Microscopic theory of resistive current states in superconducting channels

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Some features of the linear response to a iongitudinal electric field for electrons in a metal, and the role which is played in this by the electrical-neutrality condition, are analyzed. Unlike in a normal metal, where this condition expresses the continuity of the current, in a superconductor, owing to processes of "annihilation" and "creation" of electron pairs from the superconducting condensate, the electrical-neutrality condition is an independent equation determining the distribution of the electric potential. The equation of continuity of the total (normal and superconducting) current, as in the equilibrium case, is contained in the equation for the phase of the complex superconducting order parameter. Kinetic equations are constructed for a "dirty" superconductor and a solution of these equations satisfying the requirements outlined is found. These equations are employed to study the structure of the resistive current state in a superconducting channel; this structure corresponds to microscopic phase separation, with alternating superconducting normal regions along the channel.

An investigation of the Cooper instability of the normal current state at temperatures $T < T_c$ has shown[1] that, owing to its special diamagnetic properties, a narrow superconducting channel responds in different ways to electric fields established in the channel inductively and by means of an "external" EMF source. Whereas in the former case a transition to the superconducting state is impossible[2,3], in the latter case, for sufficiently small fields $E < E_{c2}$ and currents $j < j_{c2}$, superconducting centers develop in the channel and the superconductor goes over into a special dissipative state with nonzero superconducting order. This state is conserved down to a critical current $j_c < j_{c2}$, determined by the number of superconducting electrons, after which the resistance vanishes.

The object of this paper is to analyze the microscopic structure of the resistive current states on the basis of the kinetic theory of superconductivity[4]. There are a number of publications (cf., e.g.,[5]) devoted to this question[1)], the authors of which start from the so-called nonlinear time-dependent Ginzburg-Landau equations (cf.[8]). As was pointed out in[9], except in certain special cases such equations are, generally speaking, not possible when nonequilibrium corrections to the electron distribution function are taken into account. More important in the present case is the fact that in these papers the specific features of the linear response to a longitudinal field for electrons in a metal are not taken into account, and the important role which is played in this by the electrical-neutrality condition is completely ignored.

In a normal metal this situation is described by the Boltzmann transport equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\nabla f-\mathbf{v}\nabla\Phi\frac{\partial f}{\partial\varepsilon}=I(f)\equiv\frac{1}{\tau}(\langle f\rangle-f), \tag{1}$$

where $f(t, \mathbf{r}, \epsilon, \mathbf{n})$ is the electron distribution function, ϵ is the electron energy measured from the Fermi surface, and $\mathbf{n}$ is the unit vector in the direction of motion of the electron: $\mathbf{v} = v_F\mathbf{n}$, where $v_F = p_F/m$ is the velocity at the Fermi surface; $e\mathbf{E} = -\nabla\Phi$; $I(f)$ is the collision integral for collisions with impurities (for simplicity, in the τ-approximation); $\langle f\rangle = (4\pi)^{-1}\int df(\mathbf{n})$; $1/\tau$ is the collision frequency.

For small gradients and frequencies ($l\nabla \ll 1$, $\tau\partial/\partial t \ll 1$, $l = v_F\tau$), it follows from Eq. (1) that

$$f\approx f^{(0)}+l\mathrm{n}\left(\nabla\Phi\frac{\partial f^{(0)}}{\partial\varepsilon}-\nabla f^{(0)}\right), \tag{2}$$

where the isotropic function $f^{(0)}$ satisfies the equation

$$\frac{\partial f^{(0)}}{\partial t}=D\left(\nabla^2 f^{(0)}-\nabla^2\Phi\frac{\partial f^{(0)}}{\partial\varepsilon}-2\nabla\Phi\nabla\frac{\partial f^{(0)}}{\partial\varepsilon}+(\nabla\Phi)^2\frac{\partial^2 f^{(0)}}{\partial\varepsilon^2}\right), \tag{3}$$

where $D = \frac{1}{3}v_F l = \frac{1}{3}\tau v_F^2$ is the diffusion coefficient.

