The Bénard problem for hot electrons in semiconductors

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The problem analogous to the well known Bénard problem in hydrodynamics is formulated for hot electrons and solved in a linear approximation. The "quasihydrodynamical" approximation is used, which is valid when the momentum relaxation time is much smaller than the interelectron collision time and the latter in turn is smaller than the energy relaxation time. Heating of the electron gas is achieved as a result of intraband absorption of light incident on one of the surfaces of the sample. The criterion for instability of a spatially homogeneous distribution of the electron temperature is indicated; This criterion also determines the conditions under which a stationary two-dimensional spatially periodic distribution of the electron temperature appears. The period turns out to be a function of the intensity of the light incident on the sample.

1. INTRODUCTION

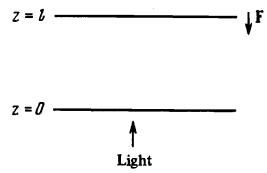
As is well known from hydrodynamics,[1] stationary free convection appears in a layer of incompressible fluid bounded by two horizontal planes in the presence of a sufficiently large temperature gradient parallel to the force of gravity. In this connection the one-dimensional (vertically) distribution of the temperature, pressure, etc., existing in the presence of small gradients, becomes unstable. In its place a three-dimensional distribution is established, which is periodic in the directions perpendicular to the force of gravity. The reason for this instability is that the pressure, which has already been increased somewhat in the lower part of the layer due to the effect of the force of gravity, increases additionally there as a result of heating. In the presence of a small temperature gradient this perturbation is dissipated by the usual thermal conductivity; with an increase of the gradient, however, this mechanism becomes inadequate, and heat transfer associated with macroscopic motion of the fluid begins.

The described phenomenon was investigated by Bénard[2] as long ago as the beginning of the present century. In recent times it has attracted attention for a number of reasons–among them the extremely fundamental characteristic: this is an example of the formation of an ordered structure as the result of an external influence, essentially deriving the system from a state of thermodynamic equilibrium.[3]

It is interesting to investigate whether it is possible to realize a somewhat similar situation in regard to the gas of charge carriers in a semiconductor.[1)] Of course, here an electric field might play the role of the gravitational field. The reasons for posing this problem are clear: a periodic distribution of the electron temperature and (or) of the concentration of charge carriers (with a period exceeding the mean free path with respect to momentum) would imply that various macroscopic characteristics of the system are also periodically distributed, among them the electrical conductivity, the light absorption coefficient, etc.–with obvious consequences.

Heating of an electron gas can be achieved even without the participation of a static field–with the aid of light (this possibility has been investigated in a different context by a number of authors[5-10]).

Thus, we arrive at the scheme represented in the figure. There the force acting from the side of the externally applied electric field is denoted by **F**. It is necessary, however, to keep in mind that the carriers redistribution due to compression of the electron gas may lead to the appearance of an additional field in the sample. The latter, of course, impedes the effect of interest to us.



The sample is bounded by the planes $z = 0$ and $z = l$ and is infinite in the x and y directions. The half-spaces $z < 0$ and $z > l$ are occupied by dielectric media.

In the present article we shall consider a monopolar semiconductor under conditions in which the characteristic times of interelectron collisions (τ_{ee}), momentum relaxation (τ_p), and energy relaxation (τ) satisfy the inequalities

$$\tau_p \ll \tau_{ee} \ll \tau. \tag{1}$$

In this connection the concept of an electron temperature T has a unique meaning, and all kinetic coefficients depend on T. The latter fact allows us to avoid the complications indicated above, which are related to the compression of a gas of charged particles. In fact, a new mechanism for variation of the pressure appears in the conditions under consideration, resulting from the temperature dependence of the energy relaxation time and the temperature dependence of the thermal conductivity κ of the electron gas: it is obvious that the pressure of the electron gas is very strongly increased in the lower part of the sample[2)] (see the figure) for $d\tau/dT > 0$ and $d\kappa/dT > 0$. In this connection the compressibility of the gas can generally be neglected, which we therefore do.[3)] Consequently the presence of an external field is no longer compulsory (although it may turn out to be of some use); here we assume $\mathbf{F} = 0$.

