On the theory of the thermomagnetic effect in a Knudsen gas

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The effect of a magnetic field on the heat flow in a free molecular gas (the thermomagnetic effect) is considered. An expression for the heat flux is derived under the assumption of an arbitrary dependence of the attachment and evaporation coefficients (α) on the directions of the velocity and angular-momentum vectors of the molecule and the normal vector to the surface. It is shown that the observed negative sign of the increment ΔQ of the heat flux when the field is applied can be accounted for if α is invariant under simultaneous change of sign of the normal vector to the surface and the normal component of the velocity vector of the molecule. Models are proposed for the interaction between a molecule and the wall and can explain the observed ratio of ΔQ for a field parallel to the wall to ΔQ for a field perpendicular to the wall.

1. INTRODUCTION

The effect of a magnetic field on heat transport in a molecular Knudsen gas in the gap between two plates (the thermomagnetic effect) was previously predicted[1,3] and detected[2,4]. This phenomenon is associated with the orientation dependence of the interaction of the gas molecules with the surface of the solid walls (plates). The change in the heat flux (ΔQ) when the field H is applied depends on the product of the molecular precession frequency $\dot{\omega}$ by the molecular mean free flight time τ from wall to wall. Since $\omega = \gamma H$ (γ is the gyromagnetic ratio of a molecule) and $\tau = L/\bar{v}$ (L is the distance between the plates and $\bar{v}$ is the mean velocity of the molecules), ΔQ is a function of the product HL. As HL increases, ΔQ rises to a limiting (saturation) value ΔQ_{sat}. The heat flux depends on the orientation of the magnetic field with respect to the plates. Moreover, it has been shown that ΔQ can sometimes be an oscillating function of HL. These heat flux oscillations, which correspond to the molecule precessing about the magnetic field vector during its flight from wall to wall through one, two, ... complete revolutions, were observed[2] in N_2, NF_3, and CO atmospheres.

The orientation dependence of the interaction of the molecules with the wall is taken into account theoretically by generalizing the well-known Maxwell boundary condition[5,6], which relates the distribution functions for the incident and reflected molecules, to the case in which the diffuse-reflection coefficient (α) is a function of the velocity and angular momentum vectors of the molecule (v and M) and the normal vector to the surface (k). It was assumed that this interaction can be described by a model in which only one nonspherical term is retained in the expansion of α in spherical functions of **v**, **M**, and **k**:

$$\alpha=\alpha_0(1+\mu\mathbf{k}[\mathbf{v}\times\mathbf{M}]) \tag{1.1}$$

(μ is a small parameter). This model is extremely plausible, for according to Eq. (1.1) the most probable direction for the angular momentum of a molecule leaving the wall is perpendicular to its velocity.

It has been found[3] that according to this model the heat flux should increase when the magnetic field is applied and that the ratio of the increment of the heat flux on applying a field perpendicular to the normal vector k ($\Delta Q_\perp$) to the corresponding increment when a field of the same strength is applied parallel to k ($\Delta Q_\parallel$) should be independent of the applied field strength and equal to one-half[1)]:

$$\Delta Q_\perp/\Delta Q_\parallel={}^1/_2. \tag{1.2}$$

Experiments with H_2, N_2, SF_6, and CO_2[4], however, showed that the heat flux decreases when the magnetic field is applied and that the ratio $\Delta Q_\perp/\Delta Q_\parallel$ is constant for strong enough fields (i.e., at saturation) and lies in the interval 1.6–2 for the gases listed above.

It is accordingly desirable to base our discussion of the effect of a magnetic field on the heat flux in a molecular Knudsen gas on more general ideas concerning the interaction of the gas molecules with the wall surfaces. As before[1,3], we shall assume the walls to have different temperatures (T_0 and T_L) but to be otherwise identical. The problem is to calculate the heat flux for an arbitrary dependence of the diffuse-reflection coefficient α on **v**, **M**, and **k**. Below we present the results of a study of the sign and anisotropy of the effect (i.e., of the sign of ΔQ and the magnitude of $\Delta Q_\perp/\Delta Q_\parallel$), using various model functions to represent the dependence of α on **v**, **M**, and **k**. In particular, we show that the heat flux should decrease when the field is applied provided the nonspherical part of α does not change when the normal component of the velocity vector and the vector normal to the wall are replaced by their opposites.