In the linear-response approximation the steady-state solution of Eq. (3) has the form

$$f^{(0)}\approx f_F-\frac{\partial f_F}{\partial\varepsilon}\frac{D\nabla^2}{\partial/\partial t-D\nabla^2}\Phi \quad f_F=\frac{1}{e^{\varepsilon/T}+1}. \tag{4}$$

With the aid of formulas (2) and (4) it is not difficult to write down expressions for the current and the change in electron density:

$$\begin{aligned}\mathbf{j}&=e\nu_F v_F\int d\varepsilon\langle\mathbf{n}f\rangle=-e\nu_F D\frac{\partial/\partial t}{\partial/\partial t-D\nabla^2}\nabla\Phi,\\ \delta N&=\nu_F\int d\varepsilon\langle\delta f\rangle\approx\nu_F\frac{D\nabla^2}{\partial/\partial t-D\nabla^2}\Phi,\end{aligned} \tag{5}$$

where $\nu_F = mp_F/\pi^2$ is the density of states at the Fermi surface ($\hbar = c = 1$).

The quantities δN and $\mathbf{j}$ (5) identically satisfy the conservation equation for the number of particles:

$$e\frac{\partial}{\partial t}\delta N+\nabla\mathbf{j}=0.$$

As can be seen from the formulas (5), the usual care must be observed in passing to the stationary case. Indeed, if we formally put $\partial\Phi/\partial t = 0$, we obtain from (5): $\mathbf{j} = 0$, $\delta N = -\nu_F\Phi$. The correct solution describing the stationary normal current,

$$\mathbf{j}=-e\nu_F D\nabla\Phi=\frac{Ne^2\tau}{m}\mathbf{E},$$

is obtained if we take into account that in a metal, owing to the high electron density, the distribution of the potential is determined by the electrical-neutrality condition: $\delta N = 0$. From this it follows that (cf. (5)) $\nabla^2\Phi = 0$, i.e., $\nabla\cdot\mathbf{j} = 0$. Thus, in this case the electrical-neutrality condition simply expresses the continuity of the current, and this is in agreement with the general conservation equation for the number of particles.

In superconductors the conservation of the number of electrons is intimately bound up with the processes of "annihilation" and "creation" of electron pairs from the superconducting condensate, which is formally a self-

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consistent "source" of electrons. As will be shown below, this leads to the result that the electrical-neutrality condition becomes a nontrivial equation determining the distribution of the potential (and of small electric charges[2]) $\sim -(4\pi e)^{-1}\nabla^2\Phi$) in the resistive current state. As regards the equation of continuity of the total (normal and superconducting) current $\nabla \cdot \mathbf{j} = 0$, it is contained, as in the equilibrium case [7], in the equation for the phase of the complex superconducting order parameter.

The study of the resistive states is of most practical interest in the case of "dirty" superconducting films: $\tau T_c \ll 1$. Therefore, the immediate problem is the derivation of kinetic equations for this case, generalizing Eq. (3), from the general quantum equations for the electron density matrix of the superconductor [4].

1. KINETIC EQUATIONS FOR "DIRTY" SUPERCONDUCTING ALLOYS

In the notation of [4], the equation for the density matrix γ has the form

$$i\partial\gamma/\partial t=[\hat{\varepsilon}+\hat{U},\ \gamma]+iL^{(2)}(\gamma). \tag{6}$$

In Eq. (6) the potential for scattering of electrons by impurities (nonmagnetic) has been separated out:

$$\hat{U}=\sum_j \hat{U}_j,\quad \hat{U}_j(\mathbf{r}_1,\mathbf{r}_2)=\sigma_z u(\mathbf{r}_1-\mathbf{r}_j)\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{7}$$

where $u(\mathbf{r}-\mathbf{r}_j)$ is the potential created by an impurity situated at the point $\mathbf{r}_j$. In Eq. (6) the last term $iL^{(2)}(\gamma)$, which describes the energy relaxation of the electrons, corresponds to the lowest frequencies $1/\tau_2 \sim 10^8$ sec^{-1} $\ll T_c \sim 10^{12}$ sec^{-1}, and needs to be taken into account only in the final stage of the relaxation of the system to a state of equilibrium.

