Bremsstrahlung instability and coherent amplification of polarized radiation in plasma located in a magnetic field

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(Submitted July 15, 1974)

Zh. Eksp. Teor. Fiz. 67, 2157–2160 (December 1974)

Because of collisions with ions, the electrons in plasma located in a magnetic field **H** can amplify radiation polarized in the (**k**, **H**) plane, where **k** is the wave vector. Radiation polarized at right angles to this plane, on the other hand, is absorbed. It is shown that when $eH/mc \gg \omega$, the amplification of radiation of frequency ω is possible even for an isotropic electron-momentum distribution, in contrast to the case of plasma without the field, when amplication is possible only for a beam-type electron distribution.

It is well known that electron-ion collisions in an equilibrium (Maxwellian) plasma lead to the absorption of radiation passing through the plasma. On the other hand, in plasma with anisotropic electron-momentum distribution, the radiation is amplified under certain conditions (see, for example,[1]). It will be shown below that amplification can occur even for an isotropic distribution function, provided the plasma is placed in a magnetic field such that $\sigma_H \gg \omega$, where $\omega_H = eH/mc$ and ω is the radiation frequency.

1. Consider the interaction between radiation and nonrelativistic electrons. Let Z and n_i be the charge and density of randomly distributed ions. We shall confine our attention to a nonquantized magnetic field ($\hbar\omega_H \ll p^2/2m$, where p is the electron momentum) and the Born approximation. Let $dW^{\pm}$ be the probability of emission (+) and absorption (−) by an electron of momentum p of a photon in a frequency interval $d\omega$, time interval dt, and solid angle element do. We then have

$$\hbar\omega dW^{\pm} = dt\, d\omega do (N_{\mathbf{k},\lambda} + {}^1/_2 \pm {}^1/_2) Q^{\pm},$$

where $N_{\mathbf{k},\lambda}$ is the number of photons in a given state, $\lambda = 1, 2$, **k** is the wave number, and e_λ the polarization vector.

The quantities $Q^{\pm}$ are, respectively, the bremsstrahlung emission and absorption power. Simple perturbation-theory calculation shows that, in the dipole approximation,

$$Q^{\pm} = \frac{2}{\pi}\frac{Z^2 e^6 n_i}{mc^3 p}\left[A^{\pm}(\xi)\cos^2\chi + MB^{\pm}(\xi)\frac{\sin^2\chi}{2}\right], \qquad (1)$$

where

$$A^{\pm}(\xi) = (1\mp\xi)^{1/2} \pm \frac{\xi}{2}\ln\frac{2\mp\xi+2(1\mp\xi)^{1/2}}{\xi},$$

$$B^{\pm}(\xi) = \ln\frac{2\mp\xi+2(1\mp\xi)^{1/2}}{\xi} - A^{\pm}(\xi),$$

$\xi = 2m\hbar\omega/p^2$, and χ is the angle between the polarization vector e and the average direction of the momentum **p** which is parallel to the magnetic field **H**. In deriving (1), we have neglected Debye screening and have set the permittivity tensor $\epsilon_{ij} = \delta_{ij}$. The parameter $M = \omega^2(\omega^2 + \omega_H^2)/(\omega^2 - \omega_H^2)^2$ characterizes the effect of the magnetic field on bremsstrahlung emission. This effect is most important at low frequencies ($\omega \ll \omega_H$) when $M = (\omega/\omega_H)^2 \ll 1$ and transverse displacements of the electron are suppressed.[1)] At high frequencies ($\omega \gg \omega_H$), we have $M = 1$ and (1) yields the well-known expression for the bremsstrahlung emitted when $H = 0$.[2]

In general, the emitted radiation is polarized. Suppose that the polarization vector $e_\parallel$ lies in the (**k**, **H**) plane where **k** is the wave vector. We then have $\cos^2\chi = \sin^2\vartheta$, where $\vartheta = \pi/2 - \chi$ is the angle between **k** and the average direction of the momentum **p**. We have from (1)

$$Q_\parallel^{\pm} = Q_0[A^{\pm}(\xi)\sin^2\vartheta + {}^1/_2 MB^{\pm}(\xi)\cos^2\vartheta]. \qquad (2)$$

Similarly, when the polarization vector $e_\perp$ is perpendicular to the (**k**, **H**) plane, we have

$$Q_\perp^{\pm} = {}^1/_2 Q_0 MB^{\pm}(\xi). \qquad (3)$$

Here and henceforth, $Q_0 = 2Z^2e^6 n_i/\pi mc^3 p$.

2. The absorption coefficient for radiation with given polarization e_λ can conveniently be written in the form

$$\mu = \frac{(2\pi)^3c^2}{\hbar\omega^3}\left\{\int_{\xi>1} d^3p\,F(\mathbf{p})Q^- - \int_{\xi<1} d^3p\,F(\mathbf{p})[Q^+ - Q^-]\right\}, \qquad (4)$$

where $F(\mathbf{p})$ is the electron distribution function, Equation (4) is based on the formula $N_{\mathbf{k},\lambda} = (2\pi)^3c^2 I/\hbar\omega^3$, which gives the number of photons $N_{\mathbf{k},\lambda}$ as a function of the radiation intensity I (see[3], Sec. 44). The intensity leaving a layer of thickness L is $I = I_0\exp(-\mu L)$, where I_0 is the incident intensity and μ the absorption coefficient.

