Generation of harmonics during scattering of a high-power electromagnetic wave by fluctuations of surface oscillations

Yu. M. Aliev, S. Vukovic,[1)] O. M. Gradov, and A. Yu. Kiriĭ

P. N. Lebedev Physics Institute, USSR Academy of Sciences, Moscow
(Submitted June 4, 1974)
Zh. Eksp. Teor. Fiz. **68**, 85–94 (January 1975)

The spectral correlation function is obtained for the electromagnetic field of surface oscillations of a stable plasma subjected to the field of a high-power electromagnetic wave. This is done without assuming that this field is weak. The presence of a strong field alters considerably the correlation spectrum and gives rise to harmonics of the external wave frequency, which are displaced by amounts equal to the natural frequencies of the surface oscillations. The fields of the low-order harmonics are localized near the plasma boundary. Spectral fluxes of the radiation emitted by the plasma are found for fields corresponding to the fairly high harmonics of the external wave frequency.

1. A theory of the dispersion properties of an inhomogeneous plasma subjected to the field of a strong high-frequency electromagnetic wave predicts changes in the surface oscillation spectra due to the oscillatory motion of electrons relative to ions under the action of this field.[1-3] Such changes in the surface oscillation spectra should have a considerable influence on the correlation function of the electromagnetic field fluctuations, which can be determined directly by experiment. A theory of quasistationary fluctuations of a stable plasma has been developed only for a homogeneous and unbounded plasma.[4,5] We shall obtain the correlation functions of the surface wave fluctuations in a plasma subjected to a strong electromagnetic wave of frequency ω_0, which is considerably higher than the plasma frequency. We shall show that the presence of this strong wave gives rise to an anisotropy in the correlation function of the fields of high-frequency surface waves so that, for example, the spectral correlation function may vanish for certain directions of propagation of surface oscillations. We shall obtain an expression for the correlation function of low-frequency surface waves whose spectrum is governed entirely by the field of the external wave. We shall find the spectral flux of the radiation emitted by the plasma as a result of noncoherent scattering of a strong electromagnetic wave by quasistatic surface oscillations whose wavelength $1/k_{\|}$ is considerably shorter than the external wavelength ω_0/c. It is precisely in this case that the correlation functions and the spectral flux of the radiation emitted by a plasma are strongly nonlinear functions of the external wave amplitude. We shall show that the interaction between a linearly polarized high-frequency electromagnetic wave end surface fluctuations gives rise to harmonics of the external field frequency ω_0, which appear in the scattered radiation spectrum and are shifted by the surface oscillation frequency $n\omega_0+\omega$. In the case of relatively low-order harmonics, $n<(\omega_0/k_{\|}c)^{-1}$, the scattered radiation is localized near the boundary of a plasma, whereas high-order harmonics $n>(\omega_0/k_{\|}c)^{-1}$ are emitted from the plasma. The spectral flux of the scattered radiation is then governed by the values of the pair correlation function of the surface wave field at the boundary of the plasma.

2. We shall consider a semi-infinite plasma with a transition layer $0<z<a$, where the density of charged particles $n_{0\alpha}(z)$ of type α varies from zero at $z=0$ to $n_{0\alpha}(a)$, and is constant in the range $z>a$. We shall assume that the field of an external electromagnetic wave

$$\mathbf{E}_0(\mathbf{r},t)=\mathbf{E}_0\sin(\omega_0 t-k_0 z)\tag{2.1}$$

is oriented along the plasma boundary and that the frequency of this field is considerably higher than the maximum Langmuir frequency of electrons $\omega_{Le}(a)=(4\pi e^2 n_{0e}(a)/m_e)^{1/2}$, so that $k_0\approx\omega_0/c$.

We shall be interested in fluctuations of plasma surface waves in the field (2.1) and we shall calculate the correlation function of the electric field of these waves:

$$G_{ij}(\mathbf{r},\mathbf{r}',t,t')=\langle E_i(\mathbf{r},t)E_j(\mathbf{r}',t')\rangle.\tag{2.2}$$

Here, the angular brackets represent averaging over an ensemble. We shall consider steady-state oscillations of a stable plasma at times t and t′ shorter than the characteristic time of the high-frequency heating of the plasma by the field (2.1) and we shall use the invariance of the function $G_{ij}(\mathbf{r},\mathbf{r}',t,t')$ of Eq. (2.2) relative to a simultaneous shift of t and t′ by any integral number of periods of the field (2.1), expressing the correlation function in the form

$$G_{ij}(\mathbf{r},\mathbf{r}',t,t')=\sum_{n,m=-\infty}^{\infty}e^{-in\omega_0 t+im\omega_0 t'}G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',t-t').\tag{2.3}$$

Equations (2.2) and (2.3) yield the following relationship between the functions $G_{ij}^{(n,m)}$ and the amplitudes $E_i^{(n)}$ of the expansion of the field $\mathbf{E}(\mathbf{r},t)$ in harmonics of the frequency ω_0 of the wave (2.1):

$$G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',t-t')=\langle E_i^{(n)}(\mathbf{r},t)E_j^{(m)*}(\mathbf{r}',t')\rangle;$$
$$\mathbf{E}(\mathbf{r},t)=\sum_{n=-\infty}^{\infty}e^{-in\omega_0 t}\mathbf{E}^{(n)}(\mathbf{r},t).\tag{2.4}$$