For the description of the relaxation of the electrons at impurities in "clean" superconductors ($\tau T_c \gg 1$), we can obtain from Eq. (6) (cf. [4]) the usual Boltzmann equation for the distribution function of the quasiparticles corresponding to Cooper pairing. For an arbitrary relationship between the mean free path $l = v_F\tau$ and the range of correlation of the superconducting electrons $\xi_0 \sim v_F/T_c$, we must start from the general solution of Eq. (6) (for $L^{(2)} = 0$):

$$t>0\quad \gamma(t)=G^{(+)}(t,0)\gamma(0)G^{(-)}(0,t), \tag{8}$$

where $G^{(+)}$ and $G^{(-)}$ are the retarded and advanced electron propagators:

$$[i\partial/\partial t_1-(\hat{\varepsilon}(t_1)+\hat{U})]G(t_1,t_2)=\delta(t_1-t_2).$$

It is necessary to average the density matrix (8) over the random positions of the impurities (7). Using for this, e.g., the well-known diagram technique of [10,11], we obtain

$$\gamma(\mathbf{r}_1,\mathbf{r}_2;t)=\left(\frac{p_F}{2\pi}\right)^2\int dO\exp[ip_F\mathbf{n}(\mathbf{r}_1-\mathbf{r}_2)]\Pi(\mathbf{n},\rho;s_1,s_2;t,t), \tag{9}$$

where s is the path covered by the electron in the direction of motion n, ρ is the "impact parameter", and the matrix $\Pi(\mathbf{n},\rho,s_1,s_2;t_1,t_2)$ satisfies the equation

$$\Pi(\mathbf{n},\rho;s_1,s_2)-\int ds\,G^{(+)}(\mathbf{n},\rho;s_1,s)\cdot nv_F^2\int dO'|f(\mathbf{n},\mathbf{n}')|^2$$
$$\times\sigma_z\Pi(\mathbf{n}',\rho';s',s')\sigma_z G^{(-)}(\mathbf{n},\rho;s,s_2)=(G^{(+)}\hat{\gamma}(0)G^{(-)})(\mathbf{n},\rho;s_1,s_2), \tag{10}$$
$$\mathbf{r}=\mathbf{n}s+\rho=\mathbf{n}'s'+\rho',\quad \mathbf{n}\rho=\mathbf{n}'\rho'=0.$$

Here contractions over the time are implied in the matrix multiplication, n is the impurity density, and $f(\mathbf{n},\mathbf{n}')$ is the amplitude for scattering of an electron by an impurity; $\hat{\gamma}(0;t_1,t_2)=\gamma(0)\delta(t_1)\delta(t_2)$.

The averaged propagator satisfies the nonlinear equation (cf. [11])

$$(\hat{\omega}^{(0)}-\hat{\varepsilon}(\mathbf{n},\rho)-M(\mathbf{n},\rho))G(\mathbf{n},\rho)=1,$$
$$\hat{\omega}^{(0)}(t_1,t_2)=i\frac{\partial}{\partial t_1}\delta(t_1-t_2),\quad M(\mathbf{n},\rho;s_1,s_2)=M(\mathbf{n},\rho,s_1)\delta(s_1-s_2), \tag{11}$$
$$M(\mathbf{n},\rho,s)=nv_F^2\int dO'|f(\mathbf{n},\mathbf{n}')|^2\sigma_z G(\mathbf{n}',\rho';s',s')\sigma_z$$
$$(\mathbf{r}=\mathbf{n}s+\rho=\mathbf{n}'s'+\rho'),$$

where the operator $\hat{\varepsilon}(\mathbf{n},\rho)$ is defined by the following formulas (cf. [4]):

$$\hat{\varepsilon}(\mathbf{n},\rho;s_1,s_2)=[\sigma_z(-iv_F\partial/\partial s_1+\Phi(s_1,\rho))$$
$$+\mathbf{v}\mathbf{p}_s(s_1,\rho)+\sigma_x\Delta(s_1,\rho)]\delta(s_1-s_2), \tag{12}$$
$$\Phi=\tfrac{1}{2}\partial\chi/\partial t+e\varphi+\varepsilon_F,\quad \mathbf{p}_s=\tfrac{1}{2}\nabla\chi-e\mathbf{A},\quad \mathbf{v}=v_F\mathbf{n}.$$

