Galvanomagnetic properties of metals with closed Fermi surfaces at low temperatures

R. N. Gurzhi

Physico-technical Institute, Ukrainian Academy of Sciences

A. I. Kopeliovich

Low-temperature Physico-technical Institute, Ukrainian Academy of Sciences
(Submitted July 11, 1974)
Zh. Eksp. Teor. Fiz. **67**, 2307–2322 (December 1974)

The low-temperature magnetoresistance of pure metals with closed Fermi surfaces is considered in conditions when the relaxation processes are entirely determined by electron collisions with phonons, and the thermal momentum of the phonons is small compared with the characteristic dimensions of the Fermi surface. Detailed account is taken of the diffusion of electrons over the Fermi surface as a result of normal collisions, the Umklapp processes, which occur only in the regions of closest approach (the lunes) of the closed Fermi surfaces, and the nonequilibrium character of the phonons (phonon drag). It is shown that in the region of strong magnetic fields the solution of the problem can be formulated in the form of Kirchoff's rules for branched electrical circuits with "resistances" of Umklapp and diffusional origin. The electrical-conductivity tensor of the metal is calculated for different topological properties of its Fermi surface, different magnetic-field orientations, and in different ranges of temperature. At temperatures that are not too low the transverse conductivity σ_{xx} displays substantial anisotropy: σ_{xx} reaches a maximum and is entirely determined by Umklapp processes if the magnetic field is oriented such that several lunes lie in the section $p_z = \text{const}$ (overlapping of the lunes); For other field directions σ_{xx} is determined by electron diffusion processes. The transverse resistivity is a maximum in the case of overlapping lunes if the number of electrons is not equal to the number of holes, and is a minimum in the case of a compensated metal. The possibility of separating the effect of the Umklapp processes in a broad range of temperature and for a large number of metals (including compensated metals) essentially distinguishes galvanomagnetic phenomena from conduction in the absence of a magnetic field. It is shown that at temperature that are not too low there exists a broad region of intermediate magnetic fields in which the conductivity depends in an unusual way on the field strength and temperature. In particular, the formation of effective open orbits (for a closed Fermi surface) is possible in this region.

INTRODUCTION

It is well known that in strong magnetic fields the asymptotic behavior of the electrical resistivity of a metal as a function of the field strength is entirely determined by the topological properties of the Fermi surface and by the direction of the field. This result can be proved rigorously, without any assumptions about the electron-scattering mechanism[1]. However, properties of the magnetoresistance such as its dependence on the temperature and on other parameters characterizing the collisions, the dependence on the orientation of the magnetic field, and, finally, the magnitude of the field at which the emergence into the asymptotic dependence occurs, are determined in an essential way by the properties of the electron-scattering mechanism. In the general case all that can be asserted is that the resistivity is a monotonically increasing function of the field[2] and that this increase becomes appreciable for $\Omega\tau_{tr} \gtrsim 1$ (τ_{tr} is the mean free time, which determines the electrical conductivity in the absence of a magnetic field: $\sigma_0 \approx (ne^2/m)\tau_{tr}$, and Ω is the Larmor frequency).

In the simplest case, when the characteristic electron-scattering angle $\Phi \sim 1$ (collisions with local crystal-lattice defects of the impurity-atom type), satisfactory results can be obtained in the relaxation-time approximation. In this case the resistivity depends on the single parameter $\Omega\tau$; in the region $\Omega\tau \ll 1$ the effect of the magnetic field is small ("weak" fields), and the asymptotic dependences are attained for $\Omega\tau \gg 1$ ("strong" fields). But if the scattering angle $\Phi \ll 1$ (electron-phonon collisions at low temperatures, scattering by dislocations, etc.), the situation becomes substantially more complicated. In conditions of small-angle scattering, in addition to τ_{tr}, which is connected with the diffusion of an electron across the Fermi surface, the characteristic time between individual collisions ($\tau' \approx \tau_{tr}\Phi^2 \ll \tau_{tr}$) is also important. As Pippard has shown[3], in a strong magnetic field the effectiveness of small-angle collisions increases substantially in the presence of certain features of the Fermi surface that lead to rapid variation of the electron distribution function in momentum space. In this case the emergence into the asymptotic dependences with increasing magnetic field is very protracted and is realized when $\Omega\tau_{tr} \gg \Phi^{-1}$ or even when $\Omega\tau_{tr} \gg \Phi^{-2}$. However, Pippard's treatment is of a highly model character and, in particular, does not take account of specific features (on which the results of the present paper are based) of the momentum-relaxation mechanism in the electron-phonon system of the metal.

In the study of the electrical conductivity of pure metals with closed Fermi surfaces, Umklapp processes in electron-phonon collisions play a fundamentally important role. At sufficiently low temperatures, when Umklapp processes in collisions between phonons can be neglected, it is precisely these processes that determine the relaxation of the momentum of the electrons. Also important is the fact that the phonons, because of their frequent normal collisions with electrons, cannot be considered to be in equilibrium (for more detail, cf.[4]). In the absence of Umklapp processes the electrical conductivity σ_0 in zero magnetic field is infinite (for $n_e \neq n_h$), and in a strong magnetic field only the off-diagonal, Hall components of the tensor of the transverse electrical conductivity are nonzero:

$$\sigma_{xy}=-\sigma_{yx}=e(n_e-n_h)cH^{-1}, \quad \sigma_{xx}=\sigma_{yy}=0.$$

(Here n_e and n_h are the electron and hole densities and z is the magnetic-field direction.)

At low temperatures, when the thermal momentum of the phonons is small compared with the characteristic dimensions of the Fermi surface, the scattering of electrons in normal collisions with phonons is small-angle and can be described in terms of diffusion of the electrons over the Fermi surface[4]. Umklapp collisions are possible only in comparatively small regions on the Fermi surface–the "lunes"[5] (the regions of closest approach of isolated electron or hole groups). The lune radius $r_0 \ll p_F$.

The present paper is devoted to the study of the galvanomagnetic properties of metals with closed Fermi surfaces at low temperatures, with allowance for the nonequilibrium character of the phonons, Umklapp processes and electron diffusion over the Fermi surface. In the first three Sections we consider the region of strong magnetic fields, and in Sec. 4 the region of intermediate fields.