2. DISTRIBUTION FUNCTION

We shall use a coordinate system such that one of the walls lies in the xy plane, and the other, in the plane z = L. The kinetic equation for the problem under consideration is

$$v_z\frac{\partial f}{\partial z}+\gamma[\mathbf{M}\times\mathbf{H}]\frac{\partial f}{\partial \mathbf{M}}=0. \tag{2.1}$$

The solution to this equation can be written in the form[3]:

$$f=\sum_{lmk}\chi_{lm}D_{mk}^{(l)*}Y_{lk}\left(\frac{\mathbf{M}}{M}\right)\exp(im\omega z/v_z). \tag{2.2}$$

Here $D_{mk}^{(l)}(\varphi_H, \theta_H, \psi)$ is the finite-rotation matrix (θ_H and φ_H are the spherical angles defining the direction of the field, and ψ is the angle of rotation about the direction of the field). The coefficients χ_{lm} are functions of v and M^2 and are to be determined from the boundary conditions.

Following[3], we write the boundary conditions relating the distribution function f^+ for molecules moving in the forward direction to the distribution function f^- for molecules moving in the backward direction in the form

$$f^+(\mathbf{v},\mathbf{M})=[1-\alpha(\mathbf{v},\mathbf{M},\mathbf{k})]f^-(\mathbf{v}-2\mathbf{k}(\mathbf{kv}),\mathbf{M})+\alpha(\mathbf{v},\mathbf{M},\mathbf{k})\, f_0(v^2,M^2),\quad z=0; \tag{2.3}$$

$$f^-(\mathbf{v},\mathbf{M})=[1-\alpha(\mathbf{v},\mathbf{M},-\mathbf{k})]f^+(\mathbf{v}-2\mathbf{k}(\mathbf{kv}),\mathbf{M})+\alpha(\mathbf{v},\mathbf{M},-\mathbf{k})\times f_L(v^2,M^2),\quad z=L,$$

where f_0 and f_L are the Maxwell distribution functions for temperatures T_0 and T_L, and α represents the attachment of evaporation coefficient.

Further, we shall assume that α does not depend strongly on v and M and can be expanded as follows[3]:

$$\alpha(\mathbf{v},\mathbf{M},\mathbf{k})=\alpha_0\Big[1+\varepsilon\sum_{l_1+l_2+l_3\neq0}\alpha_{l_1l_2l_3}\sum_m\begin{pmatrix} l_1 & l_2 & l_3\\ m & -m & 0\end{pmatrix}Y^v_{l_1m}Y^M_{l_2-m}Y^k_{l_30}\Big];$$

$$Y_{lm}{}^a=(a/a_0)^lY_{lm}(\mathbf{a}/a),\quad v_0=(2T_0/m)^{1/2},\quad M_0=(2IT_0)^{1/2}, \tag{2.4}$$

where ϵ is a small dimensionless parameter characterizing the deviation of the angular distribution of the molecules evaporated from the wall from the distribution that would obtain in the case of isotropic diffuse reflection. The dimensionless expansion coefficients $\alpha_{l_1l_2l_3}$ are functions of v^2 and M^2. The fact that k is parallel to the z axis is taken into account in expansion (2.4).

On substituting the values of the functions $f^\pm$ from Eq. (2.2) into the boundary-condition equations (2.3), multiplying by $Y^*_{s\sigma}(\mathbf{M}/M)$, and integrating over the direction of M, we obtain the following equations for the boundary conditions:

$$\chi_{s\sigma}{}^+=(1-\alpha_0)\chi_{s\sigma}{}^--\alpha_0\varepsilon\sum_{l_2lmk}\chi_{lm}{}^-B'_{l_2k}C_{s\sigma lml_2k}+(4\pi)^{1/2}\alpha_0f_0\delta_{s0}+\alpha_0\varepsilon f_0B_{s\sigma}', \tag{2.5}$$

$$\chi_{s\sigma}{}^-e^{-i\sigma\eta}=(1-\alpha_0)\chi_{s\sigma}{}^+e^{i\sigma\eta}-\alpha_0\varepsilon\sum_{l_2lmk}\chi_{lm}{}^+e^{im\eta}B''_{l_2k}C_{s\sigma lml_2k}+(4\pi)^{1/2}\alpha_0f_L\delta_{s0}+\alpha_0\varepsilon f_LB_{s\sigma}''.$$

