Electrons and phonons in strong electromagnetic fields

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(Submitted March 17, 1974)
Zh. Eksp. Teor. Fiz. **68**, 127–135 (January 1975)

It is shown that, under certain conditions, the propagation of a large-amplitude electromagnetic wave is accompanied not only by carrier heating but also by the heating and drag of long-wavelength phonons. Moreover, thermal fluxes due to the heating inhomogeneity appear in the phonon subsystem and these fluxes give rise to a "phonon" anomalous skin effect. In this effect the nonlocality of the relationship between the carrier temperature and field is due to the phonon thermal conductivity.

Nonlinear effects associated with carrier heating accompany the propagation of a strong electromagnetic wave in a conducting medium. These "self-action" effects have been studied in sufficient detail for the case when the phonon subsystem is in equilibrium.[1] However, carrier heating frequently results in the heating of phonons and in the appearance of quasiparticle fluxes in the phonon subsystem (phonon drag and thermal fluxes). These phenomena should influence strongly the nature of the propagation of a strong wave.

The published investigations of this subject (see, for example,[2-4]) have dealt only with some aspects of the problem and the leading role of the effect under consideration has either been analyzed insufficiently thoroughly or ignored completely. Several important effects have not been discussed at all.

The purpose of the present paper is to analyze (from a unified standpoint) the role of the phonon subsystem in the propagation of a strong electromagnetic wave in the case of arbitrary relationships between the parameters of the problem.

The only restriction on the theory given below is the assumption that the conducting medium under consideration has a thermal reservoir for the long-wavelength phonons and this reservoir is the short-wavelength phonon subsystem.[1)] This assumption is justified if the carrier heating has an upper limit[5]

$$\bar{\varepsilon} \ll T\frac{T}{ms^2}, \tag{1}$$

where $\bar{\epsilon}$ is the average carrier energy, m is the carrier mass, T is the temperature of short-wavelength phonons, and s is the velocity of sound.

It should be noted that the inequality (1) is not difficult to satisfy because at $T \gtrsim 10°K$ and for $T/ms^2 \sim 100$ the carrier heating may be strong ($\bar{\epsilon} \gg T$) when Eq. (1) is satisfied.

The main system of equations describing this problem is composed of the Maxwell equations and the transport equations for electrons and phonons. The transport equation for the phonon distribution function N is

$$\frac{\partial N}{\partial t}+\mathbf{s}\frac{\partial N}{\partial \mathbf{r}}=S_{fe}\{N\}+S_{ff}\{N\}+S_{fd}\{N\}. \tag{2}$$

The phonon-phonon and phonon-defect collision integrals can be expressed in the τ approximation:[5]

$$S_{ff}\{N\}=\nu_{ff}(q)(N_T-N),\quad S_{fd}\{N\}=\nu_{fd}(q)(N_0-N), \tag{3}$$

where ν_{ff} and ν_{fd} are the frequencies of collisions of long-wavelength phonons with short-wavelength phonons and with defects, respectively; N_0 is the isotropic part of the phonon distribution function; N_T is the equilibrium distribution function; q is the phonon momentum.

The phonon-electron collision integral $S_{fe}\{N\}$ in the case of a weak anisotropy of the electron distribution function (this is the case of greatest interest) can be reduced to the form

$$S_{ef}\{N\}=\nu_{fe}(N_\Theta-N)-2\pi m^2W_qN\frac{\mathbf{q}}{q}\int\limits_{q/2}^{\infty}\mathbf{V}(\varepsilon)\frac{\partial f_0}{\partial \varepsilon}d\varepsilon, \tag{4}$$

and the frequency of phonon-electron collisions ν_{fe} is

$$\nu_{fe}=2\pi m^2W_qsf_0(q/2). \tag{5}$$

The following notation is used in Eqs. (4) and (5): W_q is the probability of the scattering of electrons by phonons; N_Θ is the Planck distribution function with a temperature Θ [it should be noted that when the inequality (1) is satisfied, we have $N_\Theta \sim \Theta/sq \gg 1$]; f_0 is the symmetric part of the electron distribution function f, which can be expressed in the form

$$f(\mathbf{p},\mathbf{r},t)=f_0(\varepsilon,\mathbf{r})-\mathbf{V}(\varepsilon,\mathbf{r},t)\mathbf{p}\frac{\partial f_0(\varepsilon,\mathbf{r})}{\partial \varepsilon}, \tag{6}$$

where p is the electron quasimomentum.

