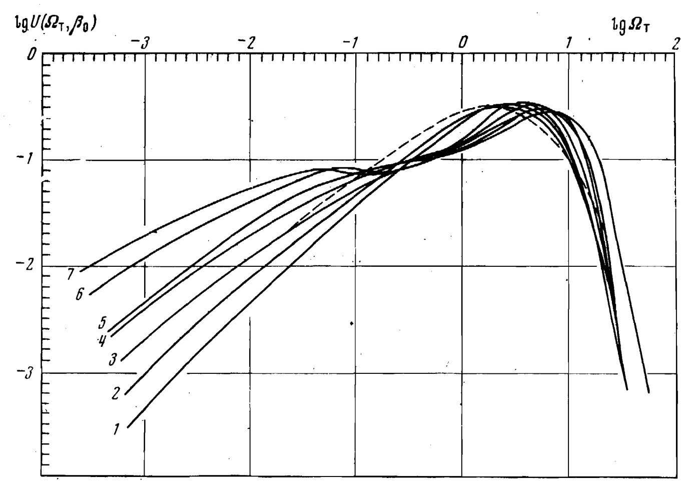

FIG. 4. Frequency spectra of intensity fluctuations as functions of the dimensionless frequency Ω_T and β_0. Dotted curve corresponds to $\lambda = 10.6\ \mu m$, $\beta_0 = 1$. Solid lines correspond to $\lambda = 0.63\ \mu m$. 1) $\beta_0 = 0.95$; 2) $\beta_0 = 1.6$; 3) $\beta_0 = 2.8$; 4) $\beta_0 = 4.9$; 5) $\beta_0 = 7$; 6) $\beta_0 = 27$; 7) $\beta_0 = 35$.

largest values of β_0 attained in the present work a second maximum in the spectral dependence $U(\Omega_T, \beta_0)$ begins to appear at frequencies of the order $\Omega_T \sim 0.05$.

As a consequence of (16) for a plane wave the ratio $U(\Omega_T, \beta_0)/\Omega_T$ must be bounded as $\Omega_T \to 0$. This condition is well satisfied in the case of experimental spectra for $\beta_0 \leq 2$. However, for greater values of β_0 due to the increase in the role played by low frequency components of intensity fluctuations it is necessary to increase appreciably the time for the observation, so as to be able to penetrate still further into the region of low frequencies, then has been possible in our measurements. But in the domain of such low frequencies yielding a fairly small (not greater than 1%) contribution to the dispersion, the effect of the different sources of interference masks the effects associated with the inhomogeneity of the field of dielectric constant.

As the result of an approximate solution of Eq. (2), Gochelashvili and Shishov[28] have proposed a characteristic high frequency transverse scale for intensity fluctuations which coincides with the range of coherence of the field. The range of coherence of the field l_C is

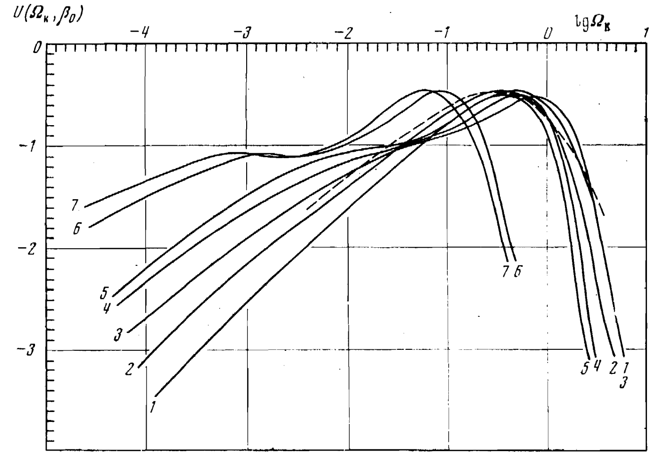

FIG. 5. Frequency spectra of intensity fluctuations as functions of the dimensionless frequency Ω_C and β_0. The notation is the same as in Fig. 4.

determined by the coherence function of the second order $\Gamma_2(\rho)$ and for the spectrum (4) can be taken to be equal to[4,5]

$$l_C = (C_\varepsilon^2 k^2 x)^{-3/5} = 0.3 l_T \beta_0^{-36/55}.$$

For large β_0 the value of l_C differs appreciably from the scale l_T introduced earlier (6). In order to determine how the scale l_C manifests itself in experimental data, Fig. 5 gives spectra of intensity fluctuations as functions of the dimensionless wave number $\Omega_C = \omega l_C / V_\perp$ and the parameter β_0. A simple comparison of the graphs shown in Figs. 4 and 5 provides evidence of the fact that the scale l_T should be preferred as a characteristic transverse high frequency scale of intensity fluctuations.

On the whole the experimental data obtained confirm the presence in the problem of the fluctuations of intensity of light in a turbulent medium of two characteristic scales: the longitudinal scale L_T and the transverse scale l_T.

The dimensionless functions represented in Figs. 1, 2, and 4 can be regarded as estimates of the universal functions (12) and (14), describing strong fluctuations in the intensity of light propagated in a turbulent medium with a Kolmogorov spectral density of inhomogeneities of the dielectric susceptibility.

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Note added in proof (October 9, 1974): Recently carried out measurements of spectra of intensity fluctuations at a wavelength of the radiation of 10.6 μm under conditions of strong fluctuations have confirmed the similarity relations (12).

1)This restriction was pointed out to us by V. I. Tatarskiĭ.

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