Here, Φ, $\mathbf{p}_S = m\mathbf{v}_S$ and Δ are gauge-invariant quantities: Φ is the "electrochemical" potential, $\mathbf{v}_S$ is the velocity of the superconducting condensate, and Δ is the modulus of the order parameter; χ is the phase of the complex order parameter, and φ and $\mathbf{A}$ are the electromagnetic potentials.

As can be verified, the asymptotic "rest point" for the matrix γ, as determined by Eqs. (9)—(12), has the form

$$\gamma^{(0)}=\int\frac{d\omega}{2\pi i}\varphi(\omega)(G^{(-)}(\omega)-G^{(+)}(\omega)), \tag{13}$$

where $G^{(\pm)}(\omega)$ are the Fourier transforms of the functions $G^{(\pm)}(t_1-t_2)$, and $\varphi(\omega)$ is an arbitrary function of the frequency (depending on the initial condition $\gamma(0)$). The matrix $\gamma^{(0)}$ (13) is obviously the impurity-averaged "rest point" $\gamma^{(0)}=\int d\omega\varphi(\omega)\delta(\omega-\hat{\varepsilon}-\hat{U})$ of the starting equation (6): $[\hat{\varepsilon}+\hat{U},\ \gamma^{(0)}]=0$, and $\varphi(\omega)$ is thus the energy distribution function that is established after a time $t \gg (1/T_c, \tau)$ has elapsed. On including the energy relaxation (the last term $iL^{(2)}(\gamma)$ in Eq. (6)), we can find the equilibrium distribution function [4]:

$$\varphi^{(0)}=\tfrac{1}{2}[1+\mathrm{th}(\omega/2T)]. \tag{14}$$

In a "dirty" superconductor ($\tau T_c \ll 1$), the metal behaves as a normal metal at frequencies $\omega \sim 1/\tau$ and at distances $r \sim l$. Therefore, it is the "low" frequencies $\omega \sim T_c \ll 1/\tau$ and "large" distances $r \sim \xi_0 \gg l$ that are of interest. With these scales the relaxation at the impurities is complete and the deviations from equilibrium in the density matrix γ and the matrix Π (9) are localized, as can be seen from formula (13), in a narrow range of frequencies $\sim T_c$. The corresponding asymptotic expansions in the parameter $\tau T_c \ll 1$[3] in Eqs. (10) and (11) resemble the well-known [12,13] expansions in the thermodynamic theory of "dirty" superconducting alloys. When written in the technique of temperature Green functions, the final results are partially equivalent to the equations of Larkin and Ovchinnikov [14] for the Green functions of coincident arguments.

To elucidate the characteristic features of these expansions, we shall consider the spatially-uniform case when the Fourier representation

$$G(\mathbf{n},\rho;s_1,s_2)=\int\frac{d\xi}{2\pi v_F}\exp\left[\frac{i\xi}{v_F}(s_1-s_2)\right]G(\mathbf{n},\xi). \tag{15}$$

is valid.

Assuming for simplicity that the scattering by the impurities is isotropic ($f(\mathbf{n},\mathbf{n}') = \mathrm{const} = f_0$), we obtain from Eqs. (11) and (12) (for $\Phi = 0$)

$$[\hat{\omega}-\mathbf{vp}_s-(\sigma_z\xi+M)]G(\mathbf{n},\xi)=1, \quad M=\int\frac{d\xi}{2\pi\tau}\sigma_z\langle G(\mathbf{n},\xi)\rangle\sigma_z,$$
$$\hat{\omega}=\hat{\omega}^{(0)}-\sigma_z\Delta, \tag{16}$$

where $1/\tau = 4\pi n v_F f_0^2$, $\mathbf{v} = v_F\mathbf{n}$, and the quantity $\hat{\omega}^{(0)}$ is defined in (11).