Here we have used the following notation:

$$\eta=\omega L/\zeta,\quad \zeta=\begin{cases} v_z & (\mathbf{vk}>0)\\ -v_z & (\mathbf{vk}<0)\end{cases}, \tag{2.6}$$

$$B_{lk}'=B_{lk}(v_x,v_y,\zeta,\mathbf{k}),\quad B_{lk}''=B_{lk}(v_x,v_y,-\zeta,-\mathbf{k}),$$

$$B_{l_2k}=\left(\frac{M}{M_0}\right)^{l_2}\sum_{l_1l_3}\alpha_{l_1l_2l_3}\sum_m\begin{pmatrix} l_1 & l_2 & l_3\\ m & -m & 0\end{pmatrix}Y^v_{l_1m}D^{(l_2)}_{k-m}Y^k_{l_30}, \tag{2.7}$$

$$C_{s\sigma lml_2k}=(-1)^{s-\sigma}i^{s+l+l_2}\begin{pmatrix} s & l & l_2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} s & l & l_2\\ -\sigma & m & k\end{pmatrix}\left[\frac{(2s+1)(2l+1)(2l_2+1)}{4\pi}\right]^{1/2} \tag{2.8}$$

(for Eq. (2.8) see[7], for example).

We can seek the solution to Eqs. (2.5) as an expansion in powers of ϵ:

$$f^\pm=\sum_n f_n{}^\pm,\quad f_n{}^\pm\sim\varepsilon^n,\quad \chi^\pm_{ns\sigma}\sim\varepsilon^n. \tag{2.9}$$

Only the difference $\chi^+_{200}-\chi^-_{200}$ will be required for the subsequent calculations. Solving Eqs. (2.5) by the method mentioned above, we obtain

$$\chi^+_{200}-\chi^-_{200}=\frac{\alpha_0{}^2\varepsilon^2}{(2-\alpha_0)^2}\frac{(f_0-f_L)}{(4\pi)^{1/2}}\sum_{l_2m}(-1)^{l_2+m}\Delta_m{}^{-1}\cdot$$

$$\cdot[B'_{l_2m}B''_{l_2-m}+B'_{l_2-m}B''_{l_2m}-(1-\alpha_0)e^{im\eta}B'_{l_2m}B'_{l_2-m}-(1-\alpha_0)e^{im\eta}B''_{l_2m}B''_{l_2-m}], \tag{2.10}$$

$$\Delta_m=e^{-im\eta}-(1-\alpha_0)^2e^{im\eta}.$$

Equation (2.10) can be simplified. We note that some of the terms in expansion (2.4) and sum (2.7) (those for which l_1+m+l_3 is even) do not change under the simultaneous substitution $v_z\to-v_z$, $\mathbf{k}\to-\mathbf{k}$, while the other ones (those for which l_1+m+l_3 is odd) change sign under that substitution. Let us denote terms of the first type by a, and terms of the second type by b. Then

$$B'_{l_2m}=a_{l_2m}+b_{l_2m},\quad B''_{l_2m}=a_{l_2m}-b_{l_2m}. \tag{2.11}$$

Substituting (2.11) into (2.10), we obtain

$$\chi^+_{200}-\chi^-_{200}=\frac{\alpha_0{}^2\varepsilon^2}{(2-\alpha_0)^2}\frac{(f_0-f_L)}{\sqrt{\pi}}\sum_{l_2m}(-1)^{l_2+m}[a_{l_2m}a_{l_2-m}\Psi_m-b_{l_2m}b_{l_2-m}\Phi_m], \tag{2.12}$$

in which

$$\Psi_m=[1-(1-\alpha_0)e^{im\eta}]/\Delta_m,\quad \Phi_m=[1+(1-\alpha_0)e^{im\eta}]/\Delta_m. \tag{2.13}$$