1. DIFFUSION EQUATION IN A STRONG MAGNETIC FIELD

In the presence of a magnetic field the diffusion equation has the form

$$-\frac{1}{v}\frac{\partial\chi_p}{\partial t}+\operatorname{div}\hat{D}_p(\nabla\chi_p-\mathbf{a}(\chi_p))+\Pi_p=-e\mathbf{E}\mathbf{n}_p,$$
$$\mathbf{a}(\chi_p)=\int\hat{A}_{pp'}\nabla\chi_{p'}\,dS'. \quad (1)$$

Here $-\chi_p\partial n/\partial\epsilon$ is the nonequilibrium correction to the electron distribution function (χ_p does not depend on the energy ϵ), $n(\epsilon)=[e^{(\epsilon-\mu)/T}+1]^{-1}$, t is the orbit period in the magnetic field, $\mathbf{v}=\partial\epsilon/\partial\mathbf{p}$ and $\mathbf{n}=\mathbf{v}/v$. One must understand by div and ∇ the corresponding two-dimensional operators in the tangent plane to the Fermi surface. The second term in the left-hand side of Eq. (1) describes normal collisions[1)]: $\hat{D}_p$ is the diffusion tensor, and the integral term a is associated with phonon drag. Explicit expressions for the tensors $\hat{D}_p$ and $\hat{A}_{pp'}$ are given in[4]. We note that $\hat{D}_p \propto T^5$, and $\hat{A}_{pp'}$ does not depend on the temperature.

The term Π_p describing the Umklapp processes can be represented in the form

$$\Pi_p=\sum_g\int\mathcal{I}_{pk}(\chi_p-\chi_k)\,dS_k, \quad \mathcal{I}_{pk}=\frac{16\pi^2}{h^4v_pv_k}M_p(\mathbf{e})\,q\omega_q\frac{dN}{d\omega_q},$$

where $\mathbf{q}=\mathbf{k}-\mathbf{p}-\mathbf{g}$. The momenta p and k are positioned in equivalent lunes, between which Umklapp processes are possible; g is the reciprocal-lattice vector corresponding to the given pair of equivalent lunes (in the case of a multiply-connected Fermi surface for transitions within the Brillouin zone, $\mathbf{g}=0$); ω_q is the energy of the phonon as a function of its quasi-momentum q; $N_q=[\exp(\omega_q/T)-1]^{-1}$. The matrix element of the electron-phonon interaction is written in the form $[qM_p(\mathbf{e})]^{1/2}$, $\mathbf{e}=\mathbf{q}/q$.

We note that in the derivation of the diffusion equation the fact that the lunes are small has already been used: in the integral term $a(\chi_p)$ terms associated with the effect of Umklapp processes on the phonon distribution function have been omitted; in the term Π_p, only transitions between isolated lunes have been retained.

The electric-current density is

$$\mathbf{j}=\frac{2e}{h^3}\int\mathbf{n}_p\chi_p\,dS_p.$$

In calculating the current in a direction perpendicular to the magnetic field it is convenient to use the formula

$$\mathbf{j}_\perp=-\frac{2c}{h^3H^2}\int[\mathbf{H}\times\mathbf{p}]\Pi_p\,dS_p+e(n_e-n_h)\mathbf{u}_H, \quad (2)$$

where $\mathbf{u}_H=cH^{-2}[\mathbf{E}\times\mathbf{H}]$ is the Hall drift velocity. This relation is easily obtained from Eq. (1) or from the original system of kinetic equations for the electrons and phonons by making use of the conservation of quasi-momentum in normal collisions.

In a strong magnetic field (the appropriate criterion will be given below), in solving Eqs. (1) it is natural to make use of the method of successive approximations. We have $\chi=\chi^{(1)}+\chi^{(2)}+\ldots$, with

$$\frac{\partial\chi^{(1)}}{\partial t}=e\mathbf{E}_\perp\mathbf{v}, \quad \chi^{(1)}=\mathbf{u}_H\mathbf{p}+f(p_z), \quad (3)$$

$$-\frac{\partial\chi^{(2)}}{\partial t}+v\operatorname{div}\hat{D}(\nabla f-\mathbf{a}(f))+v\Pi(\chi^{(1)})=-eE_zv_z. \quad (4)$$

In the latter equation we have made use of the invariance of the flux $\nabla\chi-\mathbf{a}(\chi)$ under the galilean transformation $\chi_p\to\chi_p+\mathbf{u}\cdot\mathbf{p}$[4].

In calculating the electric current in a strong magnetic field it is sufficient to confine ourselves to the first approximation (3). However, the function $f(p_z)$ is determined from the conditions that the second-approximation equation (4) be soluble:

$$\langle v\operatorname{div}\hat{D}(\nabla f-\mathbf{a}(f))\rangle+\langle v\Pi(\chi^{(1)})\rangle=-eE_z\langle v_z\rangle,$$
$$\frac{2}{h^3}\int p_zv\Pi(\chi^{(1)})\,dS=-eE_z(n_e-n_h). \quad (5)$$

The angular brackets describe averaging over the period $T(p_z)$ of the rotation

$$\langle\ldots\rangle=\left|\frac{eH}{c}\right|\int_0^T\ldots dt.$$

We next perform the replacement $f(p_z)=u_cp_z+\psi(p_z)$ and thereby transform to the co-moving reference frame[4]. In the co-moving frame, according to its definition, Eq. (5) can be solved by iterations in the integral term $a(\psi)$. We shall confine ourselves to the first iteration (i.e., we simply omit the term a in the co-moving frame). After simple transformations we finally arrive at the following equations for the function $\psi(p_z)$ and velocity u_c [2)]

$$\frac{d}{dp_z}D\frac{d\psi}{dp_z}+\langle v\Pi(\mathbf{up}+\psi)\rangle=-eE_z\langle v_z\rangle, \quad (6)$$

$$\frac{2}{h^3}\int p_z\Pi(\mathbf{up}+\psi)\,dS=-eE_z(n_e-n_h), \quad (7)$$

where

$$D(p_z)=\langle D_{\xi\xi}v_\perp^2/v\rangle, \quad \mathbf{u}_\perp=\mathbf{u}_H, \quad u_z=u_c.$$

Here $D_{\xi\xi}$ is the diagonal element of the diffusion tensor $\hat{D}$ along the direction perpendicular to the orbit. The term with the derivatives in Eq. (6) describes the diffusion of electrons over the Fermi surface in the direction perpendicular to the orbits; $-Dd\psi/dp_z$ is the total diffusion current through the section p_z = const. We emphasize that in a strong magnetic field we can neglect the diffusive displacement along the orbit, and the diffusion therefore has a one-dimensional character. The term $\langle v\Pi\rangle$ gives transitions between electron orbits passing through equivalent lunes. Eq. (7) describes the quasi-momentum balance in the direction of the magnetic field.

2. KIRCHOFF'S RULES

The character of the solution of Eqs. (6) and (7) depends essentially on the mutual disposition of the lunes on the Fermi surface, the orientation of the magnetic field, and the temperature range. We begin by considering the simplest physical situation, although, as will become clear from the following, the results obtained have a considerably wider range of applicability. Namely, we shall assume that the layers of orbits passing through each lune ("belts" on the Fermi surface' do not overlap and that the distances between the belts are considerably greater than their width.