We shall calculate the correlation functions $G_{ij}^{(n,m)}$ for a nonequilibrium (but stable) plasma with inhomogeneous distributions of the particle temperatures and we shall do this using the method of phase microdistributions,[6] in which the calculation of the correlation functions reduces to the finding of the solution of the initial problem for a microfield $\mathbf{E}^{(n)}(\mathbf{r},t)$ with specified, at the moment $t=0$, distributions of the fluctuations $\delta N_\alpha(\mathbf{r},\mathbf{p},t)=N_\alpha(\mathbf{r},\mathbf{p},t)-f_\alpha(\mathbf{r},\mathbf{p},t)$ of the phase microdistributions $N_\alpha(\mathbf{r},\mathbf{p},t)$ relative to the average value $f_\alpha(\mathbf{r},\mathbf{p},t)\equiv\langle N_\alpha\rangle$, which is a single-particle distribution function of particles of type α. The function (2.4) can then be expressed in terms of the Laplace transform of the solution of the initial problem

$$\mathbf{E}^{(n)}(\mathbf{r},\omega)=\int_0^\infty dt\,e^{i\omega t}\mathbf{E}^{(n)}(\mathbf{r},t),\quad \operatorname{Im}\omega>0\tag{2.5}$$

in the following way:

$$G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',\omega)=\lim_{\Delta\to+0}2\Delta\langle E_i^{(n)}(\mathbf{r},\omega+i\Delta)E_j^{(m)*}(\mathbf{r}',\omega+i\Delta)\rangle,$$

$$G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',t-t')=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',\omega). \qquad (2.6)$$

We shall be interested in the case when the thickness of the transition layer is considerably greater than the Debye electron radius $r_{De}=(T_e/4\pi e^2 n_{0e}(a))^{1/2}$ and when the spatial dispersion and damping of surface waves are entirely due to the inhomogeneity of the plasma in the transition region and to the high-frequency field.[2,3] Surface waves can then be described by the hydrodynamic approximation for a cold plasma. Consequently, we shall replace the equations for the fluctuations of phase microdistributions $\delta N_\alpha(\mathbf{r},\mathbf{p},t)$ with the equations for the first moments of δN_α, i.e., of the microdensity $\delta N_\alpha(\mathbf{r},t)$ and microvelocity $\delta\mathbf{V}_\alpha(\mathbf{r},t)$:

$$\delta N_\alpha(\mathbf{r},t)=\int d\mathbf{p}\,\delta N_\alpha(\mathbf{r},\mathbf{p},t),$$
$$n_{0\alpha}(\mathbf{r},t)\delta\mathbf{V}_\alpha(\mathbf{r},t)=\int d\mathbf{p}(\mathbf{v}-\mathbf{v}_{E,\alpha}(\mathbf{r},t))\delta N_\alpha(\mathbf{r},\mathbf{p},t),$$
$$n_{0\alpha}(\mathbf{r},t)=\int d\mathbf{p}\,f_\alpha(\mathbf{r},\mathbf{p},t),$$
$$n_{0\alpha}(\mathbf{r},t)\mathbf{v}_{E,\alpha}(\mathbf{r},t)=\int d\mathbf{p}\,\mathbf{v}f_\alpha(\mathbf{r},\mathbf{p},t), \qquad (2.7)$$

which—in the case of a plasma homogeneous along the x and y axes and for negligible thermal pressure and pair correlation effects—are of the form

$$\frac{\partial}{\partial t}\delta N_\alpha(\mathbf{r},t)+\frac{\partial}{\partial\mathbf{r}}[n_{0\alpha}(\mathbf{r},t)\delta\mathbf{V}_\alpha(\mathbf{r},t)+\mathbf{v}_{E,\alpha}(\mathbf{r},t)\delta N_\alpha(\mathbf{r},t)]=0, \qquad (2.8)$$

$$\frac{\partial}{\partial t}\delta\mathbf{V}_\alpha(\mathbf{r},t)+\left(\mathbf{v}_{E,\alpha}(\mathbf{r},t)\frac{\partial}{\partial\mathbf{r}}\right)\delta\mathbf{V}_\alpha(\mathbf{r},t)+\left(\delta\mathbf{V}_\alpha(\mathbf{r},t)\frac{\partial}{\partial\mathbf{r}}\right)\mathbf{v}_{E,\alpha}(\mathbf{r},t)$$
$$=\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)+\frac{e_\alpha}{m_\alpha c}[\mathbf{v}_{E,\alpha}(\mathbf{r},t)\mathbf{B}]+\frac{e_\alpha}{m_\alpha c}[\delta\mathbf{V}_\alpha(\mathbf{r},t)\mathbf{B}_0]. \qquad (2.9)$$