The absorption of warming light in the conditions under consideration must be caused by intraband transitions. In this connection energy is put into the electron gas, but new charge carriers do not appear, and the redistribution of the electrons in space also does not play an important role.

2. FUNDAMENTAL EQUATIONS

Under conditions (1) the fundamental equations of the problem are the equation of continuity, the expression for the current density **j**, the energy transport

equation, and Poisson's equation. Let us introduce the following notation: n denotes the carrier concentration;[4)] δn denotes its small fluctuation; u = j/en is the drift velocity; T_0 is the lattice temperature (expressed in energy units, just like T); α is the differential thermo e.m.f.; μ is the mobility ($\mu \sim T^r$, where r is a known number); m is the effective mass; ϵ is the dielectric constant of the sample; γ is the coefficient for the absorption of the "warming" light; J(z) denotes the flux of light energy into the sample, and J_m is its value at z = +0.

It is clear that three characteristic lengths exist in the problem: l, γ^{-1}, and $\lambda_0^{-1} = (2\kappa_0\tau_0/3)^{1/2}$ (the subscript 0 denotes the corresponding quantity in the absence of heating (for $T = T_0$)). Depending on the relationships between them, the following cases are distinguished: a) surface absorption, a "thin" sample: $\gamma^{-1} \ll l \ll \lambda_0^{-1}$; b) surface absorption, a "thick" sample: $\gamma^{-1} \ll \lambda_0^{-1} \ll l$; c) bulk absorption: $\gamma^{-1} \gg \lambda_0^{-1}$.

In cases a) and b) the absorption of light energy can be taken into consideration with the aid of the boundary condition on the equation of energy transport; in case c) the absorption must be taken into consideration in this equation itself. The latter situation is evidently encountered most frequently, and it is the only case which will be explicitly treated in this article. One can show, however, that results analogous to those indicated below are obtained for surface absorption. We shall also assume that $l \gg \gamma^{-1}$. In this connection the sample can be regarded as "infinitely thick."

Thus, in the case of a nondegenerate gas the equations for the problem have the form

$$\operatorname{div} \mathbf{u} = 0, \tag{2}$$

$$\mathbf{u} = \mu\mathbf{E} - \mu\alpha\nabla T, \tag{3}$$

$$\partial T/\partial t + {}^1/_3(5+2r)\operatorname{div}(\mathbf{u}T) - {}^2/_3 e\mathbf{u}\mathbf{E} - {}^2/_3 \operatorname{div}(\varkappa\nabla T) + (T-T_0)/\tau = {}^2/_3 n^{-1}\gamma J(z), \tag{4}$$

$$\operatorname{div}\mathbf{E} = 4\pi e\delta n/\epsilon. \tag{5}$$

Formula (3) is obtained from the well known expression for the current density (see, for example,[12], Chap. II, formula (4.18)):

$$\mathbf{j} = en\mu(T)\left\{\mathbf{E} - \frac{T}{en}\nabla n - \alpha(T)\nabla T\right\}.$$

Here the gradient ∇n is taken at constant temperature. The approximation of incompressibility assumed by us consists, as usual, of neglecting the second term inside the curly brackets (retaining the possibility of changes in n due to a variation of the temperature T). Poisson's equation (5) is the only equation where it is necessary to take the small change of the electron concentration into consideration. As is customary in such a formulation of the problem, in what follows it will be utilized in order to estimate δn and to establish the conditions for applicability of the quasineutrality approximation.

The boundary conditions on Eqs. (2)–(5) are the usual continuity conditions for E (they determine the field outside the sample), the conditions of boundedness of all quantities upon unlimited (in absolute value) growth of the coordinates x, y, and z, and the equations

$$u_z = 0, \quad z = 0, \tag{6}$$

$$-n\varkappa\partial T/\partial z = \nu n(T - T_0), \quad z = 0. \tag{7}$$

Here ν is a phenomenologically introduced positive coefficient (having the dimensions of a velocity), which characterizes the heat exchange between the electron gas in the semiconductor and in the dielectric medium adjacent to it. Its exact calculation requires a detailed investigation of the kinetics of the electronic processes in the contact region. For not too large a difference between T and T_0, in order of magnitude one has

$$\nu \approx \left(\frac{T}{m}\right)^{1/2} P, \tag{8}$$

where P is the probability for the passage of an electron through the contact. In fact, by passing into the dielectric medium the electron carries away an average energy of order T, but returns from there with an energy of order T_0.