In the case under consideration the result of solving Eqs. (6) (7) can be formulated in terms of the problem of the flow of stationary currents over branched electrical circuits (cf.[4]). The analog of the potential is the function $\psi(p_z)$. The diffusion current is

$$J_d = -D d\psi/dp_z. \tag{8}$$

First we shall consider the case of crossed fields **E** and **H**, i.e., $E_z = 0$. Then from (8) the potential difference is

$$\delta\psi = J_d R_d, \quad R_d = \int_{p_{z1}}^{p_{z2}} \frac{dp_z}{D(p_z)}. \tag{9}$$

Here the integration is taken within the layer between two neighboring belts, and R_d has the meaning of the resistance of this portion.

The Umklapp current through the given lune is

$$J_U = R_U^{-1}[\mathbf{ug} + \psi^0 - \psi^{0*}], \quad R_U^{-1} = -\int \mathcal{H}_{pk} dS_p dS_k. \tag{10}$$

Here **g** is the corresponding reciprocal-lattice vector (it is assumed that $\Delta p \ll g$, ψ^0 is the value of the function $\psi(p_z)$ in the given lune, and ψ^{0*} is its value in the equivalent lune. (In the model under consideration, the variation of the function $\psi(p_z)$ within the belt can be neglected.) The quantity R_U has the meaning of the Umklapp resistance.

By integrating Eq. (6) over a small region encompassing one of these belts, we find

$$J_{d1} + J_{d2} + J_U = 0, \tag{11}$$

where J_{d1} and J_{d2} are the diffusion currents at the boundaries of the belt; for the first lune, corresponding to the minimum p_z, $J_{d1} = 0$, and for the second lune $J_{d2} = 0$. (Current emerging from a lune is taken to be positive.)

Eqs. (9)–(11) taken together with the continuity condition on the function $\psi(p_z)$ coincide with Kirchoff's laws for a certain electrical circuit. As an example, a Fermi surface with two pairs of equivalent lunes $\alpha\alpha^*$ and $\beta\beta^*$ and the corresponding circuit are drawn in Fig. 1. (We note that equivalent lunes are not necessarily positioned centro-symmetrically.) Sources of emf $\mathscr{E} = \mathbf{u}\cdot\mathbf{g}$ are included in the circuit (cf. (10)).

If $E_z \neq 0$, the function $\psi(p_z)$ is conveniently sought in the form $\psi = \psi_1 + \psi_2$, with

$$\frac{d}{dp_z} D \frac{d\psi_1}{dp_z} = -eE_z\langle v_z\rangle, \quad \psi_1(p_z) = \pm eE_z \int \frac{S_\perp(p_z')}{D(p_z')} dp_z', \tag{12}$$

where the upper sign is taken for the case of an electron Fermi surface and the lower sign for a hole Fermi surface, and $S_\perp(p_z)$ is the area of the section of the Fermi surface made by the plane p_z = const. The function $\psi_2(p_z)$ is determined by Kirchoff's laws with an EMF $\mathscr{E} = \mathbf{u}\cdot\mathbf{g} + \psi_1^0 - \psi_1^{0*}$. The quasi-momentum balance

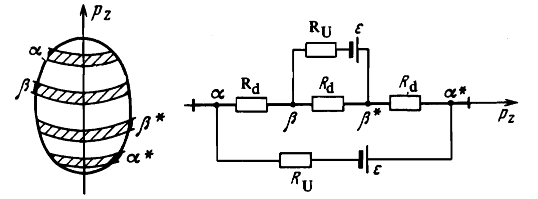


FIG. 1

equation (7) has the form

$$\frac{2}{h^3} \sum_k g_z^k J_U^k = eE_z(n_e - n_h).$$

The summation runs over all inequivalent lunes.

The relations given make it possible to find the function $\psi(p_z)$(to within an unimportant arbitrary constant) and the velocity u_c of the co-moving system. The electric-current density is

$$\mathbf{j}_\perp = -\frac{2c}{h^3H^2} \sum_k [\mathbf{g}^k \times \mathbf{H}] J_U^k + e(n_e - n_h)\mathbf{u}_H, \tag{13}$$

$$j_z = \frac{2e}{h^3} \int n_z \psi\, dS + e(n_e - n_h) u_c. \tag{14}$$

We shall discuss the physical meaning of the results obtained. In order of magnitude, $J_U = u_H \cdot g(R_d + R_U)^{-1}$, and according to (13) the transverse conductivity is

$$\sigma_{xx} \approx (c^2g^2/h^3H^2)(R_d + R_U)^{-1}. \tag{15}$$

The conductivity σ_{xx} is proportional to the number of Umklapp scatterings; however, if we completely neglect the diffusion, the concentrations of nonequilibrium electrons in equivalent lunes turn out to be equal, and $\sigma_{xx} = 0$. (For $R_d = \infty$, according to (9)–(11), $\mathbf{u}\cdot\mathbf{g} + \psi^0 - \psi^{0*} = 0$ and $J_U = 0$.) In other words, in order that $J_U \neq 0$, it is necessary that there be a diffusion current from one of the equivalent lunes to the other, closing the Umklapp flux between these lunes. Therefore, the effective relaxation time characterizing the electrical conductivity is composed of the Umklapp and diffusion times.

We note, in this connection, that the expression (15) can be represented in the form

$$\sigma_{xx} \approx n_{\text{eff}} e^2/m\Omega^2\tau_{\text{eff}},$$
$$n_{\text{eff}} \approx \frac{b}{p_F} n, \quad \tau_{\text{eff}} \approx \tau_d^{(b)} + \frac{p_F b}{r_0^2} \tau'_U. \tag{16}$$

Here τ_{eff} is the effective mean free time with respect to Umklapp collisions; n_{eff} is the number of electrons which participate in these collisions; b is the distance between belts ($b > r_0$); p_F is a characteristic dimension of the Fermi surface, $n \approx (p_F/h)^3$; $\tau_d^{(b)} \approx (p_F b/v) R_d^b \approx \tau_F (b/p_F)^2$ is the time corresponding to diffusional displacement over the distance b, where $\tau_F \propto T^{-5}$ is the usual transport time for the electron-phonon interaction, corresponding to diffusional displacement over the distance p_F; $\tau'_U \approx r_0^2 R_U/v$, and τ'^{-1}_U is the probability of an Umklapp scattering for an electron situated in a lune; r_0^2/p_F^b is the probability of finding an electron in a lune.

The diffusional resistivity of a layer of thickness b equal

$$R_d^{(b)} \approx R_F b/p_F, \quad R_F \approx p_F/D \propto T^{-5}, \tag{17}$$

where R_F corresponds to a diffusional displacement across the whole Fermi surface (cf. (9)).