We shall ignore small corrections to the distribution function $f_\alpha(\mathbf{r},\mathbf{p},t)$ and the terms in Eq. (2.8) proportional to powers of the small parameter $v_{E,\alpha}/c$, which is the ratio of the amplitude of oscillations of particles of type α in the field of the external wave (2.1) ($\mathbf{v}_{E,\alpha}=e_\alpha E_0/m_\alpha\omega_0$) to the velocity of light. The distributions functions can then be described by the expression

$$f_\alpha(\mathbf{r},\mathbf{p},t)=F_\alpha[z,(\mathbf{p}-m_\alpha\mathbf{v}_{E,\alpha}(z,t))_i^2],$$
$$\mathbf{v}_{E,\alpha}(z,t)=-\frac{e_\alpha\mathbf{E}_0}{m_\alpha\omega_0}\cos(\omega_0 t-k_0 z), \qquad (2.10)$$

corresponding to the adiabatic application of the external field at $t=-\infty$, when the particle distribution function is of the form

$$f_\alpha(\mathbf{r},\mathbf{p},t=-\infty)=F_\alpha[z,p_i^2].$$

We shall solve the initial problem of the perturbations of the electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields using the Maxwell equations together with Eqs. (2.8) and (2.9). Since the fields $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ are governed by the initial values of δN_α and $\delta\mathbf{V}_\alpha$, the correlation functions can be expressed in terms of the microvelocity and microdensity correlators, which—to within unimportant terms associated with the pair correlation between the particles—have the following form obtained using Eq. (2.10) (compare with[9]):

$$\langle\delta N_\alpha(\mathbf{r},t)\delta N_\beta(\mathbf{r}',t)\rangle=\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')n_{0\alpha}(\mathbf{r}),$$
$$\langle\delta N_\alpha(\mathbf{r},t)\delta\mathbf{V}_\beta(\mathbf{r}',t)\rangle=0,$$
$$\langle\delta V_{\alpha,i}(\mathbf{r},t)\delta V_{\beta,j}(\mathbf{r}',t)\rangle=\delta_{\alpha\beta}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\frac{T_\alpha^{(i)}(\mathbf{r})}{m_\alpha n_{0\alpha}(\mathbf{r})}. \qquad (2.11)$$

Here, the temperature $T_\alpha^{(i)}(\mathbf{r})$ is given by the formula

$$n_{0\alpha}(\mathbf{r})T_\alpha^{(i)}(\mathbf{r})=m_\alpha\int d\mathbf{p}(\mathbf{v}-\mathbf{v}_{E,\alpha}(z,t))_i^2 f_\alpha(\mathbf{r},\mathbf{p},t).$$

The homogeneity of the plasma in planes parallel to the boundary allows us to use the Fourier transform of the correlation function $G_{ij}^{(n,m)}$ with variables x and y:

$$G_{ij}^{(n,m)}(\mathbf{r},\mathbf{r}',\omega)=\int\frac{d\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel(\mathbf{r}-\mathbf{r}')}G_{ij}^{(n,m)}(z,z',\omega,\mathbf{k}_\parallel), \qquad (2.12)$$

and then Eq. (2.6) becomes

$$(2\pi)^2\delta(\mathbf{k}_\parallel-\mathbf{k}_\parallel')G_{ij}^{(n,m)}(z,z',\omega,\mathbf{k}_\parallel) \qquad (2.13)$$
$$=\lim_{\Delta\to+0}2\Delta\langle E_i^{(n)}(z,\omega+i\Delta,\mathbf{k}_\parallel)E_j^{(m)*}(z',\omega+i\Delta,\mathbf{k}_\parallel')\rangle;$$

$$\mathbf{E}^{(n)}(\mathbf{r},\omega)=\int\frac{d\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel\mathbf{r}}\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel). \qquad (2.14)$$

3. In this section, we shall find the solution of the system (2.8)—(2.9) together with the Maxwell equations. Since the frequency ω_0 of the external wave (2.1) is assumed to be considerably higher than the Langmuir frequency ω_{Le}, we shall consider fluctuations of frequencies $\omega\ll\omega_0$. Substituting into the Maxwell equations the expression for the current, obtained by solving the system (2.8)—(2.9),

$$\mathbf{j}(\mathbf{r},t)=\sum_\alpha e_\alpha[n_{0\alpha}(\mathbf{r})\delta\mathbf{V}_\alpha(\mathbf{r},t)+\delta N_\alpha(\mathbf{r},t)\mathbf{v}_{E,\alpha}(z,t)]$$

and neglecting small corrections of the order of ω/ω_0, we find that the electric field amplitudes $\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel)$ are described by the following system of equations:

$$\frac{c^2}{\omega_n^2}[\nabla_z(\nabla_z\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel))-\Delta_z\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel)]-\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel)$$
$$-\delta_{n0}[\delta\varepsilon_i\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel)+\delta\varepsilon_e J_0\vec{\mathscr{E}}(z,\omega,\mathbf{k}_\parallel)]$$
$$+ie^{ink_0z}\frac{\mathbf{r}_E}{\mathbf{k}_\parallel\mathbf{r}_E}\frac{n\omega_0}{\omega_n}J_{-n}\nabla_z\delta\varepsilon_e\vec{\mathscr{E}}(z,\omega,\mathbf{k}_\parallel)=e^{ink_0z}\mathbf{Q}^{(n)};$$
$$\omega_n=n\omega_0+\omega,\qquad \nabla_z=\mathbf{e}_z\frac{\partial}{\partial z}+i\mathbf{k}_\parallel,\qquad \Delta_z=\nabla_z^2. \qquad (3.1)$$