3. HEAT FLUX

The heat flux in the gas between the two plates is given by the equation

$$Q=\int_{\mathbf{vk}>0}v_zE(f^+-f^-)\,d\mathbf{v}\,d\mathbf{M},\quad E=\frac{mv^2}{2}+\frac{M^2}{2I}. \tag{3.1}$$

Now taking (2.2) and (2.9) into account, we have

$$Q=\sum_n Q_n,\quad Q_n\sim\varepsilon^n, \tag{3.2}$$

$$Q_n=(4\pi)^{1/2}\int_{\mathbf{vk}>0}v_zE(\chi^+_{n00}-\chi^-_{n00})M^2\,dM\,d\mathbf{v}.$$

It can be shown that the dependence of the heat flux on the strength and direction of the magnetic field is a second order effect in ϵ. Using Eqs. (3.2) and (2.12), we obtain

$$Q_2(\mathbf{H})=\frac{2\alpha_0{}^2\varepsilon^2}{(2-\alpha_0)^2}\int_{\mathbf{vk}>0}v_zE(f_0-f_L)\sum_{l_2m}(-1)^{l_2+m}\times[a_{l_2m}a_{l_2-m}\Psi_m-b_{l_2m}b_{l_2-m}\Phi_m]M^2\,dM\,d\mathbf{v}. \tag{3.3}$$

The dependence of Q_2 on the field strength (on the parameter HL) is determined by the functions Ψ_m and Φ_m (see Eqs. (2.13)), and its dependence on the field orientation angle θ_H, by the functions $D^{(l)}_{mk}(\varphi_H,\theta_H,\psi)$ that occur in the expressions for the a_{l_2m} and b_{l_2m}. We note that according to Eqs. (3.3), (2.11), and (2.7), the flux Q_2 is independent of the angles φ_H and ψ, in accordance with the symmetry of the problem.

4. GAS-WALL INTERACTION MODELS

It is not desirable to use a great many terms in expansion (2.4) to describe the effect under consideration, since in that case the number of unknown coefficients $\alpha_{l_1l_2l_3}$ in the expression for the heat flux in the field may exceed the number of parameters that can be unambiguously evaluated by comparison with experiment. In the following we shall therefore consider only a few terms in expansion (2.4). A description of the gas-wall interaction obtained in this way we shall call a model description.

For simplicity, the parameters $\alpha_{l_1l_2l_3}$ can be treated as constants, since the sign of ΔQ (i.e., the sign of the change in the heat flux on applying the magnetic field) and the quantity $\Delta Q_\perp/\Delta Q_\parallel$ are determined by the dependence of the diffuse-reflection coefficient α on the directions (but not the magnitudes) of the vectors v, M, and k. Further, we shall assume that $|T_0-T_L|\ll T_0$.

Let us first consider a model description of the gas-wall interaction in which the dependence of the fraction α of diffusely reflected molecules on the directions of the vectors v, M, and k is determined by just one of the terms in expansion (2.4) with definite values of l_1, l_2, and l_3:

$$\alpha=\alpha_0[1+\varepsilon A],\quad A=\sum_m\begin{pmatrix} l_1 & l_2 & l_3\\ m & -m & 0\end{pmatrix}Y^v_{l_1m}Y^M_{l_2-m}Y^k_{l_30}. \tag{4.1}$$

Here α_0 and ϵ are parameters to be determined by comparison with experiment. The model dependence of