To calculate R_U it is necessary to have recourse to concrete models of the lunes (cf. Sec. 3). However, certain conclusions can also be drawn in the general case. At temperatures $T \gtrsim T_0 = \Delta ps$ (s is the speed of sound and Δp is the minimum distance between the Fermi surfaces), the Umklapp processes, like the normal collisions, are effected by thermal phonons with momenta $q_T \approx T/s$. However, diffusion over the distance p_F requires $(p_F/q_T)^2$ Brownian steps, whereas an Umklapp process is realized as a result of one step and therefore requires less time. As a result it turns out that $R_U \approx (q_T/p_F)^n R_F$, $n \gtrsim 1$. (The exponent n depends on the shape of the Fermi surface in the region of the lune; cf. Sec. 3). At temperatures $T < T_0$ the Umklapp processes are effected by phonons with momenta $\Delta p > q_T$; $R_U \propto \exp(T_0/T)$ increases exponentially and is comparable with R_F at a certain temperature $T_P \ll T_0$[3)]. The temperature dependence of the ratio R_F/R_U is depicted schematically in Fig. 2.

In the absence of a magnetic field the electrical conductivity of a metal with a closed Fermi surface and unequal numbers of electrons and holes has the form[5]

$$\sigma_0 \approx ne^2\tau_{\text{eff}}/m, \quad \tau_{\text{eff}} \approx mg[R_F \ln(p_F/r_0) + R_U]$$

and, in accordance with what has been said above, the Peierls exponential dependence $\sigma \propto \exp(T_0/T)$ can be observed only at temperatures $T \lesssim T_P \ll T_0$. In real conditions, however, at such low temperatures the principal role will be played by scattering of electrons by crystal-lattice defects.

In the case of conduction in a strong magnetic field, it is possible to avoid the competition of the Umklapp and diffusion times by orienting the magnetic field in such a way that an electron can travel from the initial lune to one equivalent to it as a result of its motion along the orbit. In the scheme of Fig. 1 this means that one of the diffusional resistivities $R_d = 0$, and in the corresponding part of the circuit a large current $J_U \approx \mathbf{u} \cdot \mathbf{g} R_U^{-1}$ then arises, and $\sigma_{xx} \propto R_U^{-1}$. We emphasize that this result does not depend on the relationship between the numbers of electrons and holes.

Thus, the possibility arises that the Umklapp processes in a broad temperature range, and in particular, the dependence $\sigma_{xx} \propto \exp(-T_0/T)$ for $T < T_0$ (cf.[6]) can be investigated experimentally. In experiments one usually measures the resistivity tensor $\hat{\rho} = \hat{\sigma}^{-1}$. In order of magnitude the transverse resistivity is

$$\rho_{yy} \approx \frac{\sigma_{xx}}{\sigma_{xy}^2} \approx \frac{g^2}{e^2h^3(n_e - n_h)^2}(R_d + R_U)^{-1}, \qquad n_e \neq n_h,$$

$$\rho_{yy} \approx \sigma_{xx}^{-1} \approx \frac{h^3H^2}{c^2g^2}(R_d + R_U), \qquad n_e = n_h.$$

It can be seen from these formulas that for $T > T_0$ and for magnetic-field directions corresponding to overlap of lunes, the resistivity has a maximum if $n_e \neq n_h$ and has a minimum if $n_e = n_h$.

We shall now discuss the range of applicability of the results obtained.

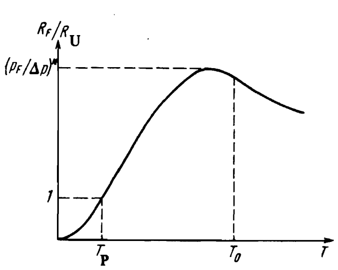


FIG. 2

Kirchoff's rules (9)–(11) are based, in essence, only on the assumption that the function $\psi(p_z)$ varies little within the limits of a lune belt and, because of this, effects from diffusion and Umklapp processes can be separated.

We note, first of all, that if the equivalent lunes are not positioned centro-symmetrically, overlap of two inequivalent lunes does not lead to overlap of the lunes equivalent to them, and the results obtained remain valid. (In this case, an electron moving out of a given lune along an orbit in the magnetic field and undergoing Umklapp processes cannot return to the initial lune without the participation of diffusion processes.)

Moreover, the results remain valid for any field direction, if for $b = r_0$ the condition $R_d \ll R_U$ is fulfilled: for small b the diffusion resistivities can be neglected, while for $b \gg r_0$ Kirchoff's rules are applicable. This is valid in any case for $T < T_P$.

We note, finally, that the relations (9)–(11) are easily generalized to the case of a multiply-connected Fermi surface. In this case, for transitions between isolated groups within the Brillouin zone, we have, in place of (10),

$$J_U = R_U^{-1}(u\Delta p + \psi_m^0 - \psi_{m+1}^{0*}),$$

where Δp is the minimum spacing between neighboring groups with labels m and m+1.

To conclude this subsection we shall give the results of solving the relations (9)–(12) for certain special cases.

1. At the lowest temperatures $T < T_P$, the resistivities R_d can be neglected in comparison with R_U for any magnetic-field direction, and the function ψ_1 can be assumed constant. Then, as is easily shown, $J_U^k = \mathbf{u} \cdot \mathbf{g}^k (R_U^k)^{-1}$, $u_c = -u_x S_{yx} S_{zz}^{-1}$; and the electrical-conductivity tensor has the form

$$\hat{\sigma} = \begin{pmatrix} \left(\frac{c}{H}\right)^2 [S_{yy} - S_{yz}^2 S_{zz}^{-1}] & \frac{\Delta nec}{H} & -\frac{\Delta nec}{H} S_{yz} S_{zz}^{-1} \\ -\frac{\Delta nec}{H} & \left(\frac{c}{H}\right)^2 [S_{xx} - S_{xz}^2 S_{zz}^{-1}] & \frac{\Delta nec}{H} S_{xz} S_{zz}^{-1} \\ \frac{\Delta nec}{H} S_{yz} S_{zz}^{-1} & -\frac{\Delta nec}{H} S_{xz} S_{zz}^{-1}, & e^2(\Delta n)^2 S_{zz}^{-1} \end{pmatrix}, \quad (18)$$

where

$$S_{\alpha\beta} = \frac{2}{h^3} \sum_k g_\alpha^k g_\beta^k (R_U^k)^{-1}, \quad \Delta n = n_e - n_h \neq 0.$$

2. For $T > T_P$ it is necessary to take into account the dependence $\psi_1(p_z)$, and only the elements of the transverse conductivity tensor have a relatively simple form.