Here, $\mathbf{r}_E=\mathbf{v}_{E,e}/\omega_0$ is the amplitude of electron oscillations in the external field (2.1), the Bessel function J_n depends on $\mathbf{k}\parallel\mathbf{r}_E$, and $\delta\epsilon_\alpha=-\omega_{L\alpha}^2(z)/\omega^2$ is the partial permittivity. The vector function $\mathscr{E}(z,\omega,\mathbf{k}_\parallel)$ is governed by a linear combination of the harmonics of the fluctuating field:

$$\vec{\mathscr{E}}(z,\omega,\mathbf{k}_\parallel)=\sum_{n=-\infty}^{\infty}J_{-n}\mathbf{E}^{(n)}(z,\omega,\mathbf{k}_\parallel)e^{-ink_0z}, \qquad (3.2)$$

and the inhomogeneity of (3.1) leads to the following expression for $\mathbf{Q}^{(n)}$ to within terms that do not contribute to the correlation functions (2.6) of the surface waves:

$$\mathbf{Q}^{(n)}=-\delta_{n0}\frac{4\pi e_i n_{0i}(z)}{\omega^2}\delta\mathbf{V}_i(t=0,z,\mathbf{k}_\parallel)-J_{-n}\frac{4\pi e}{\omega\omega_n}\Big\{n_{0e}(z)\delta\mathbf{V}_e(t=0,z,\mathbf{k}_\parallel)$$
$$-i\frac{\mathbf{r}_E}{\mathbf{k}_\parallel\mathbf{r}_E}\frac{n\omega_0}{\omega}\nabla_z n_{0e}(z)\delta\mathbf{V}_e(t=0,z,\mathbf{k}_\parallel)\Big\}. \qquad (3.3)$$

It should be noted that terms of the order of v_E/c, related to the Lorentz force in Eq. (2.9), may give rise to a parametric buildup of nonpotential oscillations.[7,8] However, we shall assume that such instabilities do not develop because of the finite thickness L of the plasma layer, which is assumed to be small compared with the characteristic growth distance of waves in such instabilities but much greater than the thickness a of the transition layer in the plasma. The last condition allows us to use the approximation of a semi-infinite plasma in an analysis of weakly damped surface oscillations which satisfy the condition $k_\parallel L\gg 1$.

We shall be interested in the range of relatively short wavelengths of surface oscillations ($k_{\|}c \gg \omega_0$), whose interaction with the strong high-frequency field results in very considerable changes in the frequency spectrum [3-5] and in the correlation function. The field harmonics $\mathbf{E}^{(n)}$ of relatively low orders $n \ll k_{\|}c/\omega_0$ are then quasistatic. Since the main contribution to the vector $\vec{\mathscr{E}}(z, \omega, \mathbf{k}_{\|})$ of Eq. (3.2) is made by these harmonics [allowance for harmonics of higher orders $n > (ck_{\|}/\omega_0)$ lead to small corrections of the order of $(\omega_0/k_{\|}c)$], the functions $\mathbf{E}^{(0)}$ and $\vec{\mathscr{E}}$ can be expressed in the form

$$\mathbf{E}^{(0)}=-\nabla_z\Phi^{(0)}, \quad \vec{\mathscr{E}}=-\nabla_z\Phi,$$

and the potentials $\Phi^{(0)}$ and Φ are described by the following system of equations obtained from Eq. (3.1):

$$k_{\|}^2[\Phi(1+\delta\varepsilon_e)+J_0\delta\varepsilon_i\Phi^{(0)}]-\frac{\partial}{\partial z}\left[(1+\delta\varepsilon_e)\frac{\partial\Phi}{\partial z}+J_0\delta\varepsilon_i\frac{\partial\Phi^{(0)}}{\partial z}\right]=\Omega_e,$$

$$k_{\|}^2[\Phi^{(0)}(1+\delta\varepsilon_i)+J_0\delta\varepsilon_e\Phi]-\frac{\partial}{\partial z}\left[(1+\delta\varepsilon_i)\frac{\partial\Phi^{(0)}}{\partial z}+J_0\delta\varepsilon_e\frac{\partial\Phi}{\partial z}\right]=\Omega_i. \quad (3.4)$$

Here, within terms which do not contribute to the correlation functions $G_{ij}^{(0,0)}$, the functions Ω_α are

$$\Omega_\alpha=\frac{4\pi e_\alpha}{\omega^2}\frac{\partial}{\partial z}n_{0\alpha}(z)\delta\mathbf{V}_{\alpha,z}(t=0, z, \mathbf{k}_{\|})$$
$$+J_0\frac{4\pi e_\beta}{\omega^2}\frac{\partial}{\partial z}n_{0\beta}(z)\delta\mathbf{V}_{\beta,z}(t=0, z, \mathbf{k}_{\|}), \quad \beta\neq\alpha. \quad (3.5)$$