In what follows we shall assume that γ does not depend on T, and consequently

$$J(z) = J_m e^{-\gamma z}. \tag{9}$$

Interesting conditions exist experimentally (see below, Sec. 4) in which the adopted assumption is satisfied. Upon giving it up, the problem becomes complicated, requiring the enlistment of the methods developed in[5-7].

Let us assume

$$T = T_0(1+\xi), \tag{10}$$

$$({}^5/_3\varkappa\tau)^{-1} = \lambda_0^2 f(\xi), \quad f = \varkappa_0\tau_0/\varkappa\tau, \quad f(0) = 1 \tag{11}$$

and let us introduce the following units of measurement: length λ_0^{-1}, time τ_0, velocity $u_0 = 3/\lambda_0\tau_0(5 + 2r)$, energy T_0, field strength $E_0 + 3T_0/2e\tau_0 u_0$, mobility $u_0E_0^{-1}$, differential thermoelectric power $e^{-1}(5/2 + r)$, and energy flux $J_0 = (3/2)nT_0\tau_0^{-1}\lambda_0^{-1}$.

Keeping the previous notation for dimensionless quantities, we may rewrite Eqs. (3)–(5) in the following form:

$$\mathbf{u} = \mu(\mathbf{E} - \alpha\nabla\xi), \tag{3'}$$

$$\frac{\varkappa_0}{\varkappa}\left[\frac{\partial\xi}{\partial t} + (\mathbf{u}, \nabla\xi - \mathbf{E})\right] - \nabla^2\xi + \xi f(\xi) - \frac{(\nabla\xi)^2}{1+\xi}\frac{d\ln\varkappa}{d\ln T} = \frac{\varkappa_0}{\varkappa}\gamma J_m e^{-\gamma z}, \tag{4'}$$

$$\eta \operatorname{div}\mathbf{E} = \delta n/n, \tag{5'}$$

where

$$\eta = ({}^5/_2 + r)eT_0\lambda_0^2/4\pi ne^2. \tag{12}$$

Here the quantity γ is still dimensionless. By definition, $\gamma \ll 1$. The approximation assumed by us of incompressibility of the gas is valid for $\eta \ll 1$.

Equation (2) formally remains without any changes, and the same is true of the boundary condition (6); condition (7) takes the following form (z = 0):

$$\frac{\varkappa}{\varkappa_0}\frac{\partial\xi}{\partial z} = -\nu'\xi, \quad \nu' = \nu\varkappa_0^{-1}\lambda_0^{-1}. \tag{7'}$$

In the case of almost complete degeneracy, it is convenient to measure the time in units of $(2/3)\tau_0$, and the velocity in units of $u_0 = 9F_0/\pi^2(4r + 5)\tau_0\lambda_0 T_0$, where F_0 is the Fermi level for $T = T_0 = 0$. In this connection Eqs. (3') and (5') remain without changes, instead of Eq. (12) we obtain

$$\eta = \frac{\pi^2}{6}(4r+5)\frac{T_0}{F_0}\frac{eT_0\lambda_0^2}{4\pi ne^2}, \tag{12'}$$

and in Eq. (4') the expression inside the square brackets should be replaced by

$$\left[(1+\xi)\left(c\frac{\partial\xi}{\partial t} + \mathbf{u}\nabla\xi\right) - \mathbf{u}\mathbf{E}\right].$$

Here $c = \pi^2 T_0/2F_0$ is the heat capacity per electron for $T = T_0$.

Under conditions (1) we have the order of magnitude relationship[13]

$$\varkappa \approx T\tau_p/m. \tag{13}$$

Formula (13) is valid in the absence of Fermi degeneracy as well as in the presence of complete degeneracy: the numerical coefficient omitted in it depends on the scattering mechanism. Therefore,

$$\nu' = P(\tau_0/\tau_p)^{1/2} \equiv P\rho^{1/2}, \tag{14}$$

where $\rho = \tau_0/\tau_p$ is the "coefficient of inelasticity." Under conditions (1) it exceeds unity noticeably. However, the probability P strongly depends on the nature of the contact and, in particular, may be extremely small. Because of this, in principle almost any value of ν' is possible—ranging from very small values to values appreciably exceeding unity.