For the case when the equivalent lunes are positioned centro-symmetrically, and for directions of the field **H** such that all $R_d \gg R_U$, σ_{xx} and σ_{yy} are determined by the same formulas as in subsection 1, in which, however,

$$S_{\alpha\beta} = \frac{2}{h^3} \sum_k (g_\alpha^k - g_\alpha^{k-1})(g_\beta^k - g_\beta^{k-1})(R_d^{k,k-1})^{-1}.$$

Here $R_d^{k,k-1}$ is the diffusion resistivity of the layer between the (k − 1)-th and k-th lunes, and R_d^{10} is the resistivity of the layer between the central section and the first lune. (The labeling of the inequivalent lunes starts from the central section.)

If, however, $R_d \ll R_U$ for some of the lunes (the lunes either overlap completely, or are close to such a position), then σ_{xx} and σ_{yy} are principally determined by such lunes. In this case, in the determination of the currents J_U the corresponding parts of the circuit can be regarded as isolated. The result has the form

$$\sigma_{xx}=\left(\frac{c}{H}\right)^2\frac{2}{h^3}\sum_{i,k}(g_y^{ik}-\overline{g_y^k})^2(R_U^{ik})^{-1},$$

$$\overline{g_y^k}=\sum_i g_y^{ik}(R_U^{ik})^{-1}\Big/\sum_i (R_U^{ik})^{-1},$$

where k labels the layers of the overlapping lunes and i labels the lunes in each layer.

It can be seen from the formulas presented that for $T \gg T_p$ the transverse conductivity is strongly anisotropic:

$$\sigma_{xx}^{max}/\sigma_{xx}^{min}\approx(R_F/R_U)(b_{max}/p_F),$$

where b_{max} is the maximum possible spacing between the lune layers.

3. The case when there is only one pair of equivalent lunes, or, more accurately, when the probability of an Umklapp process is considerably greater for one pair of lunes than for all the others, is special. In the presence of only one Umklapp vector **g** and for arbitrarily small deviations of the vectors **H** and **g** from perpendicularity, drift arises along the magnetic field, such that the resultant velocity $\mathbf{u} = \mathbf{u}_H - \mathbf{H}(\mathbf{g}\cdot\mathbf{u}_H)(\mathbf{g}\cdot\mathbf{H})^{-1}$ turns out to be perpendicular to the vector **g**. The result $[\mathbf{j} = e(n_e - n_h)\mathbf{u}$, $\sigma_{xx} = \sigma_{yy} = 0]$ is exact for any H and is not due to the diffusion approximation, since the total collision integral is made to vanish by a drift solution with velocity **u** perpendicular to the vector **g**. It is clear that for small $\mathbf{g}\cdot\mathbf{H}$, when $u_z \to \infty$, allowance for arbitrarily weak scattering mechanisms (scattering by impurities, and the influence of other lunes) will have a substantial effect on the result. By adding, in the diffusion equation, a term $(\psi + \mathbf{u}\cdot\mathbf{p})\tau_0^{-1}$ describing these scattering mechanisms, we obtain

$$\sigma_{xx}\approx\frac{2c^2}{h^3H^2}g_y^2R_U^{-1}(1+g_z^2\tau_0K^{-1}R_U^{-1})^{-1},\quad K\approx\int\frac{p_z^2}{v}dS. \tag{19}$$

We note that σ_{xx} is strongly anisotropic: $\sigma_{xx}(\varphi)/\sigma_{xx}(0) \approx [1 + (\varphi/\varphi)^2]^{-1}$, where φ is the angular deviation of the magnetic field from a direction perpendicular to **g**, $\varphi_0^2 \approx \tau_U/\tau_0$, and $\tau_U \approx \tau_U^*(g/r_0)^2$ is the effective Umklapp time in the absence of a magnetic field.

A situation similar to that described also occurs in the case when there are several pairs of equivalent lunes positioned in the same plane, with the electric field parallel to this plane. In this case,

$$\sigma_{xx}\approx\frac{2c^2}{h^3H^2}\Big(1+\sum_g g_z^2\tau_0K^{-1}R_U^{-1}\Big)^{-1}\sum_g g_y^2R_U^{-1}. \tag{20}$$

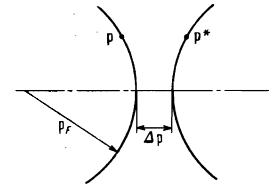
We emphasize that the results (19) and (20) are essentially associated with the phonon drag.

3. UMKLAPP PROCESSES AND ANISOTROPY OF THE CONDUCTIVITY

To determine the Umklapp resistivities and analyze the conductivity in the case of overlapping lunes, it is necessary to consider concrete models of the lunes. Below we shall consider three models: 1) "broad" lunes–the radius of curvature of the Fermi surface in the region of the lune is $r \approx p_F \gg \Delta p$; 2) "narrow" lunes–broken cylindrical necks with radius of curvature $r \lesssim \Delta p$; 3) "prolate" lunes with two appreciably different radii of curvature $r_1 \approx p_F \gg \Delta p$ and $r_2 \lesssim \Delta p$.

1. Broad lunes (Fig. 3). In this case the characteristic lune dimension is $r_0 \approx (p_F q_T)^{1/2}$ [5]. For Umklapp

FIG. 3



processes the local approximation is applicable, i.e., we can assume that the transitions occur between points p and p* situated on one horizontal[4)] (see Fig. 3). The term Π_p in the diffusion equation (1) can be written in the form[5]

$$\Pi_p\approx-\frac{D_0f(q_m/q_T)}{30\zeta(5)q_T^2}(\chi_p-\chi_{p^*})\equiv-\frac{1}{2}A(\mathbf{p})(\chi_p-\chi_{p^*}),$$

$$q_m=|\mathbf{p}-\mathbf{p}^*|,\quad f(x)=\int_x^\infty e^y(e^y-1)^{-2}y^3\,dy,$$

$$D_0=T^5\frac{30\zeta(5)}{\pi\hbar^4v^2}\frac{M}{s^5}, \tag{21}$$

where D_0 is the diffusion coefficient in the lune. Substituting this expression into (10), we obtain

$$R_U\approx\frac{30\zeta(5)}{\pi D_0}\frac{q_T}{p_F}\left[24\zeta(3)+\left(\frac{\Delta p}{q_T}\right)^3\right]^{-1}\exp\left(\frac{\Delta p}{q_T}\right). \tag{22}$$

As already noted, the results of the preceding section were essentially based on the assumption that the distribution function is constant within the lunes. If the lunes overlap or almost overlap ($b \lesssim r_0$), this assumption is fulfilled provided that the corresponding diffusion resistivity $R_d^b \ll R_U$. It can be seen from the relations (17) and (22) (see also Fig. 2; in the given case, n = 1) that such a situation occurs only at sufficiently low temperatures $T < T'$, where $T_p < T' < T_0$. At higher temperatures the character of the anisotropy of σ_{xx}, generally speaking, is such that a transition from the "Umklapp" to the "diffusion" situation occurs when there is significant overlapping of the lunes. To analyze this transition it is necessary to take into account the dependence $\psi(p_z)$ inside the lune.