We shall be interested only in the values of $\mathbf{E}^{(n)}$ related to fluctuations of the surface wave field and we shall ignore volume fluctuations of the field assuming that $\mathbf{Q}^{(n)} = 0$ when $n \neq 0$. Bearing in mind that the frequency ω_0 of the external wave exceeds considerably the electron Langmuir frequency ω_{Le}, we obtain from Eq. (3.1) the following system of equations for $\mathbf{E}^{(n)}$ when $n \neq 0$:

$$\frac{\partial^2E_{\|}^{(n)}}{\partial z^2}+k_n^2E_{\|}^{(n)}=-k_n^2A_{\|}^{(n)}, \quad E_z^{(n)}=\frac{ik_{\|}}{k_n^2}\frac{\partial E_{\|}^{(n)}}{\partial z}, \quad (3.6)$$

$$\frac{\partial^2E_{\perp}^{(n)}}{\partial z^2}+k_n^2E_{\perp}^{(n)}=-\frac{\omega_n^2}{c^2}A_{\perp}^{(n)},$$

$$\mathbf{A}^{(n)}\equiv-i\frac{\mathbf{r}_E}{\mathbf{k}_{\|}\mathbf{r}_E}e^{ink_0z}J_{-n}\nabla_z\delta\varepsilon_e\vec{\mathscr{E}}(z, \omega, \mathbf{k}_{\|}), \quad k_n^2=\frac{\omega_n^2}{c^2}-k_{\|}^2, \quad c^2k_n^2\gg\omega_{Le}^2(a).$$

Here, the components are $E_\lambda^{(n)} = \mathbf{e}_\lambda \cdot \mathbf{E}^{(n)}$, where $\mathbf{e}_{\|} = \mathbf{k}_{\|}/k_{\|}$, and $\mathbf{e}_{\perp} = [\mathbf{e}_{\|}\mathbf{e}_z]$. In solving the system (3.6), we shall use the boundary conditions corresponding to the investigated case of the scattering of an external wave by fluctuations of surface waves and we shall assume that the amplitudes of the waves traveling from $z = \pm\infty$ to the plasma boundary for $k_n^2 > 0$ and the amplitudes of the waves growing exponentially in the limit $z \to \pm\infty$ for $k_n^2 < 0$ all vanish. The solutions of the system (3.6) then become

$$E_\lambda^{(n)}=e^{ik_nz}\int_0^z dz'\,e^{-ik_nz'}\psi_\lambda^{(n)}(z')+e^{-ik_nz}\int_z^\infty dz'\,e^{ik_nz'}\psi_\lambda^{(n)}(z'), \quad (3.7)$$

$$\psi_{\|}^{(n)}(z)=(k_n/2k_{\|})e^{ink_0z}J_{-n}\nabla_z\delta\varepsilon_c\vec{\mathscr{E}}, \quad k_n^2>0,$$

$$\psi_{\perp}^{(n)}(z)=\frac{\omega_n^2}{2c^2k_{\|}k_n}\frac{\mathbf{e}_{\|}[\mathbf{e}_z\mathbf{r}_E]}{\mathbf{e}_{\|}\mathbf{r}_E}J_{-n}\nabla_z\delta\varepsilon_e\vec{\mathscr{E}}. \quad (3.8)$$

If $k_n^2 < 0$, the expressions (3.7) and (3.8) should be modified by replacing k_n with $i\kappa_n$, where $\kappa_n = (-k_n^2)^{1/2}$. Thus, it is sufficient to find expressions for Φ and $\Phi^{(n)}$ in order to calculate all the amplitudes $\mathbf{E}^{(n)}$.

As is known, [1-3] there are two branches of quasistatic surface waves in an inhomogeneous plasma subjected to a strong high-frequency electric field. One of these branches is associated with electrons and corresponds to frequencies ω close to $\omega_{Le}(a)/\sqrt{2}$, whereas the low-frequency branch is described by $\omega \approx \omega_{Li}(a) \times (1 - J_0^2)^{1/2}/\sqrt{2}$. In accordance with this situation, we shall solve first the system (3.4) on the assumption that $\omega \gg \omega_{Li}(a)$, ignoring terms of the order of m_e/m_i and allowing for the thinness of the transition layer a compared with the wavelengths of the high-frequency surface waves $1/k_{\|}$. Applying the solution method used in [9], we obtain from Eq. (3.4) the following expressions for $\Phi^{(0)}$ and Φ to within small terms of the order of $(k_{\|}a)^2$:

$$\Phi^{(0)}(z, \omega, \mathbf{k}_{\|})=J_0\Phi(z, \omega, \mathbf{k}_{\|}), \quad (3.9)$$

$$\Phi(z)=\Phi(0)h(z)-\int_0^z\frac{dz'}{\varepsilon(z', \omega)}\int_0^{z'}dz''\Omega_e(z''), \quad 0<z<a. \quad (3.10)$$

Here, the value of $\Phi(0)$ and the function $h(z)$ are of the form

$$\Phi(z=0, \omega, \mathbf{k}_{\|})=\frac{1}{D(\omega, \mathbf{k}_{\|})}\int_0^a\frac{dz}{\varepsilon(z\ \omega)}\int_0^z dz'\Omega_e(z'), \quad (3.11)$$

$$h(z)=1+k_{\|}\int_0^z\frac{dz'}{\varepsilon(z', \omega)}+\int_0^z\frac{dz'}{\varepsilon(z', \omega)}\int_0^{z'}dz''\,k_{\|}^2\varepsilon(z'', \omega),$$

and the condition that the dispersion function

$$D_{\mathrm{hf}}(\omega, \mathbf{k}_{\|})=1+\frac{1}{\varepsilon(a, \omega)}+k_{\|}\int_0^a\frac{dz}{\varepsilon(z, \omega)}+\frac{k_{\|}}{\varepsilon(a, \omega)}\int_0^a dz\,\varepsilon(z, \omega) \quad (3.12)$$

should vanish, to within nonpotential corrections of the order of $\omega^2/k_{\|}^2c^2$ and terms proportional to m_e/m_i, determines the spectrum of high-frequency quasistatic surface waves in a plasma subjected to a high-frequency electric field. [2,3] In this case, the quantity $\nabla_z\delta\epsilon_e\vec{\mathscr{E}}$, which determines—in accordance with Eqs. (3.7) and (3.8)—the amplitudes of the electric field harmonics, is given by the following expression obtained from the system (3.4):

$$\nabla_z\delta\varepsilon_e\vec{\mathscr{E}}=\frac{1}{I}[\Delta_z\Phi^{(0)}+\Omega_e]. \quad (3.13)$$