In the lowest approximation Eqs. (16) are written in the form

$$-(\sigma_z\xi+M_0)G_0(\xi)=1, \quad \sigma_zM_0\sigma_z=\oint\frac{d\xi}{2\pi\tau}G_0(\xi),$$

whence it follows that

$$G_0(\xi)=-\frac{\sigma_z\xi+i_s\hat{u}/2\tau}{\xi^2+(1/2\tau)^2}, \quad M_0=-\frac{i_s}{2\tau}\sigma_z\hat{u}\sigma_z \tag{17}$$

($i_\pm = \pm i$, $s = \pm 1$), where the matrix $\hat{u}$ satisfies the condition

$$\hat{u}\sigma_z\hat{u}=\sigma_z \quad \left(\int dt\,\hat{u}(t_1,t)\sigma_z\hat{u}(t,t_2)=\sigma_z\delta(t_1-t_2)\right). \tag{18}$$

A further equation determining this matrix follows from consideration of higher approximations in Eqs. (16):

$$M_1=0, \quad G_1(\mathbf{n},\xi)=G_0(\xi)\mathbf{vp}_sG_0(\xi),$$
$$G_2=G_0(M_2-\hat{\omega})G_0+G_0\mathbf{vp}_sG_0\mathbf{vp}_sG_0.$$

Substituting these expressions into Eq. (16) for the mass operator M and integrating over ξ taking formulas (17) and (18) into account, we obtain

$$\sigma_zM_2\sigma_z+\hat{u}M_2\hat{u}=\hat{u}\hat{\omega}\hat{u}-\sigma_z\hat{\omega}\sigma_z-{}^1/_2\,i_sD\,(\hat{u}p_s^2+p_s^2\hat{u}$$
$$+\sigma_z\mathbf{p}_s\hat{u}\mathbf{p}_s\sigma_z-3\hat{u}\mathbf{p}_s\hat{u}\mathbf{p}_s\hat{u}). \tag{19}$$

We multiply Eq. (19) by $\hat{u}\sigma_z$ on the left and by $\sigma_z\hat{u}$ on the right. Subtracting the result from (19) and taking the relation (18) into account, we obtain the condition for solubility of Eq. (19):

$$\hat{u}\hat{\Omega}\hat{u}=\sigma_z\hat{\Omega}\sigma_z, \quad \hat{\Omega}=\hat{\omega}+i_sD\mathbf{p}_s\hat{u}\mathbf{p}_s. \tag{20}$$

Equations (18) and (20), together with the boundary conditions $\hat{u}_\pm(t_1, t_2) \propto \theta(\pm(t_1 - t_2))$, where $\theta(x)$ is the Heaviside function, uniquely determine the matrix $\hat{u}$, and, thereby, the propagators G_0 (17), G_1, G_2, etc.

The expansion of the matrices γ (9) and Π (10) proceeds in parallel with the expansion in Eqs. (16). Here it is sufficient to consider the homogeneous equation (10), since the effect of the initial conditions $\gamma(0)$ is "washed out" in the time-scale considered (low frequencies are important in the resolvent):

$$\Pi(\mathbf{n},\xi)=\tau^{-1}G^{(+)}(\mathbf{n},\xi)\sigma_zR\sigma_zG^{(-)}(\mathbf{n},\xi),$$
$$R-\int\frac{d\xi}{2\pi\tau}\langle G^{(+)}\sigma_zR\sigma_zG^{(-)}\rangle=0, \tag{21}$$

In the zeroth approximation it follows from these equations (21) and formulas (17) and (18) that

$$\Pi_0=\frac{1}{\tau}R_0\left[\xi^2+\left(\frac{1}{2\tau}\right)^2\right]^{-1}, \quad R_0=\hat{u}_+\sigma_zR_0\sigma_z\hat{u}_-. \tag{22}$$