We must now consider the conditions under which the absorption coefficient μ may be less than zero. When this is so, we have the coherent amplification of radiation, the system operates as a maser, and we can speak of bremsstrahlung instability in the system. Suppose the distribution $F(\mathbf{p})$ is such that electrons with energy $p^2/2m < \hbar\omega$ are either absent altogether or are very few. In that case, (4) can be written in the form

$$\mu = -\frac{(2\pi)^3c^2}{\hbar\omega^3}\int_{\xi<1} d^3p\,F(\mathbf{p})[Q^+ - Q^-]. \qquad (5)$$

In this case, the sign of μ is determined by the sign of the difference $\Delta Q = Q^+ - Q^-$ and, correspondingly, when $\Delta Q > 0$, we have negative reabsorption ($\mu < 0$).

We must now consider in greater detail the conditions under which $\Delta Q > 0$. For different polarizations $e_\parallel$ and $e_\perp$, we have respectively,

$$\Delta Q_\parallel = Q_0\{{}^1/_2\xi\alpha(\xi)[1 - {}^1/_2(2+M)\cos^2\vartheta] + M\beta(\xi)\cos^2\vartheta\}, \qquad (6)$$

$$\Delta Q_\perp = Q_0 M[\beta(\xi) - {}^1/_4\xi\alpha(\xi)], \qquad (7)$$

where

$$\alpha(\xi) = \ln\frac{2-\xi+2(1-\xi)^{1/2}}{\xi} + \ln\frac{2+\xi+2(1+\xi)^{1/2}}{\xi} - \frac{4}{(1-\xi)^{1/2}+(1+\xi)^{1/2}}$$

$$\beta(\xi) = \ln\frac{1+(1-\xi)^{1/2}}{1+(1+\xi)^{1/2}}.$$

It is readily shown that for all values of ξ we have $\Delta Q_\perp < 0$ and, consequently, radiation with polarization $e_\perp$ will always experience absorption. On the other hand, $\Delta Q_\parallel > 0$ when $\alpha(\xi) > 0$ (for this, it is necessary that $\xi < 0.92$) and

$$\cos^2\vartheta < \frac{\xi\alpha(\xi)/2}{\xi\alpha(\xi)/2+M[\xi\alpha(\xi)/4-\beta(\xi)]}, \quad (8)$$

i.e., radiation with polarization $e_\parallel$ is amplified under these conditions.

When $\xi \equiv 2m\hbar\omega/p^2 \ll 1$, the condition given by (8) assumes the form

$$\cos^2\vartheta < 2/(2+M). \quad (9)$$

At low frequencies ($\omega \ll \omega_H$), we have $M = (\omega/\omega_H)^2 \ll 1$ and $\cos^2\vartheta < 1 - M/2$. This means that amplification is possible practically throughout the angular range with the exception of a narrow cone with aperture angle $\vartheta_{min} = \omega/\sqrt{2}\omega_H \ll 1$ near the direction of **H** and **−H**. In the high-frequency region ($\omega \gg \omega_H$), when $M = 1$, we have from (9) the well-known expression for the amplification of bremsstrahlung,[1] namely, $\vartheta > \vartheta_{min} = \cos^{-1}(2/3)^{1/2} = 0.62\ r \approx 35°$, where ϑ is the angle between the wave vector **k** and the momentum **p**.

It follows from the foregoing description that for one of the polarizations ($e_\parallel$), amplification in a magnetic field is possible even when the electron distribution function is isotropic. At the same time, when there is no magnetic field, plasma with isotropic electron distribution can only absorb radiation.

It is clear that plasma will absorb radiation whether the magnetic field is present or not for any electron distribution function for which $\partial F(p)/\partial p < 0$ and, in particular, in the case of the equilibrium distribution [see (4)]. This is to say, as before, the necessary condition for coherent amplification is that the electron distribution be of the nonequilibrium type, with the one difference that, in the absence of the magnetic field, we must additionally demand that the distribution be anisotropic (beam-type distribution), and in a nonzero magnetic field the amplification will occur even for an isotropic distribution (spherical-layer type).

3. Analysis of the conditions under which the maser effect is possible is particularly important for astrophysics because many cosmic sources exhibit high radiation temperatures at low frequencies (in the radio band), which also indicates the coherent nature of this emission. The coherent emission mechanism discussed here requires no exotic conditions, since it relies on bremsstrahlung emission in a magnetic field for which $\hbar\omega_H \ll p^2/2m$. Applications of this mechanism (it may be designated as the magnetic bremsstrahlung maser) in astrophysics will be discussed elsewhere.

We are indebted to V. N. Tsytovich for valuable discussions.

1)We shall confine our attention throughout this paper to frequencies well away from resonance, in which case $\omega \approx \omega_H$.

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Translated by S. Chomet
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