$(l_1l_2l_3)$	Sign of $\Delta Q_{\parallel}$	Sign of $\Delta Q_{\perp}$	$(\Delta Q_{\perp}/\Delta Q_{\parallel})_{\xi\to\infty}$ for $0<\alpha_0<1$	$l_1l_2l_3$	Sign of $\Delta Q_{\parallel}$	Sign of $\Delta Q_{\perp}$	$(\Delta Q_{\perp}/\Delta Q_{\parallel})_{\xi\to\infty}$ for $0<\alpha_0<1$
(011)		+	∞	(122)	−	−	1
(110)	−	+	$(-\infty, -1/2)$	(221)	+	±	$(-1/8, 5/8)$
(111)	+	+	$1/2$	(222)	+	±	$(-\infty, 1)$
(211)	−	+	$(-\infty, -1/6)$	$(0ll)$, l even		−	$-\infty$
(112)	−	+	$(-\infty, -7/2)$	$(0ll)$, l odd		+	∞
(212)	+	+	$1/2$	$(1ll)$, l even	−	−	1
(022)		−	$-\infty$	$(1ll)$, l odd	+	+	$(0, 1)$
(220)	+	+	$(5/8, 1)$	$(l1l)$, $l=3,4\ldots$	+	+	$1/2$
(121)	+	+	$(0, 1)$				

α on **v**, **M**, and **k** described by Eq. (4.1) we shall call the $(l_1l_2l_3)$ model.

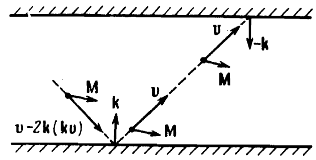
Below we shall consider various terms in expansion (2.4) up to the term with $l_1 = l_2 = l_3 = 2$, the terms with $l_1 = 0$ or $l_1 = 1$ and $l_2 = l_3 = 3, 4, \ldots$, and the terms with $l_2 = 1$ and $l_1 = l_3 = 3, 4, \ldots$. These terms are listed in the table. The terms with $l_2 = 0$ (i.e., that do not depend on M) are not discussed below since for these terms the dependence on the magnitude and direction of the magnetic field drops out of Eq. (3.3).

Let us investigate the sign of ΔQ as given by various models of the type of (4.1). The simplest of these models are those for which A contains only terms with one specific value of the modulus m:

$$A=\left(1-\frac{1}{2}\delta_{m0}\right)\left[\begin{pmatrix} l_1 & l_2 & l_3 \\ m & -m & 0\end{pmatrix} Y^{v}_{l_1m}Y^{M}_{l_2-m}+\begin{pmatrix} l_1 & l_2 & l_3 \\ -m & m & 0\end{pmatrix} Y^{v}_{l_1-m}Y^{M}_{l_2m}\right]Y^{k}_{l_30}. \tag{4.2}$$

These are the models $(0ll)$, $(1ll)$, and $(l1l)$ with $l = 1, 2, \ldots$; all the other models contain two or more terms of the type of (4.2). The sign of ΔQ as predicted by the models of the type of (4.2) can be determined for all allowable values of l_1, l_2, l_3, and m. As the calculations showed, ΔQ is negative (the heat flux decreases when the magnetic field is applied) for models of the type of (4.2) having even values of $l_1 + m + l_3$, when α is invariant under the substitution $v_z \to -v_z$, $\mathbf{k} \to -\mathbf{k}$ (terms of type a in Eq. (3.3)), and ΔQ is positive for models of the type of (4.2) with odd values of $l_1 + m + l_3$, when A changes sign under that substitution (terms of type b in Eq. (3.3)).

These results have a simple physical meaning. First let us consider the models (4.2) with even $l_1 + m + l_3$. From the fact that α is invariant under the substitution $v_z \to -v_z$, $\mathbf{k} \to -\mathbf{k}$ it follows that the probability that a (**v**, **M**) molecule (i.e., a molecule having velocity **v** and angular momentum **M**) will stick to the wall is equal to the probability that a $(\mathbf{v} - 2\mathbf{k}\mathbf{k}\cdot\mathbf{v}, \mathbf{M})$ molecule will stick to the other wall (see the figure). In accordance with the boundary conditions (2.3), the probability that a (**v**, **M**) molecule will evaporate from a wall is equal to the probability that a $(\mathbf{v} - 2\mathbf{k}\mathbf{k}\cdot\mathbf{v}, \mathbf{M})$ molecule will stick to that same wall. Thus, the probability that a (**v**, **M**) molecule will evaporate from one wall is equal to the probability that a (**v**, **M**) molecule will stick to the other wall. In the absence of a magnetic field, the molecule travels from wall to wall without any change in its orientation (in the direction of M). Hence a molecule leaving one wall with the most probable velocity and orientation will have the highest probability of sticking to the other wall. In the presence of a magnetic field the molecule will precess during its flight between the walls and will therefore arrive at the second wall



with an orientation that differs in general from that with which it left the first wall, so that the correspondence between the maximum probabilities for sticking and evaporating is violated. Hence the heat flux, which will obviously be greatest when the maximum probabilities for evaporation and sticking of the molecules correspond to one another, will decrease when the magnetic field is applied.