In what follows we shall use the same calculation scheme as in hydrodynamics.[1] Namely, we shall first find the one-dimensional static solution, assuming $\mathbf{u} = 0$, $\xi = \xi_S(z)$, $\mathbf{E} = \mathbf{E}_S(z) \parallel Oz$, and then we apply to it a small perturbation $\delta\xi$, $\delta\mathbf{E}$ depending on all three spatial variables and, perhaps, on the time.

3. STATIC, ONE-DIMENSIONAL SOLUTION

For $\xi = \xi_S(z)$ and $\mathbf{E} = \mathbf{E}_S(z)$ Eqs. (3′) and (4′) give

$$E_s = \alpha(\xi_s)\xi_s', \tag{15}$$

$$-\xi_s'' + \xi_s f(\xi_s) - \frac{\xi_s'^2}{1+\xi_s}\left(\frac{d\ln\varkappa}{d\ln T}\right)_{\xi=\xi_s} = \frac{\varkappa_0}{\varkappa(\xi_s)}\gamma J_m e^{-\gamma z}. \tag{16}$$

Linearizing (16) with respect to ξ_S, we obtain

$$\xi_{s\,\mathrm{lin}} = \frac{\gamma J_m}{1-\gamma^2}\left(e^{-\gamma z} - \frac{\nu'+\gamma}{\nu'+1}e^{-z}\right) \tag{17}$$

In virtue of the smallness of γ, the second term on the right hand side of Eq. (17) is important only in a comparatively small region near the surface of the sample: the right hand side of (17) reaches a maximum at the point

$$z_m = \frac{1}{1-\gamma}\ln\frac{\nu'+\gamma}{\gamma(\nu'+1)}.$$

For $\nu' \gg \gamma$ and $\nu' \ll \gamma$ this gives $z_m \approx \ln(1/\gamma)$ and $z_m \approx 0$, respectively. On the other hand, the important values of z for what follows turn out to be of order $(2/\gamma)\ln(2/\gamma)$ (see Sec. 5). Because of this, in what follows expression (17) can be replaced by the simpler expression

$$\xi_{s\,\mathrm{lin}} \approx \gamma J_m e^{-\gamma z}. \tag{17′}$$

It is difficult to find the value of ξ_S in the nonlinear problem, without specializing the form of the functions $\tau(T)$ and $\kappa(T)$. However, here one can verify that in the overwhelming part of the sample the characteristic length, over which the quantity ξ_S changes noticeably, is γ^{-1}, and the derivatives ξ_S' and ξ_S'' are small with regard to the parameters γ and γ^2. Thus, in the case of almost complete degeneracy, when (see Sec. 4 below) $\tau \sim T$, $\kappa \sim T$, and for $\gamma J_m < 1$ (but not necessarily $\ll 1$), From Eq. (16) we obtain

$$\xi_s \approx \frac{\gamma J_m e^{-\gamma z}}{1-\gamma J_m e^{-\gamma z}}. \tag{18}$$

4. THE KINETIC COEFFICIENTS AS FUNCTIONS OF THE ELECTRON TEMPERATURE

Let us introduce the notation

$$\dot{\varkappa} = \left(\frac{d\ln\varkappa}{d\ln T}\right)_0, \quad \dot{\tau} = \left(\frac{d\ln\tau}{d\ln T}\right)_0 \tag{19}$$

and so forth. The values of $\dot{k}$, $\dot{\mu}$, $\dot{\alpha}$, $\dot{\tau}$, and $\dot{\gamma}$ depend on the degree of degeneracy of the electron gas. Under the conditions of strong degeneracy we have the following result, independently of the mechanism for the scattering of energy and momentum:

$$\dot{\varkappa} = \dot{\alpha} = 1, \quad \dot{\mu} = 0, \quad \dot{\gamma} = 0, \quad \dot{\tau} = 1. \tag{20}$$

The first three of these equations are well known; the fourth is also quite obvious; a derivation of the latter is given in the Appendix. For a nondegenerate gas the well known results, cited in the Table (for a quadratic dispersion law), are obtained instead of Eqs. (20).