Taking into account the algebraic properties (18) of the matrix $\hat{u}$, it is not difficult to find the general solution of the latter equation (22) for R_0:

$$R_0={}^1/_2(\hat{u}_+\hat{\varphi}+\hat{\varphi}\hat{u}_-), \quad [\sigma_z,\ \hat{\varphi}]=0, \tag{23}$$

where $\hat{\varphi}$ ($\varphi(t_1, t_2) + \sigma_Z\varphi_Z(t_1, t_2)$ in the coordinate representation) is an arbitrary two-component function of the variables t_1 and t_2, normalized in such a way that at the "rest point" the Fourier transform $\varphi(\omega)$ of the function $\varphi(t_1 - t_2)$ ($\varphi_Z = 0$) goes over into the frequency distribution function in formula (13).

An equation for the function $\hat{\varphi}$, like Eq. (20) for the matrix $\hat{u}$, is obtained from the condition for solubility of Eq. (21) for R in the second approximation (in the first approximation, $R_1 = 0$). Omitting the detailed calculations, we shall present immediately the equation for $\hat{\varphi}$ and the corresponding generalization of Eqs. (20) for the spatially-nonuniform case. From formulas (15), (17) and (22) it is not difficult to discern the presence, in the distribution functions and matrices γ and Π, of the Dingle factor $\exp(-|s - s_2|/2l)$ in the coordinate representation. Hence it follows that to study the slow variation of the functions G, γ and Π at distances $r \gg l$ in the spatially-nonuniform case it is necessary in the initial equations (10) and (11) to go over to the mixed Wigner representation

$$G(\mathbf{n},\rho;s_1,s_2)=\int\frac{d\xi}{2\pi v_F}\exp\left(\frac{i\xi}{v_F}(s_1-s_2)\right)G\left(\mathbf{n},\xi;\rho,\frac{s_1+s_2}{2}\right) \tag{24}$$

and, besides expanding in the small "frequencies" $\hat{\omega}$, to expand $\mathbf{vp}_s$ in powers of the spatial gradients corresponding to the variable $(s_1 + s_2)/2$. The final system of equations has the following form:

$$\gamma_0(\xi,\mathbf{r},t)=\frac{1}{2\tau}\frac{(\hat{u}_+\hat{\varphi}+\hat{\varphi}\hat{u}_-)(\mathbf{r};t,t)}{\xi^2+(1/2\tau)^2}; \tag{25}$$

$$\hat{u}\sigma_z\hat{u}=\sigma_z; \tag{26}$$

$$\sigma_z\hat{\omega}\hat{u}-\hat{u}\hat{\omega}\sigma_z=i_sD\{\hat{u}[{}^1/_2\sigma_z\nabla^2\hat{u}\sigma_z+\mathbf{p}_s\hat{u}\mathbf{p}_s$$
$$+i(\mathbf{p}_s\nabla\hat{u}\sigma_z-\sigma_z\nabla\hat{u}\mathbf{p}_s)]\sigma_z-\sigma_z[\ldots]\hat{u}+i(\hat{u}\nabla\mathbf{p}_s\hat{u}-\nabla\mathbf{p}_s)\}, \tag{27}$$

$$\hat{\omega}=\hat{\omega}^{(0)}-\sigma_z\Phi-\sigma_z\Delta, \quad \hat{\omega}^{(0)}(t_1,t_2)=i\frac{\partial}{\partial t_1}\delta(t_1-t_2), \quad i_\pm=\pm i; \tag{27a}$$

$$\hat{u}_+\{[\hat{\omega},\hat{\varphi}]-iD[{}^1/_2(\sigma_z\hat{u}_+\nabla\hat{e}+\nabla\hat{e}\hat{u}_-\sigma_z)+(\sigma_z\nabla\hat{u}_+\hat{e}+\hat{e}\nabla\hat{u}_-\sigma_z)+i(\mathbf{p}_s\hat{u}_+\hat{e}$$
$$-\hat{e}\hat{u}_-\mathbf{p}_s)]\}\sigma_z+\sigma_z\{\ldots\}\hat{u}_-=0, \tag{28}$$

$$[\sigma_z,\hat{\varphi}]=0, \quad \hat{e}=\sigma_z\nabla\hat{\varphi}+i[\mathbf{p}_s,\hat{\varphi}]. \tag{28a}$$