The signs of $\Delta Q_{\perp}$ and $\Delta Q_{\parallel}$ were determined for all the models listed in the table. In the general case Eq. (4.1), the model equation for the gas-wall interaction, contains terms that give positive contributions to ΔQ, as well as terms that give negative contributions to it. On the whole, such models lead to complicated relationships between the flux and the field. Hence, models (110), (211), and (112) lead to a decrease in the heat flux on applying the field for the case $\mathbf{H} \parallel \mathbf{k}$, and to an increase for the case $\mathbf{H} \perp \mathbf{k}$, while the sign of $\Delta Q_{\perp}$ for models (221) and (222) depends on the value of α_0.

Now let us determine the dependence of the heat flux on the field vector H and the anisotropy of the effect, as predicted by various of the models described by Eq. (4.1), and let us first consider model (122). Using Eq. (3.3), (2.11), and (2.7), we obtain

$$Q_2^{(122)}(\mathbf{H})=\frac{15}{128\pi^{5/2}v_0^2M_0^4}\frac{\alpha_0^2\varepsilon^2}{(2-\alpha_0)^2}\int\limits_{\mathbf{vk}>0}E(f_0-f_L)v^3M^4 \tag{4.3}$$

$$\times\cos\theta_v\sin^2\theta_v\sum_m(-1)^{m+1}\Psi_m[d^{(2)}_{m1}d^{(2)}_{-m-1}+d^{(2)}_{m-1}d^{(2)}_{-m1}]M^2\,dM\,d\mathbf{v}.$$

From this equation we can obtain the following expressions for the changes in the heat flux incident to the application of fields parallel and perpendicular, respectively, to the vector **k**:

$$\begin{aligned}\Delta Q_{\parallel}^{(122)}&=Q_{2\parallel}^{(122)}-Q_2^{(122)}(0)=-I_0^{(122)}+I_1^{(122)},\\ \Delta Q_{\perp}^{(122)}&=Q_{2\perp}^{(122)}-Q_2^{(122)}(0)=-I_0^{(122)}+{}^1/_2I_1^{(122)}+{}^1/_2I_2^{(122)},\\ Q_2^{(122)}(0)&=I_0^{(122)},\end{aligned} \tag{4.4}$$

where

$$I_k^{(122)}=\frac{675\sqrt{2}}{2^{12}\pi^4}\frac{\alpha_0^2\varepsilon^2}{(2-\alpha_0)^2}n_0v_0(T_c-T_L)J_k^{(122)},$$

$$J_k^{(122)}=\int\limits_0^\infty(x^3+5{,}5x)e^{-x^2}\frac{(1-\alpha_0)^3+\alpha_0(2-\alpha_0)\cos(k\xi/x)-(1-\alpha_0)\cos(2k\xi/x)}{1-2(1-\alpha_0)^2\cos(2k\xi/x)+(1-\alpha_0)^4}dx, \tag{4.5}$$

$$\xi=\gamma HL/v_0=\omega\tau.$$

The ξ dependence of the integrals $J_k^{(122)}$ (k = 1, 2) is similar to the previously investigated[3] dependence of $J_k^{(111)}$. In the absence of a field, we have $J_k^{(122)} = J_0^{(122)}$. As the field strength increases ($\xi \sim H$), $J_k^{(122)}$ decreases, and as ξ increases further, it oscillates with decreasing amplitude. In the limit $\xi \to \infty$, we have $|J_k^{(122)}| \to 0$ (k = 1, 2), so it follows from (4.4) that

$$(\Delta Q_{\perp}^{(122)}/\Delta Q_{\parallel}^{(122)})_{\xi\to\infty}=1. \tag{4.6}$$