Generally speaking it is impossible to use the results of the kinetic equation for a calculation of $\dot{\gamma}$, because the case when $T \lesssim \hbar\omega$ is quite possible, where ω is the frequency of the absorbed light. In the case when the quasimomentum is primarily scattered on a charged impurity, we have[14-16]

$$\dot{\gamma} = -3/2 \text{ for } T \gg 1/2\hbar\omega; \quad \dot{\gamma} = 0 \text{ for } T < 1/2\hbar\omega. \tag{21}$$

In accordance with what was said earlier, we shall confine the investigation to rather low temperatures, when either the second of inequalities (21) is realized or the gas of charge carriers is strongly degenerate (in both cases $\dot{\gamma} = 0$). We note that it is precisely this temperature range which is obviously of greatest experimental interest: the phenomena considered below can be observed only for a sufficiently long energy relaxation time.

5. INSTABILITY OF THE ONE-DIMENSIONAL SOLUTION AND THE PERIODIC DISTRIBUTION OF THE ELECTRON TEMPERATURE

Let us assume

$$\xi = \xi_s + \delta\xi, \quad \mathbf{E} = \mathbf{E}_s - \nabla\varphi, \quad \mathbf{u} = \mathbf{u}(x, y, z, t), \tag{22}$$

$$\delta\xi = f_1(z)e^{st+i\mathbf{kr}}, \quad \varphi = f_2(z)e^{st+i\mathbf{kr}}, \quad u_z = f_z e^{st+i\mathbf{kr}}, \quad \mathbf{u}_\perp = f_\perp e^{st+i\mathbf{kr}}. \tag{23}$$

Here $\mathbf{k} = \{k_x, k_y\}$, $\mathbf{r} = \{x, y\}$, and $\mathbf{u}_\perp = \{u_x, u_y\}$ are two-dimensional vectors with real components, and s is a complex parameter.

Varying Eqs. (2′), (3′), and (4′) about the static solution and discarding the terms which are small with regard to the parameters γ and γ^2, we obtain (for the nondegenerate gas)

$$f_z = -\mu(\xi_s)[f_2' + \alpha(\xi_s)f_1'], \tag{24}$$

$$\mathbf{f}_\perp = -i\mathbf{k}\mu(\xi_s)[f_2 + \alpha(\xi_s)f_1], \tag{25}$$

$$f_z'' - k^2 f_2 + \alpha(\xi_s)[f_1'' - k^2 f_1] = 0, \tag{26}$$

$$f_1'' - [k^2 + 1 + s - (\dot{\varkappa} + 2\dot{\tau} + s\dot{\varkappa})\xi_s]f_1 = 0. \tag{27}$$

The boundary condition (7′) now takes the form (z = 0):

$$(1 - \dot{\varkappa}\xi_s)f_1' = -\nu' f_1. \tag{7″}$$

It is convenient to introduce the function

$$\Phi(z) = f_2 + \alpha(\xi_s)f_1. \tag{28}$$

According to Eqs. (24)–(26)

$$f_z \approx -\mu(\xi_s)\Phi', \quad \mathbf{f}_\perp \approx -i\mathbf{k}\mu(\xi_s)\Phi, \tag{29}$$

$$\Phi'' - k^2\Phi = 0. \tag{30}$$

For $\mathbf{k} \neq 0$ Eq. (30) with (6) taken into consideration has only the trivial solution: $\Phi = 0$. In this connection

$$f_2 = -\alpha(\xi_s)f_1, \quad \mathbf{f}_\perp = f_z = 0. \tag{31}$$

Scattering mechanism	$\dot{\varkappa}$	$\dot{\mu}$	$1/\dot{\alpha}$	$\dot{\tau}$
Acoustic phonons (deformation potential)	$1/2$	$-1/2$	$-(1+2T_0/\|F\|)$	$-1/2$
Acoustic phonons (piezoelectric potential	$3/2$	$1/2$	$-(1+3T_0/\|F\|)$	$1/2$
Optical nonpolar and intervalley phonons				$1/2$, $T_0 > 1/2\hbar\omega_0$ $-\hbar\omega_0/2T_0$, $T_0 \ll 1/2\hbar\omega_0$
Optical polar phonons				$3/2$, $T_0 > 1/2\hbar\omega_0$ $-\hbar\omega_0/2T_0$, $T_0 \ll 1/2\hbar\omega_0$
Charged impurity	$5/2$	$3/2$	$-(1+4T_0/\|F\|)$	—
Neutral impurity	1	0	$-(1+5/2\,T_0/\|F\|)$	—