Equation (4) is derived on the assumption that the particle control condition is satisfied,[1] i.e., that f_0 is a Fermi function with a time-independent temperature Θ.[2)]

Assuming that the anisotropy of the phonon distribution function is weak (the relevant criteria will be given later), we find that Eq. (2) can easily be solved if we use Eqs. (3) and (4). The solution is

$$N=\begin{cases} N_0-\dfrac{\mathbf{q}}{q}\boldsymbol{\psi}, & q\leqslant 2p,\\ N_T, & q>2p,\end{cases} \tag{7}$$

where the isotropic part of the distribution function is

$$N_0=\frac{N_\Theta\nu_{fe}+N_T\nu_{ff}}{\nu_{fe}+\nu_{ff}}, \tag{8}$$

and the anisotropic part is

$$\boldsymbol{\psi}=\frac{s}{\nu_f}\frac{dN_0}{d\mathbf{r}}+\frac{\nu_{fe}}{\nu_f}N_0\frac{1}{s}\int\limits_{q/2}^{\infty}\mathbf{V}_c\frac{\partial f_0}{\partial\varepsilon}d\varepsilon+\frac{\nu_{fe}}{\nu_f-i\omega}N_0\frac{1}{s}\int\limits_{q/2}^{\infty}\mathbf{V}_t\frac{\partial f_0}{\partial\varepsilon}d\varepsilon,$$

$$\nu_f=\nu_{fe}+\nu_{ff}+\nu_{fd}. \tag{9}$$

Equation (9) is derived by representing, as in[6], the function $\mathbf{V}(\epsilon, \mathbf{r}, t)$ as a sum of a static component $[\mathbf{V}_c(\epsilon, \mathbf{r}, t)]$, representing the presence—in the electron subsystem—of static thermal fluxes due to the dependence of Θ on r, and a high-frequency component $[\mathbf{V}_t(\epsilon, \mathbf{r}, t) \propto e^{-i\omega t}]$, due to a high-frequency electromagnetic wave of frequency ω.

If

$$\delta/\lambda = 1/\mathrm{Re}, \tag{10}$$

it follows from Eq. (8) (see also [2,5]) that the symmetric part of the phonon distribution function is in equilibrium ($N_0 = N_T$) and the anisotropic part is small [see Eq. (9)]. Since there is no temperature inhomogeneity in the phonon subsystem, the anisotropy of the phonon distribution function is then related only to the static and high-frequency drag of phonons by electrons [second and third terms in Eq. (9)].

However, if

$$\nu_{ff} \ll \nu_{fe}, \tag{11}$$

we find that $N_0 \sim N_\Theta$. In this case the term describing in Eq. (9) the anisotropy of the phonon distribution function due to the thermal force $d\Theta/d\mathbf{r}$ is found to be of the order of $N_0 l_f / l_\Theta$, where $l_f = s/\nu_f$ is the mean free path of phonons and l_Θ is the characteristic temperature-variation length. The smallness of this term compared with N_0 is ensured by the inequality

$$l_f \ll l_\Theta, \tag{12}$$

which—as shown below—is satisfied automatically.

The anisotropy due to the drag of phonons by electrons may be weak if the inequality (11) is satisfied in one of the two cases: a) when phonons are scattered strongly by various defects ($\nu_{fd} \gg \nu_{fe}$); b) when the electric field has an upper limit [$V_{c,t}(E) \ll s$].

