Neutral weak interaction currents and the Josephson effect

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Parity-nonconserving electron-nucleon interactions might be detected through some effects in superconductors with polarized nuclei. In particular, the flux quantization condition and current flowing through a double Josephson junction are altered. Both effects depend on the relative orientation of the polarization direction of the nuclei and the magnetic field linked by the circuit. For heavy metals these effects can exceed $10^{-6}L$, where L is the length of the circuit in cm.

1. INTRODUCTION

The observation of weak electron-nucleon interactions would be of great interest for elementary particle physics. In particular, so-called neutral currents, which would lead to such effects, are predicted by a number of renormalizable models that describe in a unified manner the weak and electromagnetic interactions.

Since the cross sections of these processes increase with the energy, the most natural way of observing neutral currents seem to be experiments with high-energy particles. It was in such experiments that neutral currents involving neutrinos were observed [1,2]. The problem of detecting weak interaction effects in electron scattering is much more complicated, since at the presently accessible energies the weak interactions are indeed much weaker than the electromagnetic interactions.

In this connection there arises the question whether one could not observe the weak electron-nucleon interactions at energies of the order of atomic energies, making use of the high degree of accuracy of experiments in that region. It is natural to consider qualitative effects produced by parity-nonconservation in weak interactions. Possible manifestations of parity-nonconservation in atomic transitions have been discussed in [3-6].

In the present paper, a brief announcement of which has been made in [7], we discuss the possibility, in principle, of observing parity-nonconserving electron-nucleon interactions by means of the Josephson effect in a superconductor with polarized nuclei. The nuclei may be polarized by an external magnetic field, with the superconducting state setting in after the field is switched off. We assume that the relaxation time of the nuclear spins is sufficiently long so that the effects under discussion can be observed. The relaxation time increases as the temperature gets lower and may reach a value of several seconds (cf., e.g., [8,9]).

It should be noted that experiments on the observation of parity nonconservation in superconductors with oriented nuclei and in atomic transitions [5] are in some sense complementary to each other, since in the first case what is involved is the correlation between the electron momentum and the spin of the same electron, whereas in the second the correlation is between the electron spin and the nuclear spin. In relativistic language the first effect is described in the interaction Lagrangian by the product of the electronic vector current with the nucleonic axial-vector current, whereas the second effect is described by the product of the electronic axial-vector current with the nucleonic vector current.

2. THE BASIC EQUATIONS

The Hamiltonian describing an odd-P contact interaction between two fermions to first order in v/c can be written in the form:

$$\mathscr{H}=\frac{G\hbar^3}{\sqrt{2}c^2}\frac{1}{2m}[(\beta_1\sigma_1+\beta_2\sigma_2)\{\mathbf{p},\delta(\mathbf{r})\}_+ + i\beta_3[\sigma_1\times\sigma_2]\{\mathbf{p},\delta(\mathbf{r})\}_-]. \quad (1)$$

In this equation $G = 10^{-5}m_p^{-2}$ is the weak coupling constant, m is the reduced mass of the fermions, $\mathbf{r}$ and $\mathbf{p}$ are their relative coordinate and momentum, $\sigma_{1,2}$ are the fermion spin matrices, β_1, β_2, β_3 are dimensionless parameters, and the subscripts plus and minus denote respectively the anticommutator and commutator.

In the case of electron-nucleon interaction which interests us we shall consider σ_2 to be the electron spin and σ_1 to be the nucleon spin. We neglect terms of the order m_e/m_p. If, for example, the relativistic interaction Lagrangian is represented as a product of currents having the V − A structure and characterized by the weak coupling constant G, then $\beta_1 = -\beta_2 = -\beta_3 = -1$. In the currently popular Weinberg model [10] the constants β_i are

$$\beta_{1p}=-\beta_{3p}=-{}^1\!/_2 g_A(1-4\sin^2\theta),\quad \beta_{2p}={}^1\!/_2(1-4\sin^2\theta),$$
$$\beta_{1n}=-\beta_{3n}={}^1\!/_2 g_A(1-4\sin^2\theta),\quad \beta_{2n}=-{}^1\!/_2. \quad (2)$$

The subscripts p and n denote the constants that characterize the interaction of the electron with the proton and neutron, $g_A = 1.2$ is the axial-vector constant for beta decay and θ is a mixing angle which is the parameter of the model (it follows from the experimental data that $\sin^2\theta \approx 0.35$ [1]).