Note. The limiting energy of the corresponding phonons is denoted by $\hbar\omega_0$. The values of $\dot{\kappa}$, $\dot{\mu}$, and $\dot{\alpha}$ associated with scattering by optical (or intervalley) phonons are not indicated since these mechanisms for the scattering of quasimomentum will not be of interest to us.

The last equation indicates that the drift velocity turns out to be a quantity of higher order in smallness. This was to be expected: according to Eq. (29) Φ is the velocity potential in the assumed approximation, whereas the motion should be rotational. On the other hand, upon taking account of terms of the next order in γ in Eqs. (25) and (26), the connection of Φ with f_z and $f_\perp$ would turn out to be more complicated, and instead of Eq. (30) we would obtain:

$$\Phi''-k^2\Phi=-\alpha(\xi_s)\dot{\alpha}f_1'\xi_s'. \tag{30'}$$

One can substitute the solution (27) into the right hand side of Eq. (30'), as a result of which in the stationary case the quantities Φ and u turn out to be of the order of $\gamma|\alpha|k|\delta\xi|$.

Equation (27) is easily solved if ξ_S is approximated by expression (17').[5] Introducing the notation

$$q^2=(2\dot{\tau}+\dot{\varkappa}+s\dot{\varkappa})\gamma J_m, \tag{32}$$

we obtain

$$f_1=AJ_p(t)+BJ_{-p}(t). \tag{33}$$

Here $J_{\pm p}$ are Bessel functions, A and B are constants,

$$p=2\gamma^{-1}(1+s+k^2)^{1/2},\quad t=2q\gamma^{-1}e^{-\gamma z/2} \tag{34}$$

(it exists in the form of the branch of the square root corresponding to its positive value for $\arg(1+k^2+s)=0$).

For the investigation of a possible fluctuation instability of the static one-dimensional solution, it is sufficient to consider the case of purely real values of s, and moreover values such that (for real values of p)

$$1+s>0. \tag{35}$$

In fact, the latter condition also satisfies, in particular, the inequality $s > 0$, corresponding to the indicated instability. Under conditions (35) B = 0, and Eq. (7'') takes the form

$$J_{p-1}\left(\frac{2q}{\gamma}\right)-\frac{\gamma p}{2q}J_p\left(\frac{2q}{\gamma}\right)=-\frac{\nu'}{1-\dot{\varkappa}\gamma J_m}\frac{\gamma}{2q}J_p\left(\frac{2q}{\gamma}\right). \tag{36}$$

According to Eqs. (32) and (34), the argument $2q/\gamma$ and the order p are very large. Let us set

$$2q/\gamma=\zeta p \tag{37}$$

and let us assume that $\zeta = 1-\eta$, $0 \le \eta \ll 1$. This assumption will be justified by the subsequent calculation; it essentially reduces to a condition which is imposed on the energy flux J_m. The following asymptotic representation of the Bessel function is valid for such values of the argument and order:

$$J_p(\zeta p)=\frac{\zeta^p\exp(p(1-\zeta^2)^{1/2})}{(2\pi p)^{1/2}(1-\zeta^2)^{1/4}[1+(1-\zeta^2)^{1/2}]^p},$$

and for $J_{p-1}(\zeta p)$ we have, correct to within quantities of order η, the expression

$$J_{p-1}(\zeta p)=\frac{\zeta^{p-1}\exp[(p-1)(1-\zeta^2)^{1/2}]}{(2\pi(p-1))^{1/2}(1-\zeta^2)^{1/4}[1+(1-\zeta^2)^{1/2}]^{p-1}}.$$