The transport equation for the anisotropic part of the electron distribution function [the equation for $\mathbf{V}(\epsilon, \mathbf{r}, t)$] is of the form

$$\frac{\partial \mathbf{V}}{\partial t} + \omega_H[\mathbf{V}, \mathbf{h}] = \frac{\Theta}{m}\nabla_r\left(\frac{\varepsilon-\mu}{\Theta}\right) + \frac{e}{m}\mathbf{E} + \mathbf{S}_{ef}^{(1)}. \tag{13*}$$

Here, $\omega_H = |e|H/mc$; e is the electron charge; c is the velocity of light; $\mathbf{H}$ is the external magnetic field; $\mathbf{h} = \mathbf{H}/H$; μ is the chemical potential; $\mathbf{E} = \mathbf{E}_c + \mathbf{E}_t$; $\mathbf{E}_t \propto e^{-i\omega t}$ is the alternating electric field; $\mathbf{E}_c$ is the thermoelectric field due to the damping of the wave $\mathbf{E}_t$; $\mathbf{S}_{ef}^{(1)}$ is the anisotropic part of the electron-phonon collision integral which can be represented in the following form using Eq. (7):

$$\mathbf{S}_{ef}^{(1)} = -\mathbf{V}(\varepsilon, \mathbf{r}, t)\frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0\, dq - \frac{\pi m}{p^3}\int_0^{2p} W_q q^2 s \boldsymbol{\psi}\, dq. \tag{14}$$

Separating the static and alternating terms in Eq. (13), we obtain the following expressions for $\mathbf{V}_t$ and $\mathbf{V}_c$ using Eq. (9):

$$-i\omega\left[\mathbf{V}_t + \frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0 \frac{\nu_{fe}}{\nu_f^2+\omega^2}\,dq \int_{q/2}^{\infty} \mathbf{V}_t\left(-\frac{\partial f_0}{\partial \varepsilon}\right)d\varepsilon\right]$$

$$+\mathbf{V}_t\frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0\, dq - \frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0 \frac{\nu_{fe}\nu_f}{\nu_f^2+\omega^2}\,dq \int_{q/2} \mathbf{V}_t\left(-\frac{\partial f_0}{\partial \varepsilon}\right)d\varepsilon$$

$$+\nu_{ed}\mathbf{V}_t + \omega_H[\mathbf{V}_t, \mathbf{h}] = \frac{e}{m}\mathbf{E}_t,$$

$$\mathbf{V}_c\frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0\, dq - \frac{\pi m}{p^3}\int_0^{2p} W_q q^3 N_0 \frac{\nu_{fe}}{\nu_f}\,dq \int_{q/2}^{\infty} \mathbf{V}_c\left(-\frac{\partial f_0}{\partial \varepsilon}\right)d\varepsilon + \nu_{ed}\mathbf{V}_c$$

$$+\omega_H[\mathbf{V}_c\mathbf{h}] = \frac{e}{m}\mathbf{E}_c + \frac{\Theta}{m}\nabla_r\left(\frac{\varepsilon-\mu}{\Theta}\right) - \frac{\pi m}{p^3}\int_0^{2p} W_q q^3 \frac{s^2}{\nu_f}\frac{dN_0}{dr}\,dq, \tag{15}$$

where ν_{ed} is the frequency of collisions of electrons with various defects.

It follows from Eq. (14) that the most convenient materials for investigation are degenerate semiconductors because the carrier degeneracy simplifies considerably the calculation procedure. However, we can easily show that the main results are applicable also to nondegenerate semiconductors.

The system (15) can be solved easily using the "sharpness" of the function $\partial f_0/\partial\epsilon$. It is sufficient to use the zeroth approximation with respect to Θ/μ in solving the equation for $\mathbf{V}_t$, whereas the solution for $\mathbf{V}_c$ should include terms up to Θ/μ (this is necessary because of the nature of the boundary conditions used below).

Bearing in mind these points, we find that $\mathbf{V}_t$ is given by

$$\mathbf{V}_t = \frac{e}{m^*(\nu_e^* - i\omega)}\,\frac{\mathbf{E}_t - \Omega[\mathbf{E}_t\mathbf{h}] + \Omega^2\mathbf{h}(\mathbf{E}_t\mathbf{h})}{1+\Omega^2}, \qquad \Omega = \omega_H^*/(\nu_e^* - i\omega). \tag{16}$$