Consider a superconductor with polarized nuclei. Since the electrons are not polarized, only the term proportional to β_1 is essential. We assume that during the experiment the spin states of the nuclei do not change. Then the Hamiltonian for the odd-P electron-nucleus interaction can be represented in the form

$$\mathscr{H}=\frac{G\hbar^3}{\sqrt{2}c^2}\frac{\beta}{2m}\sum_i \xi_i\{\mathbf{p},\delta(\mathbf{r}-\mathbf{r}_i)\}_+, \quad (3)$$

where ξ_i is a unit vector in the spin direction of the i-th nucleus, $\mathbf{r}_i$ is the coordinate of that nucleus and the constant β has the following expression in terms of the constants β_{1p} and β_{1n}:

$$\beta=\left|\left\langle \beta_{1p}\sum_p \sigma_p+\beta_{1n}\sum_n \sigma_n\right\rangle\right|. \quad (4)$$

In this equation the averaging is over the nucleon states in the nucleus.

By averaging the Hamiltonian (3) over the wave functions of the electrons of a Cooper pair we obtain the following expression for the odd-P addition to the effective Hamiltonian describing the motion of the pair as a whole:

$$\mathcal{H}_{\text{eff}} = \frac{G\hbar^3 n\beta K}{\sqrt{2}\,c^2}\frac{1}{2m_e}\{\mathbf{p},\boldsymbol{\zeta}(\mathbf{r})\}_+. \quad (5)$$

Here m_e is the effective mass of the electron, n is the density of nuclei, $\boldsymbol{\zeta}(\mathbf{r})$ is the polarization vector of the nuclei (its absolute value is the degree of polarization), and the factor K takes into account the difference between the electron current in the vicinity of the nucleus and the average current in the crystal:

$$K = \frac{|\operatorname{Im}\psi_{\mathbf{k}}^*\nabla\psi_{\mathbf{k}}|_{r=r_{\text{nuc}}}}{|\operatorname{Im}\psi_{\mathbf{k}}^*\nabla\psi_{\mathbf{k}}|_{\text{av}}}. \quad (6)$$

In this equation $\psi_{\mathbf{k}}(\mathbf{r})$ is the electron wave function in the crystal, the quasimomentum $\mathbf{k}$ being regarded as small. In Eqs. (5) and (6) we neglect the anisotropy of the crystal.

The interaction (5) is of a form similar to the electromagnetic interaction, so that taking it into account can be easily verified to be equivalent (to first order in G) to the substitution

$$\frac{2e}{c}\mathbf{A} \to \frac{2e}{c}\mathbf{A} - \frac{2G\hbar^3 n\beta K}{\sqrt{2}\,c^2}\boldsymbol{\zeta}. \quad (7)$$

Let us analyze the changes brought about by this substitution in the electrodynamics of the superconductors. We restrict our attention to superconductors of the London type, although it seems that this restriction is not essential for the conclusions to be reached below.

The Maxwell equations for a static magnetic field now take the form

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} = \frac{4\pi e\rho}{m_e c}\left[\hbar\nabla\varphi - \frac{2e}{c}\mathbf{A} + \frac{2G\hbar^3 n\beta K}{\sqrt{2}\,c^2}\boldsymbol{\zeta}\right]. \quad (8)$$

Here ρ is the density of Cooper pairs and φ is the phase of their wave function. Taking the curl of Eq. (8) we obtain

$$\left(-\Delta + \frac{8\pi e^2\rho}{m_e c^2}\right)\mathbf{H} = \frac{8\pi e\rho G\hbar^3 n\beta K}{\sqrt{2}\,m_e c^3}\operatorname{rot}\boldsymbol{\zeta}. \quad (9)$$

Thus, in the presence of polarized nuclei neither the magnetic field nor the current vanishes, in general, deep inside the superconductor. However, with the usual method of polarization by means of an external magnetic field $\mathbf{H}_0(\mathbf{r})$, the polarization vector $\boldsymbol{\zeta}(\mathbf{r})$ is obviously proportional to $\mathbf{H}_0(\mathbf{r})$, so that $\operatorname{curl}\boldsymbol{\zeta}(\mathbf{r}) = 0$, and neither the magnetic field, nor the current penetrate into the interior of the superconductor. In the sequel we restrict our attention to just this case, for the sake of simplicity.

3. PHYSICAL EFFECTS

What is the influence of the interaction (5) on the physical effects in superconductors? From the analogy between $\mathbf{A}$ and $\boldsymbol{\zeta}$ (cf. (7)) it is clear that the quantization condition for the magnetic flux Φ linked by a superconducting ring changes. It will now take the form

$$\frac{2e}{\hbar c}\Phi - \frac{2G\hbar^2 n\beta K}{\sqrt{2}\,c^2}\oint d\mathbf{r}\,\boldsymbol{\zeta}(\mathbf{r}) = 2\pi m \quad (m=0,1,2,\ldots). \quad (10)$$

We call attention to the rather deep analogy between the vector potential $\mathbf{A}$ and the polarization $\boldsymbol{\zeta}$. In particular, the relation (10) differs from the usual quantization condition for the magnetic flux by the additional flux of the "quasimagnetic" field proportional to $\operatorname{curl}\zeta$ (in the situation considered by us $\operatorname{curl}\zeta \neq 0$ at the boundary of the superconductor).