Substituting these expressions into formula (36), one can easily find

$$\eta=\frac{1}{2p}\left(1+\frac{2\nu'}{1-\dot{\varkappa}\gamma J_m}\right).$$

It is seen that the adopted assumption is actually valid whenever

$$2\nu'/(1-\dot{\varkappa}\gamma J_m)<1. \tag{38}$$

The meaning of the last condition is clear: it will not be possible to heat the electron gas in the semiconductor at all if the heat transfer at the illuminated boundary of the sample is too large.[6)]

Neglecting quantities of order γ, from Eq. (37) we obtain

$$1+s+k^2=q^2. \tag{39}$$

It is obvious that, in the conditions under consideration this is possible only if $q^2 > 0$, i.e., if

$$2\dot{\tau}+\dot{\varkappa}+s\dot{\varkappa}>0. \tag{40}$$

This inequality, in conjunction with the table, determines the mechanisms for the scattering of energy and momentum and the experimental conditions under which the phenomena considered below can take place.

Setting k = 0 in Eq. (39) (a one-dimensional distribution) and taking Eq. (32) into account, we obtain

$$s=\frac{(2\dot{\tau}+\dot{\varkappa})\gamma J_m-1}{1-\dot{\varkappa}\gamma J_m} \tag{41}$$

Here condition (40) takes the form

$$\dot{\tau}(1-\gamma\dot{\varkappa}J_m)^{-1}>0. \tag{40'}$$

We see that $s > 0$ and the static one-dimensional distribution of the electron temperature turns out to be fluctuationally unstable upon fulfilment of one of the following two systems of inequalities:

$$\dot{\varkappa}^{-1}>\gamma J_m>(2\dot{\tau}+\dot{\varkappa})^{-1},\quad \dot{\tau}>0, \tag{42a}$$

$$(2\dot{\tau}+\dot{\varkappa})^{-1}>\gamma J_m>\dot{\varkappa}^{-1},\quad \dot{\tau}<0. \tag{42b}$$

We shall only be interested in the first case, (42a). Upon fulfilment of the indicated inequalities, the one-dimensional distribution, having become unstable, is replaced by the steady-state, three-dimensional, temperature distribution of the electrons. In fact, setting s = 0 in Eqs. (32) and (39) and changing to the usual units, we obtain the following result:

$$k=(3/2\varkappa_0\tau_0)^{1/2}(J_m/J_c-1)^{1/2},\quad J_m\geqslant J_c. \tag{43}$$

Here J_c denotes the critical value of the energy flux, the value at which the considered change in the state of the electron gas takes place:

$$J_c=3nT_0/2\gamma\tau_0(2\dot{\tau}+\dot{\varkappa}). \tag{44}$$

We note that $\gamma/n = \sigma$, where σ is the cross section for photon absorption.

Similar results are also obtained in the case of a degenerate gas: it is only necessary to replace the denominator in (41) by a constant c, and out of the inequalities (42a) to keep the last two (here the case (42b) does not arise at all). In this connection formulas (43) and (44) remain without changes. The numerical factor in for-

mula (44) is changed for surface absorption, and λ_0 appears in the denominator in place of γ.

As is clear from formulas (33), (34), (39), and (43), the function $f_1(z)$ differs significantly from zero for

$$z \lesssim z_\infty = \frac{2}{\gamma} \ln \left(\frac{2}{\gamma} \left(\frac{J_m}{J_c} \right)^{1/2} \right). \quad (45)$$

At the same time, for $q^2 = 1$ formula (17′) turns out to be valid even for $z \lesssim \gamma^{-1}$ (compare with Eq. (18)). Thus, our calculation is valid under the conditions of smallness of γ.

6. CONCLUDING REMARKS. ESTIMATES

The dependence of the critical value J_c on the parameters of the material and on the experimental conditions, expressed by formula (44), has a clear meaning. In fact, heating of the electron gas implies an increase of the average energy per charge carrier. Hence, an increase of the factor n. Comments on the role of γ and τ_0 are not required;[7)] finally, T_0 is the only independent quantity having the dimensions of an energy which exists in the problem in the conditions under consideration. The meaning of the inequality $2\dot{\tau} + \dot{k} > 0$ is also clear from what was said in Sec. 1. The conditions under which this inequality is satisfied are clear from the table. In this respect either the degenerate case ($\dot{k} + 2\dot{\tau} = 3$) or the case when the energy is scattered by piezoelectric (acoustic) vibrations but the quasimomentum is scattered by charged impurities ($\dot{k} + 2\dot{\tau} = 7/2$ in the absence of degeneracy) is especially interesting. It is clear that such a situation exists in n-InSb at liquid helium temperatures,[17] and also $\tau_0 \approx 10^{-7}$ sec.