The following notation is used in Eq. (16):

$$\omega_H^* = \frac{|e|H}{m^* c}, \quad m^* = m(1+\gamma(\varepsilon)), \quad \nu_e^* = \nu_e(1+\gamma(\varepsilon))^{-1},$$

$$\nu_e = \nu_{ed} + \nu_{ef}, \quad \nu_{ef} = \frac{\pi}{p^3}\int_0^{2p} W_q q^3 N_0\left(1 - \frac{\nu_{fe}\nu_f}{\nu_f^2+\omega^2}\right)dq,$$

$$\gamma(\varepsilon) = \frac{T}{ms^2}\,\frac{1}{2p^3}\int_0^{2p} \frac{\nu_{fe}^2}{\nu_f^2+\omega^2}\,\frac{\nu_{fe}\Theta/T + \nu_{ff}}{\nu_{fe}+\nu_{ff}}\,q^2\,dq. \tag{17}$$

It follows from Eqs. (16)–(17) that the influence of heating and phonon drag on the high-frequency anisotropic part of the electron distribution function can be described as follows: first of all, the frequency of collisions between electrons and phonons ν_{ef} changes compared with the equilibrium phonon case and the change is due to the phonon heating [see Eq. (8) for N_0], which increases ν_{ef}, and also due to the high-frequency drag of phonons [second term in Eq. (17) for ν_{ef}], which naturally reduces ν_{ef}; secondly, because of the drag, the expression for $\mathbf{V}_t$ includes a new electron mass m* and a new collision frequency ν_e^*, which are related to the original mass and frequency by the factor $(1+\gamma)$.

If the relationship (10) applies, the influence of the phonon heating and drag on ν_{ef} is weak, in accordance with our expectations. However, this is insufficient to ensure the smallness of γ. In fact, the order of magnitude of this factor is

$$\gamma \sim \frac{T}{ms^2}\,\frac{\nu_{fe}^2}{\nu_f^2+\omega^2}\,\frac{\nu_{fe}\Theta/T + \nu_{ff}}{\nu_{fe}+\nu_{ff}}$$

which, subject to the inequality (10), leads to

$$\gamma \sim \frac{T}{ms^2}\,\frac{\nu_{fe}^2}{\nu_f^2+\omega^2}.$$

However, since $T/ms^2 \gg 1$, the quantity γ for $\omega \lesssim \nu_{ff}$ may be of the order of unity. Thus, if $\nu_{ff} \gg \nu_{fe}$ ($\omega \lesssim \nu_{ff}$), the phonon heating and drag are weak but they may affect the redefinition of the mass and frequency ν_e. Since $\mathbf{V}_t$ governs the high-frequency conductivity, we find that in the $\nu_{ff} \gg \nu_{fe}$ case the phonons have no influence on this conductivity only if

$$\frac{T}{ms^2}\,\frac{\nu_{fe}^2}{\nu_{ff}^2+\omega^2} \ll 1. \tag{18}$$

If the phonons relax mainly as a result of interaction with electrons [see Eq. (11)], we find that

$$\gamma \sim \frac{\Theta}{ms^2}\,\frac{\nu_{fe}^2}{\nu_{fe}^2+\omega^2}$$

and if $\omega \lesssim \nu_{fe}$ the phonon heating and drag are the strongest influence on the high-frequency conductivity ($\gamma \gg 1$). The influence of the drag is weak only for $\omega^2/\nu_{fe}^2 \gg \Theta/ms^2 \gg 1$.

We shall solve Eq. (15) for $\mathbf{V}_c$ by the method of successive approximations with respect to Θ/μ:

$$\mathbf{V}_c(\varepsilon)=V_c(\mu)-\frac{\varepsilon-\mu}{\mu}V_1(\varepsilon).$$

Then, we find that $\mathbf{V}_c(\mu)$ and $\mathbf{V}_1(\epsilon)$ are given by

$$\mathbf{V}_c(\mu)=\frac{e}{m\nu_e^c}\frac{\mathbf{E}_c'-(\omega_H/\nu_e^c)[\mathbf{E}_c'\mathbf{h}]+(\omega_H^2/\nu_e^c)\mathbf{h}(\mathbf{E}_c'\mathbf{h})}{1+(\omega_H/\nu_e^c)^2}$$

$$\mathbf{V}_1(\varepsilon)=\frac{\mu}{m\nu\Theta}\frac{\nabla_r\Theta-(\omega_H/\nu)[\nabla_r\Theta,\mathbf{h}]+(\omega_H/\nu)^2\mathbf{h}(\nabla_r\Theta,\mathbf{h})}{1+(\omega_H/\nu)^2}. \tag{19}$$