On the other hand, in spite of the presence of currents and magnetic fields near the boundary of the superconductor, altering the orientation of the nuclei in that region, for the derivation of Eq. (10) it suffices that there exist a closed loop for all points of which $\mathbf{j} = 0$, $\mathbf{H} = 0$, $\operatorname{curl}\boldsymbol{\zeta} = 0$.

As another example we consider the flow of current through two Josephson junctions connected in parallel. As is well known [11], the expression for the maximal current is of the form (for simplicity we restrict our attention to identical junctions)

$$I_{max} = 2I_c\left|\cos\frac{e\Phi}{\hbar c}\right|, \quad (11)$$

where I_c is the maximal current through one junction and Φ is the magnetic flux linked by the circuit. Allowance for the odd-P interaction (5) leads, like in the case of flux quantization, to the following modification of Eq. (11):

$$I_{max} = 2I_c\left|\cos\left(\frac{e\Phi}{\hbar c} - \frac{G\hbar^2 n\beta K}{\sqrt{2}\,c^2}\oint d\mathbf{r}\,\boldsymbol{\zeta}(\mathbf{r})\right)\right|. \quad (12)$$

Thus, the magnitude of I_{max} changes when the sign of Φ is changed, i.e., when the relative orientation of the circuit and the external magnetic field is reversed. Owing to this we can make the effect periodically time-dependent by rotating the installation, since the orientation of the nuclei in space is conserved.

4. QUANTITATIVE ESTIMATES

Let us now estimate the magnitude of the effect. The contribution of the weak interactions is determined by the dimensionless parameter γ:

$$\gamma = \frac{G\hbar^2 n\beta K}{\sqrt{2}\,c^2}\oint d\mathbf{r}\,\boldsymbol{\zeta}(\mathbf{r}) \sim \frac{G\hbar^2 n\beta K|\boldsymbol{\zeta}|L}{\sqrt{2}\,c^2}, \quad (13)$$

where L is the length of the circuit. It is assumed that the nuclei are polarized along the circuit.

The nontrivial part of the estimate is that of the factor K defined in Eq. (6). The electron wave function $\psi_{\mathbf{k}}(\mathbf{r})$ can be found by means of the Wigner-Seitz method [12]. To first order in $\mathbf{k}$ it is of the form

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}}\left\{u_0 + \frac{1}{m}\sum_{\alpha\neq 0}\frac{\langle\alpha|\mathbf{k}\mathbf{p}|0\rangle}{E_0 - E_\alpha}u_\alpha\right\}, \quad (14)$$

where $\mathbf{p}$ is the momentum operator, and u_α is the wave function of the lowest state of the zone α. The functions u_α satisfy the condition that their normal derivative vanish at the boundary of the unit cell. In calculations the potential is chosen the same as for the atomic problem, so that the difference is only in the boundary conditions. One can neglect this difference in the estimate of the factor K.

As can be seen from Eq. (14) and the definition (6) of K, in the small-k approximation this factor differs from zero only in the case of the s and p conduction bands (i.e., when u_0 describes an s or a p state). For heavy atoms the wave functions of the external electrons are all of the order $Z^{1/2}$ at distances of the order Z^{-1} from the nucleus (in atomic units) [13], so that $u_{s|r\to 0} \sim Z^{1/2}$ and $u_{p|r\to 0} \sim Z^{3/2}r$ (the subscripts s and p refer to the appropriate states). As a result we obtain for K the estimate $K \sim Z^2$.

In heavy atoms there appears an additional enhancement of the effect, due to the fact that the motion of the electrons near the nucleus is relativistic [5]. In our case the enhancement factor κ equals

$$\varkappa=\frac{4(2r_0Z/a_0)^{2(\nu-1)}}{\Gamma^2(2\nu+1)}\frac{2\nu+1}{3}, \qquad (15)$$

where r_0 is the nuclear radius, a_0 is the Bohr radius, $\nu = (1 - Z^2\alpha^2)^{1/2}$. The quantity κ increases with Z and attains a value of eight for lead.

Thus, for heavy metals the parameter γ, at $\beta \sim 1$ and $|\zeta| \sim 1$ may exceed 10^{-6}L/1 cm.

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1