Consequently, in the low-frequency limit when $\omega \ll \omega_{Le}(a)$, the functions $\vec{\mathscr{E}}$ and $\Phi^{(0)}$ are related by the following expression obtained from Eq. (3.4)

$$\nabla_z\delta\varepsilon_e\vec{\mathscr{E}}=-\frac{1}{1-J_0^2}[J_0\Delta_z\Phi^{(0)}+\Omega_e+J_0\Omega_i], \quad (3.14)$$

and the expression for $\Phi^{(0)}$ and D_{Lf} can be obtained from Eqs. (3.10) and (3.12) by the following substitutions:

$$\Phi\to\Phi^{(0)},$$
$$\varepsilon(z, \omega)\to\varepsilon_i(z, \omega)\equiv1+(1-J_0^2)\delta\varepsilon_i(z, \omega),$$
$$\Omega_e\to(1-J_0^2)\frac{4\pi e_i}{\omega^2}\frac{\partial}{\partial z}n_{0i}(z)\delta\mathbf{V}_{i,z}(t=0, z, k_{\|}). \quad (3.15)$$

The expressions obtained for the amplitudes $\mathbf{E}^{(n)}$ allow us to calculate the correlation functions $G_{ij}^{(n,m)}(z, z', \omega, \mathbf{k}_{\|})$ and the energy fluxes of the radiation of frequencies $n\omega_0 + \omega$ emerging from the plasma.

4. We shall first calculate the correlation functions $G_{ij}^{(n,m)}$ for harmonics $n, m \ll ck_{\|}/\omega_0$ when the oscillations of frequencies $n\omega_0 + \omega$ are quasistatic. The tensor correlation functions can then be expressed in terms of the derivaties of the scalar functions

$$G_{ij}^{(n,m)}(\mathbf{r}, \mathbf{r}', \omega)=\frac{\partial^2}{\partial r_i\partial r_j}\int\frac{d\mathbf{k}_{\|}}{(2\pi)^2}e^{i\mathbf{k}_{\|}(\mathbf{r}-\mathbf{r}')}G^{(n,m)}(z, z', \omega, \mathbf{k}_{\|})$$

and—as can be shown using Eqs. (3.7) and (3.10)—the functions $G^{(n,m)}(z, z')$ are expressed in terms of the values of these quantities corresponding to $z = z' = 0$, in the same way as in the absence of an external field. [9]

For example, outside the plasma ($z, z' < 0$), we find from Eq. (3.7) that

$$G^{(n,m)}(z, z', \omega, \mathbf{k}_\parallel) = e^{k_\parallel (z+z')} G^{(n,m)}(\omega, \mathbf{k}_\parallel).$$

The correlation functions at the plasma boundary $G^{(n,m)}(\omega, \mathbf{k}_\parallel) \equiv G^{(n,m)}(z = z' = 0, \omega, \mathbf{k}_\parallel)$, obtained from Eqs. (3.6), (2.9), and (3.10), are of the form

$$G^{(n,m)}(\omega, \mathbf{k}_\parallel) = \eta^{(n,m)} G^{(0,0)}(\omega, \mathbf{k}_\parallel), \tag{4.1}$$

where the high- and low-frequency values of the quantities $\eta^{(n,m)}$ are given by

$$\eta_{\text{hf}}^{(n,m)} = \frac{J_{-n}J_{-m}}{J_0^2}, \quad \eta_{\text{lf}}^{(n,m)} = \frac{[\delta_{n0} - J_0 J_{-n}][\delta_{m0} - J_0 J_{-m}]}{(1-J_0^2)^2}. \tag{4.2}$$

The correlation functions $G^{(0,0)}$ of the surface wave potential are obtained, to within corrections of the order of m_e/m_i, from Eqs. (2.11) and (3.9)—(3.15):

$$G_{\text{hf}}^{(0,0)}(\omega, \mathbf{k}_\parallel) = \frac{4\pi J_0^2}{|\omega||D_{\text{hf}}(\omega + i\Delta, \mathbf{k}_\parallel)|^2} \int_0^a dz\, \delta(\varepsilon(z, \omega))\, T_e^{(z)}(z); \tag{4.3}$$

$$G_{\text{lf}}^{(0,0)}(\omega, \mathbf{k}_\parallel) = \frac{4\pi(1-J_0^2)}{|\omega||D_{\text{lf}}(\omega + i\Delta, \mathbf{k}_\parallel)|^2} \int_0^a dz\, \delta(\varepsilon_i(z, \omega))\, T_i^{(z)}(z). \tag{4.4}$$