In the system (19) the frequency ν_e^c is identical with ν_c [see Eq. (17)] at $\omega = 0$,

$$\mathbf{E}_c'=\mathbf{E}_c-\left[\frac{1}{2p_\mu^3}\int_0^{2p_\mu}\frac{\nu_{fe}}{\nu_f}\frac{\nu_{fe}}{\nu_{fe}+\nu_{ff}}q^2\,dq-\frac{\pi^2}{6}\frac{\Theta}{\mu}\right]\frac{1}{e}\nabla_r\Theta,$$

$$\nu=\nu_{cd}+\frac{\pi m}{p^3}\int_0^{2p}W_q q^3 N_0\,dq$$

is the frequency of collisions of electrons with defects and hot phonons in the absence of drag ($\nu = \nu_e$ for $\omega \to \infty$), and p_μ is the Fermi momentum $[p_\mu = (2m\mu)^{1/2}]$.

We are interested in the propagation of a large-amplitude electromagnetic wave incident on a semi-infinite sample ($z > 0$). It is clear that in that case we have $E_{x,y} = 0$ and there is no static current along the z axis ($j_{cz} = 0$).

Since the current $\mathbf{j}$ can be expressed in terms of $\mathbf{V}$ by

$$\mathbf{j}=\frac{2}{(2\pi\hbar)^3}\frac{4\pi e}{3}(2m)^{3/2}\int_0^\infty\mathbf{V}(\varepsilon)\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\varepsilon^{3/2}\,d\varepsilon,$$

the condition $j_{cz} = 0$ is the relationship which should be used in the determination of E_{cz}'.

Multiplying the transport equation for electrons by ϵ and integrating with respect to d^3p, we obtain the following equation for the electron energy balance:

$$\frac{dQ_z^e}{dz}=\bar{B}_{ik}E_iE_k^*-P_{ef}. \tag{20}$$

We shall now omit the index t of the alternating components.

In Eq. (20), the quantity $\bar{B}_{ik}$ is the high-frequency heating tensor which is formally identical with $\bar{B}_{ik}$ calculated on the assumption of phonon equilibrium.[7] However, the quantity ν occurring in the formulas given in[7] now understood to be $\nu_e^*(\mu, \Theta)$ and m is now $m^*(\mu, \Theta)$;[3)]

$$\mathbf{Q}^e=\frac{2}{(2\pi\hbar)^3}\frac{4\pi}{3}(2m)^{3/2}\int_0^\infty\mathbf{V}_c\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\varepsilon^{5/2}\,d\varepsilon$$

is the thermal flux in the electron subsystem. Substituting $\mathbf{V}_c$ from Eq. (19) and using the condition $j_{cz} = 0$, we can represent $\mathbf{Q}^e$ in the form

$$\mathbf{Q}^e=-\frac{\pi^2}{3}n\frac{\Theta}{\mu}\mathbf{V}_1(\mu), \tag{21}$$

where n is the carrier concentration.

It follows from Eq. (21) that $\mathbf{Q}^e$ is proportional to Θ/μ and this is why it is necessary to calculate $\mathbf{V}_c$ to within Θ/μ.

Substituting in Eq. (21) the relationship for $\mathbf{V}_1$ from Eq. (19) and expressing Q_z^e in the form $Q_z^e = -\kappa(\Theta)d\Theta/dz$, we find that the electron component of the thermal conductivity is

$$\varkappa_e=\frac{\pi^2 n\Theta}{3m\nu(\Theta)}\frac{1+(\omega_H/\nu(\Theta))^2\cos^2\varphi}{1+(\omega_H/\nu(\Theta))^2}, \tag{22}$$

where φ is the angle between the magnetic field and the z axis.