It is necessary to supplement these equations with formulas for the macroscopic responses of the system, i.e., of the order parameter Δ, the current j and the change δN in the electron density. Using the corresponding expressions for these quantities in terms of the density matrix γ [4] and performing simple calculations with the aid of formulas (9), (24) and (25), we obtain

$$\Delta(t)=\frac{|g|\nu_F}{2}\pi\cdot\frac{1}{2}\mathrm{Tr}\,\sigma_x(\hat{u}_+\hat{\varphi}+\hat{\varphi}\hat{u}_-)(t,t); \tag{29}$$

$$\mathbf{j}(t)=e\nu_FD\pi\cdot\frac{1}{2}\mathrm{Tr}[\hat{u}_+\sigma_z\nabla\hat{u}_+\hat{\varphi}+\hat{\varphi}\nabla\hat{u}_-\sigma_z\hat{u}_-+i(\hat{u}_+\mathbf{p}_s\hat{u}_+-\mathbf{p}_s)\hat{\varphi}$$
$$-i\hat{\varphi}(\hat{u}_-\mathbf{p}_s\hat{u}_--\mathbf{p}_s)+(\hat{e}+\hat{u}_+\hat{e}\hat{u}_-)](t,t), \tag{30}$$
$$\hat{e}=\sigma_z\nabla\hat{\varphi}+i[\mathbf{p}_s,\hat{\varphi}];$$

$$\delta N(t)=-\nu_F\pi\cdot\frac{1}{2}\mathrm{Tr}\,\sigma_z(\hat{u}_+\hat{\varphi}+\hat{\varphi}\hat{u}_-)(t,t)-\nu_F\Phi. \tag{31}$$

Here, to calculate the current (30) it is necessary to find the first anisotropic correction to the matrix γ_0 (25).

In the integro-differential equation for the function $\hat{\varphi}(t_1, t_2)$, which is what Eq. (28) is, it is not difficult to discern the presence of the differential operator $\partial/\partial t_1 + \partial/\partial t_2$; thus, for this function, unlike the reduced "propagation functions" $\hat{u}_\pm(t_1, t_2)$, the Cauchy problem should be formulated (i.e., initial conditions, e.g., on the line $t_0 = (t_1 + t_2)/2$, should be specified). In the mixed representation

$$\hat{\varphi}(t_1,t_2)=\int\frac{d\omega}{2\pi}\exp[-i\omega(t_1-t_2)]\hat{\varphi}\left(\omega,\frac{t_1+t_2}{2}\right) \tag{32}$$

the function $\hat{\varphi}(\omega, t)$ clearly plays the role of the nonequilibrium distribution function of the excitations.

Despite the outwardly cumbersome form of Eqs. (26)—(28), their structure is transparent. We note that, as in Eq. (6), they conserve the general property of invariance under gauge transformations on introduction of a phase χ into the order parameter Δ. One can convince

oneself of this by direct calculations, e.g., by eliminating the potential Φ in Eqs. (26)—(28) and introducing it into the phase of the parameter Δ:

$$\sigma_x\Delta\to\hat{\Delta}=e^{i\sigma_z\alpha}\sigma_x\Delta e^{-i\sigma_z\alpha},\quad \partial\alpha/\partial t=\Phi,$$
$$\mathbf{p}_s\to\mathbf{a}=\mathbf{p}_s-\nabla\alpha,\quad \frac{\partial\mathbf{a}}{\partial t}=e\mathbf{E}=\frac{\partial\mathbf{p}_s}{\partial t}-\nabla\Phi, \tag{33}$$
$$\hat{u}\to e^{i\sigma_z\alpha}\hat{u}e^{-i\sigma_z\alpha},\quad \hat{\varphi}\to e^{i\sigma_z\alpha}\hat{\varphi}e^{-i\sigma_z\alpha}.$$

In the normal state, in this case, $\hat{\Delta}=0$ and $\hat{u}=1$, and Eq. (28) (in the representation (32)) goes over essentially into Eq. (3)[4] describing the diffusion of normal excitations in coordinate space and energy space in the presence of an electric field (after relaxation at the impurities). In the superconducting state Eqs. (26)—(28) are a system of kinetic equations for the unified description of the dynamics of the excitations and of the superconducting condensate.