The expression (4.3), describing fluctuations of the electron surface oscillations, reduces to that obtained in [9] for $E_0 = 0$. It is clear from Eq. (4.3) that the application of a high-power external field to a plasma gives rise to an anisotropy of the correlation function of the high-frequency surface waves, in spite of the fact that the frequency of these waves changes by a value of the order of m_e/m_i. The relationship (4.4) describes the correlation of the potential of the low-frequency surface waves, whose frequencies are close to $\omega_{Li}(a)(1 - J_0^2)^{1/2}/\sqrt{2}$. Such oscillations exist only in a sufficiently strong external field when its pressure exceeds the thermal pressure, i.e., when $r_E \gg r_{De}$. It should be noted that, in the short-wavelength limit $|\mathbf{k}_\parallel \mathbf{r}_E| \to \infty$ the low-frequency fluctuation correlation (3.10) is identical, to within the mass ratio m_c/m_i, with the high-frequency surface oscillation correlator (4.3) in the absence of the high-frequency field if the electron parameters are replaced with the ion parameters. The spectrum of the high-frequency surface fluctuations is then characterized by the frequency $\omega_{Li}(a)/\sqrt{2}$, which corresponds to independent oscillations of ions against a homogeneous background of rapidly oscillating electrons. In the absence of the high-frequency field, a similar situation occurs in the case of volume short-wavelength oscillations in a plasma when there is no screening of the ion density perturbations by electrons.

When the harmonic number n is comparable with $ck_\parallel/\omega_0$, the quasistatic approximation ($k_n \approx ik_\parallel$) becomes invalid. In this case, the correlators $G_{ij}^{(n,m)}$ are expressed in accordance with Eqs. (3.6), (3.7), (3.12), and (3.13) in terms of integrals of the function $G^{(0,0)}(z, z', \omega, k_\parallel)$, whose coordinates are z and z'. We shall not calculate the value of $D^{(n,m)}$ for arbitrary numbers n and m but simply give an expression for the spectral density of the radiation energy $\mathbf{S}^{(n)}(\omega, k_\parallel)$ emitted from a plasma at frequencies $n\omega_0 + \omega$ by harmonics whose numbers are $n > ck_\parallel/\omega_0$:

$$S^{(n)}(\omega, \mathbf{k}_\parallel) = \frac{c}{4\pi} \mathbf{e}^{(n)}(\omega, \mathbf{k}_\parallel)\, g^{(n)}(\omega, \mathbf{k}_\parallel). \tag{4.5}$$

Here, the unit vector

$$\mathbf{e}^{(n)}(\omega, \mathbf{k}_\parallel) = \frac{c}{\omega_n}[\mathbf{k}_\parallel - \mathbf{e}_z k_n]$$

is related to the direction of propagation of radiation and the function

$$g^{(n)}(\omega, \mathbf{k}_\parallel) = \sum_i G_{ii}^{(n,n)}(z=0, z'=0, \omega, \mathbf{k}_\parallel),$$

governing the average value of the square of the electric field of the emitted n-th harmonic follows from Eqs. (3.7), (3.8), (3.13), and (3.14)

$$g^{(n)}(\omega, \mathbf{k}_\parallel) = J_n^2 \frac{\theta\omega_n^2}{4c^2 k_\parallel^2} \left[1 + \frac{\omega_n^2}{c^2 k_n^2} \operatorname{tg}^2\varphi\right] \int_0^\infty dz \int_0^\infty dz'\, e^{i(z-z')(k_n + nk_0)} \Delta_z \Delta_z \times G^{(0,0)}(z, z', \omega, \mathbf{k}_\parallel). \tag{4.6}$$

Here, φ is the angle between the vectors $\mathbf{k}_\parallel$ and $\mathbf{E}_0$, and the high- and low-frequency values of the quantity θ are

$$\theta_{\text{hf}} = \frac{1}{J_0^2}, \quad \theta_{\text{ef}} = \frac{J_0^2}{(1-J_0^2)^2}, \tag{4.7}$$

where the function $G^{(0,0)}$ is given by Eqs. (4.3) and (4.4). The first term in the brackets of Eq. (4.6) represents the contribution of the emitted linearly polarized H waves, whereas the second term describes the contribution of the emitted E waves.

We shall integrate Eq. (4.6) using the expressions (3.9) which govern the dependence of the function $G^{(0,0)}$ on the coordinates z and z', and we shall also employ the smallness of the parameter $k_\parallel a$. The function $G^{(n)}$ then becomes

$$g^{(n)}(\omega, \mathbf{k}_\parallel) = J_n^2 \frac{\theta\omega_n^2 \xi}{4c^2} |M|^2 G^{(0,0)}(\omega, \mathbf{k}_\parallel), \tag{4.8}$$

$$\xi = \left[1 + \frac{\omega_n^2}{|ck_n|^2} \operatorname{tg}^2\varphi\right],$$

$$M = \frac{\varepsilon(a, \omega + i\Delta) - 1}{\varepsilon(a, \omega + i\Delta)} + i(k_n + nk_0) \int_0^a dz\, e^{iz(k_n + nk_0)} \left[\frac{1}{\varepsilon(z, \omega + i\Delta)} - \frac{1}{\varepsilon(a, \omega + i\Delta)}\right].$$