The term P_{ef} in Eq. (20), which describes the transfer of heat from electrons to long-wavelength phonons, should be found from the phonon energy balance equation, which is easily obtained if we multiply Eq. (2) by sq and integrate with respect to d^3q [using Eq. (7)]:

$$dQ_z^f/dz=P_{fe}-P_{ff}. \tag{23}$$

Here, P_{fe} is the energy acquired by phonons from electrons (it is obvious that $P_{fe} = P_{ef}$), P_{ff} is the energy transferred from short- to long-wavelength phonons,

$$P_{ff}=\frac{4\pi}{(2\pi\hbar)^3}(\Theta-T)\int_0^{2p_\mu}\frac{\nu_{ff}\nu_{fe}}{\nu_{fe}+\nu_{ff}}q^2\,dq, \tag{24}$$

$\mathbf{Q}^f$ is the thermal flux in the long-wavelength phonon subsystem, which can be represented in the form $\mathbf{Q}^f = -\kappa_f(\Theta)\nabla_r\Theta$, where the phonon thermal conductivity is given by

$$\varkappa_f(\Theta)=\frac{4\pi}{3(2\pi\hbar)^3}\int_0^{2p_\mu}\frac{s^2}{\nu_f}\frac{\nu_{fe}}{\nu_{fe}+\nu_{ff}}q^2\,dq. \tag{25}$$

Equation (25) is derived bearing in mind that the contribution to $\kappa_f(\Theta)$ due to the drag of phonons by electrons is Θ/μ times smaller than the contribution due to the thermal diffusion of phonons.

It should be pointed out that the phonon thermal conductivity described by Eq. (25) does not agree with the conductivity which usually occurs in the theory of thermomagnetic effects,[8] because in the case considered here the short-wavelength phonons are in equilibrium [see Eq. (7)] and, consequently, they do not participate in the energy transfer process.

Eliminating $P_{fe} = P_{ef}$ from Eqs. (20) and (23), we obtain the energy balance equation for the system of electrons and long-wavelength phonons and this equation relates Θ and E:

$$\frac{d}{dz}\left[\varkappa_e(\Theta)+\varkappa_f(\Theta)\right]\frac{d\Theta}{dz}=\bar{B}_{ik}E_iE_k^*-P_{ff}. \tag{26}$$

Equation (26) should be supplemented by boundary conditions at infinity (it is clear that in the limit $z \to \infty$ the value of Θ tends to T) and in the plane $z = 0$. The latter condition is similar to that used in[1]:

$$(\varkappa_e+\varkappa_f)\frac{d\Theta}{dz}\bigg|_{z=0}=\eta(\Theta)(\Theta-T)\bigg|_{z=0},$$

where $\eta(\Theta)$ is the parameter describing the rate of the surface energy relaxation in the system of electrons and long-wavelength phonons (if $\nu_{ff} \gg \nu_{fe}$ this condition reduces to the expression given in[1]).

Following[1], we can easily show that when the inequality $l_\Theta \gg \tilde{l}_{e,f}$ is satisfied $[\tilde{l}_{e,f} = (\kappa_{e,f}\Theta/P_{ff})^{1/2}$ is the cooling length of electrons and long-wavelength phonons],[4)] the left-hand side of Eq. (26) can be ignored, and this demonstrates that $l_\Theta \sim L$ is the penetration depth of the field. Thus, when

$$L\gg\max\{\tilde{l}_f,\tilde{l}_e\} \tag{27}$$

there is a local relationship between the temperature and field, i.e., the normal skin effect is observed.

In the theory discussed in[1] the condition for the normal skin effect is $L \gg \tilde{l}_e$. However, it follows from Eqs. (22) and (25) that

$$\frac{\tilde{l}_f}{\tilde{l}_e}\sim\frac{\nu_{fe}}{\nu_{fe}+\nu_{ff}}\left[\frac{\nu}{\nu-\nu_{ed}}\frac{\nu_{fe}+\nu_{ff}T/\Theta}{\nu_{fe}+\nu_{ff}}\frac{1+(\omega_H/\nu)^2}{1+(\omega_H/\nu)^2\cos^2\varphi}\right]^{1/2} \tag{28}$$

and if $\nu_{ff} \gg \nu_{fe}\Theta/T$, $\nu \gtrsim \nu_{ed}$, we find that $\tilde{l}_e \gg \tilde{l}_f$ provided the magnetic field is not orthogonal to the z axis. These last inequalities correspond to the situation in which heat is transferred mainly by electrons, which is indeed assumed in [1]. Moreover, it follows from the above discussion that the inequality (18) should be satisfied to ensure the validity of the results given in [1].