Now let us consider the (022) model. Using Eq. (3.3) to calculate the change in the heat flux when the field is applied leads to the following result:

$$\Delta Q_{\perp}^{(022)}=-I_0^{(022)}+I_2^{(022)},\quad \Delta Q_{\parallel}^{(022)}=0, \tag{4.7}$$

in which the integrals $I_k^{(022)}(k = 0, 2)$ are analogous to integrals (4.5). We note that according to any of the models $(0ll)$ with $l = 1, 2, 3, \ldots$ the heat flux remains unchanged when a field is applied parallel to the temperature gradient. This result has a simple physical meaning. For these models α is independent of the azimuthal angle φ_M which, together with the polar angle θ_M, specifies the direction of the angular momentum of the molecule. When $\mathbf{H} \parallel \mathbf{k}$, the angle θ_M is not altered by the precession of the molecule about the direction of the magnetic field, so that the distribution of the molecules with respect to θ_M also remains unchanged. Hence the application of magnetic field in this direction does not change the heat flux.

Now let us consider the results obtained by using other models to calculate the anisotropy of the effect. For models (111) and (212), $\Delta Q_\perp/\Delta Q_\parallel$ is constant and equal to one-half, regardless of the strength of the field. For the models (110), (211), (112), (220), (121), (221), and (222), the quantity A (see Eqs. (4.1)) contains terms that are invariant under the substitution $v_z \to -v_z$, $\mathbf{k} \to -\mathbf{k}$, as well as terms that change sign under that substitution. Since the corresponding type-a and type-b terms in Eq. (3.3) for the heat flux depend differently on the quantity α_0 (see Eq. (2.13)), the quantity $(\Delta Q_\perp/\Delta Q_\parallel)_{\xi\to\infty}$ will depend on α_0 for these models. The range throughout which this ratio varies for $0 < \alpha_0 < 1$ is given in the table for each of these models.

The anisotropy for the $(1ll)$ models can be determined for arbitrary values of l $(l > 2)$. The following results are obtained for the change in the heat flux on applying the field:

$$\Delta Q_\parallel^{(1ll)} = -I_0^{(1ll)} + I_1^{(1ll)},$$
$$\Delta Q_\perp^{(1ll)} = -(1-c_0) I_0^{(1ll)} + c_1 I_1^{(1ll)} + \ldots + c_l I_l^{(1ll)}, \tag{4.8}$$

where

$$c_0 = [d_{01}^{(l)}(\pi/2)]^2. \tag{4.9}$$

The coefficients c_i $(i = 1, \ldots, l)$ are easily calculated for each specific case. It can be shown that the integrals $I_k^{l_1 l_2 l_3}$ in the expression for ΔQ for any of the models $(l_1 l_2 l_3)$ of (4.1) and, in particular, the integrals $I_k^{(1ll)}$, differ from $I_k^{(122)}$ (see Eq. (4.5)) only in a numerical factor, in the form of the polynomial, and in the sign of the second term in the numerator of the integrand. It is therefore clear that as functions of ξ, the $I_k^{(l_1 l_2 l_3)}$ will behave in qualitatively the same way as the $I_k^{(122)}$. In particular, they will tend to zero as $\xi \to \infty$ $(k \neq 0)$, so that from Eqs. (4.8) and (4.9) we obtain

$$(\Delta Q_\perp^{(1ll)}/\Delta Q_\parallel^{(1ll)})_{\xi\to\infty} = 1 - [d_{01}^{(l)}(\pi/2)]^2. \tag{4.10}$$

It follows from this equation that for even l we have

$$(\Delta Q_\perp^{(1ll)}/\Delta Q_\parallel^{(1ll)})_{\xi\to\infty} = 1, \tag{4.11}$$

while for odd l this ratio lies between 0 and 1.

It is not difficult to show that the $(l1l)$ models for arbitrary values of l $(l > 2)$ lead to the value $\Delta Q_\perp/\Delta Q_\parallel = 1/2$ regardless of the strength H of the magnetic field.