The expressions (4.6) and (4.8), obtained with the aid of Eq. (3.6) are only valid for relatively large values of the quantity k_n, when $c|k_n| \gg \omega_{Le}(a)$. We can show that allowance for corrections of the order $\omega_{Le}^2(z)/c^2$ to the quantity k_n^2 results in the replacement, in Eqs. (4.6) and (4.8), of the factor ξ with the quantity

$$4|1 + k_n(0)/k_n(a)|^{-2}\xi,$$

$$k_n^2(z) = \frac{\omega_n^2}{c^2} \varepsilon(z, \omega_n) - k_\parallel^2. \tag{4.9}$$

If $k_n^2(a) < 0$, the quantity $k_n(a)$ in Eq. (4.9) should be replaced with $i[-k_n^2(a)]^{1/2}$. We can then ignore the second term in the expression for the function M in Eq. (4.8).

It should be noted that, in addition to the scattering of a strong electromagnetic wave by fluctuations of surface oscillations considered in the present paper, a plasma may also emit radiation due to scattering by fluctuations of volume waves. It is known [10] that frequencies of volume oscillations in a plasma subjected to a strong high-frequency field are

$$\omega_{\text{hf}}^2 = \omega_{Le}^2(a) + J_0^2 \omega_{Li}^2(a),$$

$$\omega_{\text{lf}}^2 = \omega_{Li}^2(a)(1 - J_0^2).$$

Using these expressions and Eq. (3.6) for k_n, we can show that, along a given direction of propagation of the emitted harmonics of the external field frequency, the scattering by volume and surface oscillations produces harmonics whose frequencies $n\omega_0 + \omega$ differ by the values of ω.

The spectral energy density $\mathbf{S}^{(n)}(\omega, \mathbf{k}_\parallel)$ of Eqs. (4.5) and (4.8) governs the energy flux $dI^{(n)}$ resulting from the scattering of a strong electromagnetic wave by surface oscillations in a frequency interval $d\omega$ and a wave vector interval $d\mathbf{k}_{\|}$:

$$dI^{(n)}(\omega, \alpha, \varphi) = S^{(n)}(\omega, \mathbf{k}_{\|}) \frac{d\omega\, d\mathbf{k}_{\|}}{(2\pi)^3},$$

where α is the angle between the normal to the surface of the plasma and the wave vector. Allowing for the relationship between the unit vector of the direction of the scattered radiation flux $\mathbf{e}^{(n)}(\omega, \mathbf{k}_{\|})$ of Eq. (4.5) and the quantities ω and $\mathbf{k}_{\|}$, we obtain the following expression for the energy emitted into a solid angle do in a frequency interval $d\omega$:

$$\frac{dI^{(n)}(\omega, \alpha, \varphi)}{do\, d\omega} = \frac{1}{(2\pi)^3} \frac{\omega_n^2}{c^2} \cos\alpha S^{(n)}(\omega, k_{\|}, \varphi). \tag{4.10}$$

Here, $S^{(n)}(\omega, k_{\|}, \varphi) \equiv S^{(n)}(\omega, \mathbf{k}_{\|})$, and the wave number is governed by the values of the frequency and angle $k_{\|} = \omega_n \sin\alpha/c$. Thus, using the relationship (4.10), we find that Eqs. (4.5) and (4.8) give the energy flux per unit solid angle and frequency interval, which can be measured directly by experiment.

[1)]Visitor from Physics Institute, Belgrade, Yugoslavia.

1. Yu. M. Aliev and É. Perlengi, Zh. Eksp. Teor. Fiz. **57**, 1623 (1969) [Sov. Phys.-JETP **30**, 877 (1970)].
2. Yu. M. Aliev, O. N. Gradov, and A. Yu. Kiriĭ, Zh. Eksp. Teor. Fiz. **63**, 1254 (1972) [Sov. Phys.-JETP **36**, 663 (1973)].
3. Yu. M. Aliev, O. M. Gradov, and A. Yu. Kiriĭ, Zh. Prikl. Mekh. Tekh. Fiz. No. 1, 11 (1973).
4. D. F. DuBois and M. V. Goldman, Phys. Rev. **164**, 207 (1967).
5. V. Yu. Bychenkov, V. P. Silin, and V. T. Tikhonchuk, Kratk. Soobshch. Fiz. No. 8, 27 (1972).
6. Yu. L. Klimontovich and V. P. Silin, Dokl. Akad. Nauk SSSR **145**, 764 (1962) [Sov. Phys.-Dokl. **7**, 698 (1963)].
7. L. M. Gorbunov and V. P. Silin, Zh. Eksp. Teor. Fiz. **50**, 1095 (1966) [Sov. Phys.-JETP **23**, 729 (1966)].
8. L. M. Gorbunov, Zh. Eksp. Teor. Fiz. **55**, 2298 (1968) [Sov. Phys.-JETP **28**, 1220 (1969)].
9. Yu. M. Aliev, S. Vukovich, O. M. Gradov, and A. Yu. Kiriĭ, Plasma Phys. (in press).
10. Yu. M. Aliev and V. P. Silin, Zh. Eksp. Teor. Fiz. **48**, 901 (1965) [Sov. Phys.-JETP **21**, 601 (1965)].

Translated by A. Tybulewicz
13