For simplicity, we shall first consider the case when only two equivalent lunes overlap. According to (6) and (21), we have (for $E_z = 0$)

$$\frac{d}{dp_z}D\frac{d\psi}{dp_z}-\frac{1}{2}\mathscr{A}\left(p_z-\frac{b}{2}\right)[\mathbf{ug}+\psi(p_z)-\psi(p_z-b)]$$
$$-\frac{1}{2}\mathscr{A}\left(p_z+\frac{b}{2}\right)[-\mathbf{ug}+\psi(p_z)-\psi(p_z+b)]=0. \tag{23}$$

Here $\mathscr{A}(p_z - p_{z0}) = \langle vA(\mathbf{p})\rangle$, where p_{z0} is the center of the corresponding lune and p_z is measured from the mean section, equidistant from the centers of the two lunes.

It is easy to show that for $b \ll r_0$ Umklapp processes lead in Eq. (23) to additional diffusion with coefficient $d = 1/2b^2\mathscr{A}(p_z)$. We have

$$\frac{d}{dp_z}D\frac{d\psi}{dp_z}+\frac{d}{dp_z}d\left(\frac{d\psi}{dp_z}+\frac{\mathbf{ug}}{b}\right)=0,\quad (D+d)\frac{d\psi}{dp_z}+\frac{d}{b}\mathbf{ug}=\text{const}. \tag{24}$$

The constant in the latter equation is the diffusion current beyond the boundaries of the overlapping lunes. This current is inversely proportional to the large resistance to diffusion to the neighboring lunes, and can therefore be taken to be const = 0. Now, making use of (13), it is not difficult to obtain the electric current

$$j_x\approx\frac{4cg_y(\mathbf{ug})}{h^3H}\int_{-\infty}^{\infty}dp_z\mathscr{A}(p_z)\left[1+\left(\frac{b}{b_0}\right)^2\frac{\mathscr{A}(p_z)}{\mathscr{A}(0)}\right]^{-1}, \tag{25}$$

$$b_0^2=\frac{2D_0}{\mathscr{A}(0)}\approx r_0^2\left(\frac{q_T}{p_F}\right)^{1/2}\begin{cases}1, & T>T_0\\ (T/T_0)^3\exp(T_0/T), & T<T_0\end{cases}. \qquad (26)$$

In the calculation of the transverse conductivity σ_{xx} in (25) we can put $u = u_x$. (Taking into account the effect of the nonoverlapping lunes, we can show that $u_H\cdot g \gg u_c g_z$.) For $b \ll b_0$ the conductivity is determined by the Umklapp resistivity and, in accordance with (15),

$$\sigma_{xx}\infty 2\int_{-\infty}^{\infty}\mathscr{A}(p_z)\,dp_z=R_U^{-1}\infty\begin{cases}T^4, & T>T_0\\ TT_0^3\exp(-T_0/T), & T<T_0\end{cases}. \qquad (27)$$

In the region $r_0 \gg b \gg b_0$ we have $\sigma_{xx} \propto D_0 b^{-2}\ln(b/b_0) \propto T^5 b^{-2}$. Finally, for $b \gg r_0$ we have, in accordance with (15), $\sigma_{xx} \propto (R_d^b)^{-1} \propto T^5 b^{-1}$.

The angular dependence of the conductivity is shown in Fig. 4a (the solid line). (We recall that b/p_F is the angle of inclination of the magnetic field to the direction corresponding to exact overlap of the lunes.) Qualitatively, such a dependence follows from formula (16), in which we must take

$$\tau_{\text{eff}}\approx\tau_d^b+\frac{p_F}{r_0}\tau_U,\quad n_{\text{eff}}\approx n\begin{cases}b/p_F, & b\gg r_0\\ r_0/p_F, & b\ll r_0\end{cases}.$$

Although the change of p_z as a result of Umklapp processes is small ($\Delta p_z = b$), the electron flux through the overlapping lunes that is associated with Umklapp processes is large ($\propto b^{-2}$) and for $b \ll r_0$ the overlapping lunes play a decisive role in the balance of the z-component of the quasi-momentum. In view of this, the velocity u_c of the co-moving system, and with it the quantity σ_{zx} (cf. (14), turn out to be substantially anisotropic. A contribution to this effect, of the same order of magnitude, results from the "potential drop" (cf. 24))

$$\delta\psi\approx u_x g r_0 b^{-1}[1+(b_0/b)^2]^{-1}.$$

that arises between the edges of the overlapping lunes. The result is shown in Fig. 4b; the dashed curve corresponds to the case when, for exact overlap of the lunes, $\sigma_{zx} = 0$ because of the symmetry of the problem.

In the case of overlap of several pairs of lunes the situation is more complicated. We shall not carry out the calculations here, but confine ourselves to formulating the main results.

If all the overlapping lunes lie in the same, central section, the conductivity is

$$\sigma_{xx}\infty\int dp_z\left\{\frac{D}{d(D+d)}\left(\sum_k g_{ky}b_k\mathscr{A}_k\right)^2\right.$$
$$\left.+2\left[\sum_k g_{ky}^2\mathscr{A}_k-\left(\sum_k g_{ky}b_k\mathscr{A}_k\right)^2\Big/\sum_k b_k^2\mathscr{A}_k\right]\right\},$$
$$d=\frac{1}{2}\sum_k b_k^2\mathscr{A}_k.$$

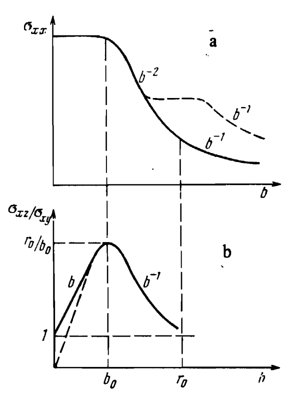


FIG. 4

It can be seen from this formula that in comparison with the case of one pair of overlapping lunes an additional term arises in σ_{xx} (the second term in the curly brackets), which does not vanish as $D \to 0$, i.e., in the absence of ordinary diffusion. We note that this term is nonnegative (by virtue of the Cauchy-Bunyakovskiĭ inequality) and vanishes only when b_k/g_{ky} = const = φ, i.e., on rotation, about the x-axis, of the central section in which all the vectors g_k lie (φ is the angle of rotation)[5]. In the latter case the angular dependence of the conductivity has approximately the same form as for overlap of one pair of lunes. On rotation about any other axis, inside the limits corresponding to the overlap of the lunes the additional term depends weakly on the angle and obviously decreases for $b \gtrsim r_0$ (see Fig. 4a (the dashed curve); if the dimensions r_0 of the overlapping lunes differ appreciably, several steps will appear in the graph).