If $\nu_{ff} \ll \nu_{fe}$, then $\tilde{l}_f \gtrsim \tilde{l}_e$, and a rise of a transverse magnetic field and of ν_{ed} both strengthen the latter inequality. In this case the role of phonons in heat transfer is at least as important as the role of electrons and the condition for the normal skin effect is more difficult to satisfy ($L \gg \tilde{l}_f$).

A similar situation is considered in [2] ignoring the drag effect. A general treatment of the normal skin effect is given in [9] but the criteria for neglecting the divergent terms in Eq. (27) are not obtained in [2] or in [9].

It is clear that if

$$L < \max\{\tilde{l}_f, \tilde{l}_e\}, \tag{29}$$

the relationship between the temperature and field is nonlocal [see Eq. (26)] and we are dealing with the anomalous skin effect considered first in [7] on the assumption that phonons are in equilibrium.

Clearly, when the inequality (29) is satisfied, the characteristic temperature-variation length l_Θ is governed by the subsystem with the higher thermal conductivity, i.e., $l_\Theta \sim \max\{\tilde{l}_f, \tilde{l}_e\}$. Therefore, if $\tilde{l}_e > \tilde{l}_f$, then $l_\Theta \sim \tilde{l}_e$ and we are dealing with a situation already discussed in [1].

If the opposite inequality applies then $l_\Theta \sim \tilde{l}_f$,[5)] and even when $L \gg \tilde{l}_e$ ($L < \tilde{l}_f$) the anomalous skin effect is observed and this effect is due to thermal fluxes in the phonon subsystem. Obviously, this effect appears more easily than the "electron" anomalous skin effect.

If the "phonon" anomalous skin effect is strong ($L \ll \tilde{l}_f$), it can be investigated in a similar way as in [1,7] replacing κ_e with κ_f and allowing for Eq. (17).

Since κ_e depends strongly on the magnitude and direction of the magnetic field and κ_f is independent of H, we can use the magnetic field in order to determine the mechanism (electron or phonon) of the anomalous skin effect.

It should be noted that, on the one hand, Θ decreases with rising z because of the wave damping and, on the other, κ_e and κ_f depend in different ways on Θ. Therefore, it is possible to have $\kappa_e \gg \kappa_f$ and the anomalous skin effect due to electrons in one part of a sample, whereas in another part of a sample we may have $\kappa_e \ll \kappa_f$ and the phonon-dominated heat flow.

Our discussion applies to the scattering of electrons by acoustic phonons. We can show that if the main relaxation mechanism is the scattering of electrons by optical phonons, the phonon thermal conductivity is usually (see below) ineffective compared with the electron mechanism. This is due to the low group velocity of the optical phonons because of their weak dispersion.

However, in some cases such as the total control given by $\nu_{ee} \gg \nu$, where ν_{ee} is the electron-electron collision frequency, presence of a strong magnetic field, etc., the heat flow in the electron subsystem tends to zero and the phonon heat conduction is the only mechanism (this applies to acoustic and optical phonons).

The authors are grateful to F. G. Bass for useful discussions.

*[Vh] ≡ V × h.

1)We shall consider only acoustic phonons.

2)The criterion for the time independence of Θ is given in [1].

3)It should be noted that in the absence of the phonon heating and drag in degenerate semiconductors there is no dependence of ν on Θ.

4)Since, $\tilde{l}_f \gg l_f$, Eq. (12) is satisfied automatically when $l_\Theta \gg \tilde{l}_f$.

5)Since $l_\Theta \sim \max\{\tilde{l}_f, \tilde{l}_e\}$ and $\tilde{l}_f \gg l_f$, the inequality (12) is satisfied automatically.

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Translated by A. Tybulewicz
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