According to[18], in this material $\sigma \approx 2.3 \times 10^{-17} (\lambda/9\mu)^2$ for a wavelength $\lambda \gtrsim 9\ \mu$. For $\lambda = 300\ \mu$ this gives $\sigma \approx 2.5 \times 10^{-14}$ cm^2, and therefore (at T = 3°K)

$$J_c \approx \frac{2.5}{2\dot{\tau}+\dot{\kappa}} \cdot 10^{-2} \left[\frac{\text{W}}{\text{cm}^2} \right].$$

In addition to n-InSb, the other narrow band semiconductors might also be of certain experimental interest in regard to the relation under consideration. In connection with this, we note that for a sufficiently large hole mass we can also apply the calculation set forth above to bipolar material, provided that the holes are simply treated as scattering centers (in this connection their role is indistinguishable from the role of charged impurities).

APPENDIX

DEPENDENCE OF THE ENERGY RELAXATION TIME ON THE ELECTRON TEMPERATURE UNDER THE CONDITIONS OF STRONG DEGENERACY

The energy relaxation time used above is determined by (for a sufficiently small drift velocity) the equation

$$^2/_3 \langle dW/dt \rangle = (T - T_0)/\tau. \quad (A.1)$$

Evaluating the left hand side of Eq. (A.1) by the usual method,[11] we obtain

$$\frac{1}{\tau} = \int dq\, dk\, n_F(W_k) [1 - n_F(W_k + \hbar\omega_q)] N_q(\ldots) \cdot \times \frac{1}{T - T_0} \left\{ \exp \left[\frac{\hbar\omega_q}{TT_0} (T - T_0) \right] - 1 \right\}. \quad (A.2)$$

Here q and k are the quasiwave vectors of the phonon and the electron; n_F and N_q are the Fermi and Planck distribution functions, which depend on T and T_0, respectively; ω_q denotes the frequency of the phonon for the branch under consideration. The matrix elements and the other factors, which do not contain T and T_0, are denoted by the symbol (. . .). Under the conditions of strong degeneracy, the corrections to τ^{-1}, which are related to temperature broadening of n_F, will be of the order of T^2/F_0^2. Correspondingly, the electron temperature only enters into the last factor in Eq. (A.2). It is immediately seen that, as $T \to T_0$ we have $\tau \sim T$ and, according to Eq. (19), $\dot{\tau} = 1$ for any scattering mechanism, any type of anisotropy of the isoenergy surfaces in any kind of phonon spectrum.

[1)] The problem of the formation of a static, spatially periodic distribution of the temperature and of the concentration of carriers in the presence of any external flux has been investigated in a number of articles. [4] In these articles, however, heating of the electron gas, which will play a major role below, was not taken into consideration.

[2)] As is well known (see, for example, [11], in the presence of a sufficiently strong current passing through the sample the condition $d\tau/dT > 0$ may lead to the emergence of S-type current-voltage characteristics. The effects considered below represent, in fact, the appearance of this same instability in static conditions and in the absence of a continuous current through the sample.

[3)] In a monopolar semiconductor this is obviously equivalent to the approximation of quasineutrality.

[4)] In what follows it is convenient to talk about electrons, but the equations are written for particles with positive charge e.

[5)] This is justified by the subsequent calculation: the important values of z turn out to be rather large (see below, formula (45)).

[6)] Formally the denominator in (38) may also be negative. Then the inequality (38) is automatically satisfied: the thermal conductivity of the electrons grows so rapidly under the influence of heating that the energy is not able to escape from the sample into the dielectric medium adjacent to it. In this connection, however, the condition of small overheating turns out to be poorly justified, and we shall not investigate this possibility.

[7)] Of course, γ depends on n. In the case of a nondegenerate gas $\gamma \sim n$, and J_c actually does not depend on the concentration of free electrons.

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