It can be shown that what has been said above about the anisotropy of the quantity σ_{xz} (see Fig. 4b) remains valid in the case under consideration. Finally, we point out that the results cited are qualitatively conserved in the case when overlapping of lunes occurs in several planes.

We remark that in the case of Na and K, for which $\Delta p \approx p_F/3$, we should hardly expect large anisotropy of the conductivity. (In particular, the condition $b_0 \ll r_0$, as can be seen from (26), cannot be fulfilled, and therefore the region $\sigma_{xx} \propto b^{-2}$ is absent.) In these metals σ_{xx} is probably a smooth function of the magnetic-field direction, reaching maxima when the lunes overlap. The temperature dependence of the conductivity in the region of the maxima has the form (27).

2. Narrow lunes (Fig. 5). It is assumed that the thermal momentum q_T of the phonons is small compared with the length of the protrusion on the Fermi surface. (Otherwise, the presence of the protrusions is unimportant and the lune can be regarded as "broad.") The characteristic area of the lune is $S_0 \approx q_T r$.

It is easy to show that for overlap of lunes $R_d/R_U \lesssim q_T/p_F \ll 1$, and the equality $R_d = R_U$ is achieved under conditions in which the lunes are far from overlapping. This means that the variation of the function χ_p within a lune can be neglected and for any magnetic-field direction the Kirchoff rules obtain in the preceding Section are valid. It is clear that the character of the anisotropy of the conductivity can depend substantially on the shape of the protrusion (especially on its length) and on the shape of the rest of the Fermi surface. By making use of formula (10), it is not difficult to calculate the Umklapp resistivity:

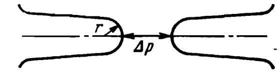
$$R_U\approx\frac{h^4v^2}{16\pi^2Mr^2q_T(q_T+\Delta p)^2}\exp\left(\frac{\Delta p}{q_T}\right). \qquad (28)$$

Thus, the temperature dependence of the conductivity for directions close to overlap has the form

$$\sigma_{xx}\infty\begin{cases}T^3, & T>T_0\\ TT_0^2\exp(-T_0/T), & T<T_0\end{cases}.$$

3. Prolate lunes. In this case the large characteristic dimension of the lune is $r_{01} \approx (p_F q_T)^{1/2}$ and the smaller dimension is $r_{02} \approx [q_T(r_2 + q_T)]^{1/2}$. From the same ar-

FIG. 5



guments as in the preceding subsection it follows that Kirchoff's rules are valid for any field direction. The resistivity of the prolate lune is

$$R_U \approx \frac{h^4 v^2 \exp(\Delta p/q_T)}{16\pi^2 M q_T (q_T+\Delta p)^2 (q_T+r_2)[p_F(q_T+\Delta p)]^{1/2}}. \tag{29}$$

The temperature dependence of the conductivity for directions close to overlap has the form

$$\sigma_{xx} \propto \begin{cases} T^{4,5}, & T>T_0 \\ T^2 T_0^{2,5} e^{-T_0/T}, & r_2 s<T<T_0. \\ T T_0^{2,5} r_2 s e^{-T_0/T}, & T<r_2 s \end{cases}$$

We note that formulas (22), (28) and (29) can be used for a direct determination of the matrix element M of the electron-phonon interaction in the lune from the experimental data.

4. REGION OF INTERMEDIATE MAGNETIC FIELDS

As we have seen, the effective relaxation times determining the electrical conductivity in the region of strong magnetic fields ($\tau_{\text{eff}}^{\infty}$) and in zero magnetic field (τ_{eff}^{0}) can differ substantially. This fact is most strongly manifested in conditions of total overlap of lunes at temperatures $T > T_P$. In this case, the time $\tau_{\text{eff}}^{\infty}$ associated with Umklapp processes is small compared with the time τ_{eff}^{0} associated with diffusion across the Fermi surface:

$$\tau_{\text{eff}}^{\infty} \approx \tau_U' p_F/r_0, \quad \tau_{\text{eff}}^{0} \approx \tau_F \gg \tau_U \approx \tau_U (p_F/r_0)^2 \gg \tau_{\text{eff}}^{\infty}.$$

The region of strong magnetic fields ($\Omega\tau_{\text{eff}}^{\infty} \gg 1$) does not adjoin the weak-field region ($\Omega\tau_{\text{eff}}^{0} \ll 1$), and there arises at broad intermediate region: $(p_F/r_0)\tau_U' \ll \Omega^{-1} \ll \tau_F$.

In this section we shall not present the calculations but shall confine ourselves to formulating the principal qualitative results for the case when exact overlap of one pair of broad lunes occurs.

In the diagram in Fig. 6 the regions in which the conductivity behaves in qualitatively different ways are drawn schematically in the variables T, $\Omega^{-1} \propto H^{-1}$, and the dependences of σ_{xx} on H are indicated. It is assumed that the lune dimension $r_0 \approx (p_F q_T)^{1/2} \ll p_F$; however, factors of order $\ln(p_F/r_0)$ are not taken into account. The regions I and VI correspond to strong and weak magnetic fields respectively, and the regions II–V correspond to intermediate fields.

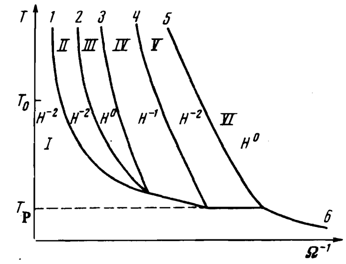


FIG. 6

The curves 1–6 in Fig. 6 correspond to the relationships

$$\Omega^{-1} \approx \tau_U' \frac{p_F}{r_0}, \left(\tau_F \tau_U' \frac{r_0}{p_F}\right)^{1/2}, \tau_F \left(\frac{r_0}{p_F}\right)^2, \tau_F \frac{r_0}{p_F}, \tau_F, \tau_U' \left(\frac{p_F}{r_0}\right)^2.$$

The conductivities in the regions I, . . . , VI respectively have the forms

$$\sigma_{xx}\frac{m}{ne^2} \approx \frac{1}{\Omega^2\tau_U'}\left(\frac{r_0}{p_F}\right)^2, \frac{1}{\Omega^2\tau_U'}\left(\frac{r_0}{p_F}\right)^2, \tau_F\left(\frac{r_0}{p_F}\right)^3, \frac{1}{\Omega}\frac{r_0}{p_F}, \frac{1}{\Omega^2\tau_F}, \tau_F+\tau_U'\left(\frac{p_F}{r_0}\right)^2. \tag{30}$$

Coefficients of order unity have been omitted.