We have now considered the behavior of the heat flux in a magnetic field for the $(l_1 l_2 l_3)$ models of (4.1) with $l_1, l_2, l_3 \leq 2$, and for the models $(0ll)$, $(1ll)$, and $(l1l)$ with $l > 2$. These models are obtained by retaining only one term with definite values of l_1, l_2, and l_3 in the expansion (2.4) of the diffuse-reflection coefficient α in spherical functions of the vectors $\mathbf{v}$, $\mathbf{M}$, and $\mathbf{k}$. The examination of these models has revealed some for which ΔQ is negative. Thus, the sign of ΔQ observed for H_2, N_2, CO_2, and SF_6 can be accounted for provided we make the additional assumption that α depends in the same way on the directions of $\mathbf{v}$ and $\mathbf{M}$ for both of the surfaces (copper and gold) used in[4].

However, none of the simple models for the gas-wall interaction discussed above leads to the observed[4] values of $(\Delta Q_\perp/\Delta Q_\parallel)_{\xi\to\infty}$, which lie between 1.6 and 2 for the investigated gases. Such values of $(\Delta Q_\perp/\Delta Q_\parallel)_{\xi\to\infty}$ can be obtained by retaining two nonspherical terms of expansion (2.4) in the model expression for α and choosing the expansion coefficients $\alpha_{l_1 l_2 l_3}$ appropriately. In this case the model expression for α will involve three parameters.

For example let us consider the model equation

$$\alpha = \alpha_0\{1 + \varepsilon[Y_{00}^v Y_{20}^M Y_{20}^k + \gamma(Y_{11}^v Y_{1-1}^M - Y_{1-1}^v Y_{11}^M) Y_{20}^k]\}, \tag{4.12}$$

which is obtained from expansion (2.4) by retaining the terms $l_1 = 0$, $l_2 = l_3 = 2$, and $l_1 = 1$, $l_2 = l_3 = 2$. In this case the expressions for $\Delta Q_\perp$ and $\Delta Q_\parallel$ will be linear combinations of expressions (4.4) and (4.7). For $\gamma = 0.21$, we obtain, for example,

$$(\Delta Q_\perp/\Delta Q_\parallel)_{\xi\to\infty} = 2. \tag{4.13}$$

We note that all the simple models for the gas-wall interaction that we have investigated lead to damped oscillations of the heat flux as a function of the field strength H. These oscillations correspond to the molecule executing one, two, ... complete revolutions as it precesses about the magnetic field vector during its flight from one wall to the other, and their amplitude amounts to ~30% of the saturation value of $\Delta Q_\perp$. No oscillations were observed in[4], although the experimental errors were only ~5%. The plates used in these experiments were made of various materials, but the hot plate was coated with gold, and the cold plate, with copper. Recent experiments in which the cold plate was of glass revealed oscillations of the heat flux in nitrogen with an amplitude of ~10% of the saturation value of $\Delta Q_\perp$ (private communication from V. S. Laz'ko et al.). Thus, the observed amplitude of the oscillations is lower than the predicted amplitude. We have generalized the problem treated above to the case of different plates and have found that the amplitude of the heat-flux oscillations in the magnetic field can be small provided the values of α_0 (see Eq. (2.4)) for the two plates differ considerably or α has different angular dependences on $\mathbf{v}$, $\mathbf{M}$, and $\mathbf{k}$ for the two plates.

We also note that the amplitude of the oscillations can be small even in the case of identical plates provided the diffuse-reflection coefficient α has a complicated angular dependence on $\mathbf{v}$, $\mathbf{M}$, and $\mathbf{k}$ described by several (more than two) terms in expansion (2.4). In this case the expressions for $\Delta Q_\perp$ and $\Delta Q_\parallel$ will contain a number of integrals analogous to $J_k^{(122)}$, which depend on $\omega\tau$ (i.e., on HL). These integrals reach their maxima at different values of $\omega\tau$, and it is natural to suppose that the oscillations of $\Delta Q_\perp$ and $\Delta Q_\parallel$ will be small.

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[1)]There is a misprint in [3]: the subscripts $\perp$ and $\parallel$ on $\Delta Q_\perp$ and $\Delta Q_\parallel$

should be understood as referring to the normal vector k, and not to the plane of the surface.

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226