We shall discuss the physical meaning of the results given. In the region I of strong magnetic fields the probability of an Umklapp process during the time of passage through the lune is low: $\Omega^{-1} \ll \tau_U' p_F/r_0$. As the magnetic field is decreased (to the right of curve 1) the electron has time to undergo an Umklapp process several times during the time of passage through the lune. Each time the electron hops from one equivalent lune to the other it changes the direction of its motion along the p_x-axis (the p_y-axis is directed along the reciprocal-lattice vector corresponding to the overlapping lunes). It is clear that such a process is a distinctive type of diffusion along the p_x-axis, with elementary step-length $\Delta \approx p_F \Omega \tau_U'$ and step-time τ_U'. An electron that starts its diffusion path at one edge of a lune (say, with $p_x > 0$) will, with high probability, emerge from the equivalent lune on the same side, i.e., will be found in the neighboring cell in the repeated-zone scheme. Thus, the electron moves along an effective open orbit, and this will continue until either there is some small probability of passing right through the lune (region II) or the electron moves away from the lune belt as a result of ordinary diffusion along the p_z-axis (region III).

The electrical conductivity of a layer of open orbits of width r_0 has the form

$$\sigma_{xx} \approx n\frac{e^2}{m}\frac{r_0}{p_F}\tau.$$

In region II the time τ is equal to the half-period $(2\Omega)^{-1}$ multiplied by the number of returns of the Brownian particle to its starting point during the time of the displacement over the distance r_0, i.e., multiplied by r_0/Δ: $\tau \approx r_0(p_F\Omega^2\tau_U')^{-1}$. (It is curious that, although the electrical-conduction mechanisms are essentially different in regions I and II, the final results are the same. However, this situation is connected with the model used: e.g., it can be shown that in the case of overlap of two pairs of equivalent lunes the dependence of the electrical conductivity on Ω in region II differs from that in region I.) In region III we have $\tau \approx \tau_F(r_0/p_F)^2$ and, as is characteristic for open orbits, the electrical conductivity does not depend on the magnetic field.

On further decrease of the magnetic field, in a half-period an electron has time to diffuse over a distance $\delta^* \sim p_F(\Omega\tau_F)^{-1/2} \gg r_0$ along p_z, and therefore an open orbit does not arise. (In regions IV and V, $r_0 \ll \delta^* \ll p_F$.) Nevertheless, for an estimate of the conductivity in region IV we can make use of the expression (31), taking $\tau \approx \Omega^{-1}$. We note that in this region the transverse resistivity $\rho_{yy} \propto H$. This result has been obtained, with slightly different assumptions, by Young[7].

In region V the situation becomes more complicated. We note that the diffusional displacement $\delta(t) \approx p_F(t/\tau_F)^{1/2}$, and, therefore, for sufficiently short times t the rate of displacement of the electron ($\dot{\delta} \propto t^{-1/2}$) exceeds the rate $p_F\Omega$ of its motion along the orbit in the magnetic field. The corresponding critical distance $\delta_0 \approx p_F(\Omega\tau_F)^{-1}$ ($\delta_0 \equiv \delta(t_0)$; $\dot{\delta}(t_0) = p_F\Omega$), if it is greater than r_0, plays the role of the size of the lune. Precisely this situation arises in region V; the result $\sigma_{xx} \propto (\Omega^2\tau_F)^{-1}$ follows from the corresponding expression for region IV after r_0 is replaced by δ_0.

[1)]Strictly speaking, normal collisions within a lune can be treated in the diffusion approximation under the condition that $r_0 \gg \Delta p$, where Δp is the minimum distance between the isolated Fermi surfaces [5]. However, as a detailed analysis shows, this restriction is unimportant, since for $r_0 \lesssim \Delta p$ the variation of the distribution function within the lune can be neglected.

[2)]In the general case of a multiply-connected Fermi surface Eq. (6) must be replaced by a system of equations, in each of which the averaging is performed within one given electron or hole group. (In particular, the diffusion coefficients in these equations are different.) The quantities sought in this case will be the functions $\psi_m(p_z)$ (m is the label of the group) and the velocity u_c.

[3)]As shown in [5], the ratio T_P/T_0 falls as the parameter $p_F/\Delta p$ increases, but even in the case of Na and K, when $p_F/\Delta p \approx 3$, $T_P/T_0 \approx 1/10$.

[4)]The extent to which the Umklapp processes are nonlocal is characterized by the size of that region on the Fermi surface to which transitions from the given joint **p** are possible ($r^* \approx q_T + \Delta p$). For $r^* \ll r_0$ the nonlocal character leads to additional diffusion in the lune, with diffusion coefficient $D^* \approx (r^*)^2/\tau'_U v_F$. However, as one can easily convince oneself, in a strong magnetic field the ordinary diffusion associated with normal collisions plays a more important role. In the opposite limiting case $r^* \gg r_0$, the variation of the function χ within the lune can be neglected and therefore the nonlocal character of the Umklapp processes is unimportant.

[5)]The condition b_k/g_{ky} = const means that in being displaced to infinity along the p_y-axis (in the extended p-space) as a result of Umklapp processes and motion in the magnetic field, an electron is also inevitably displaced to infinity along the p_z-axis, i.e., goes outside the lune. But if the ratio b_k/g_{ky} is not the same for different lunes, then, as is not difficult to show, the electron can always move away to infinity along p_y while remaining within the limits of the lune layer (for $b_k \ll r_0$).

[1]I. M. Lifshitz, M. Ya. Azbel' and M. I. Kaganov, Elektronnaya teoriya metallov (Electron Theory of Metals), Nauka, M., 1971.

[2]M. I. Kaganov and V. G. Peschanskiĭ, in the Collection Issledovanie ênergeticheskogo spektra elektronov v metallakh (Investigation of the Energy Spectrum of Electrons in Metals), Naukova Dumka, Kiev, 1965.

[3]A. B. Pippard, Proc. Roy. Soc. A305, 291 (1968).

[4]R. N. Gurzhi and A. I. Kopeliovich, Zh. Eksp. Teor. Fiz. 61, 2514 (1971) [Sov. Phys.-JETP 34, 1345 (1972)].

[5]R. N. Gurzhi and A. I. Kopeliovich, Zh. Eksp. Teor. Fiz. 64, 380 (1973) [Sov. Phys.-JETP 37, 195 (1973)].

[6]R. N. Gurzhi and A. I. Kopeliovich, ZhETF Pis. Red. 19, 630 (1974) [JETP Lett. 19, 327 (1974)].

[7]R. A. Young, Phys. Rev. 175, 813 (1968).

Translated by P. J. Shepherd
244