Closely related to the above-mentioned gauge-invariance property is the fact that the conservation of the number of particles is an exact consequence of Eqs. (26)—(31):

$$e\frac{\partial}{\partial t}\delta N+\nabla\mathbf{j}=-\pi e\nu_F\Delta\,\mathrm{Tr}\,\sigma_y(\hat{u}_+\hat{\varphi}+\hat{\varphi}\hat{u}_-)(t,t). \tag{34}$$

According to this formula (34), in the general nonequilibrium case the continuity equation is contained in the equation for the phase of the complex order parameter (cf. [4]).

2. SOLUTION OF THE KINETIC EQUATIONS

Having it in mind in this paper to treat principally the theoretical aspects of the problem of the resistive current states, we shall make a number of simplifying assumptions and shall assume, first of all, that it is possible to separate the problem of the heat transfer from the dynamics of the condensate and of the excitations. In practice, this means that the temperature should be close enough to the critical temperature for the Joule heating to be small and for the energy distribution of the excitations to be an equilibrium distribution in the first approximation. According to [1], the critical field E_{c2} near T_c is given in order of magnitude by $eE_{c2}/T_c \sim\eta^2(T_c/D)^{1/2}$, where $\eta=((T_c-T)/T_c)^{1/2}$. Substituting this estimate into Eq. (3) and taking into account the characteristic orders of the energy-relaxation frequencies $1/\tau_2\sim 10^8\ \mathrm{sec}^{-1}$, we find that if $\eta<10^{-1}$ the term quadratic in the electric field in this equation can be neglected. In these conditions the solution of Eq. (28) can be sought in the form $\hat{\varphi}=\varphi^{(0)}+\hat{\varphi}'$, where $\hat{\varphi}'$ is a small nonequilibrium correction to the distribution function $\varphi^{(0)}$ (14).

With the aim of further simplifications, in this paper we shall consider stationary current states. In the first approximation this assumption corresponds to the experimental situation that obtains in the measurement of static volt-ampere characteristics. Thus, in Eqs. (26)—(28) we can assume that all quantities are uniform in time, and it is necessary only to take into account the above-mentioned features of the passage to the static limit in the electrical-neutrality equation $\delta N=0$. For this it is sufficient to assume that the potential Φ varies infinitely slowly in time, and to retain the time-differentiation in the equation for the nonequilibrium contribution $\hat{\varphi}'$ to the distribution function (cf. formula (4)).

We first note that the phase transformations (33), as can be seen from formula (32), displace the energy distribution function by an amount $\sigma_z\Phi$. Since the potential Φ describes a nonequilibrium situation with an electric field weakly perturbing the equilibrium state, the unperturbed distribution function has the form (14) only in the representation (33). In the representation in which Eqs. (26)—(28) are written, the unperturbed distribution is displaced by $\sigma_z\Phi$, and therefore it is necessary to substitute into these equations a distribution function of the form

$$\hat{\varphi}(\omega,\mathbf{r})=\varphi^{(0)}(\omega-\sigma_z\Phi(\mathbf{r}))+\hat{\varphi}'(\omega,\mathbf{r}). \tag{35}$$

First we shall calculate the contributions of the unperturbed distribution function to Eq. (29) for Δ and to the reality condition for the parameter Δ (cf. (34)). The corresponding frequency integrals

$$\int\frac{d\omega}{2}\cdot\frac{1}{2}\,\mathrm{Tr}\,\sigma_x[\hat{u}_+(\omega)\varphi^{(0)}(\omega-\sigma_z\Phi)+\varphi^{(0)}(\omega-\sigma_z\Phi)\hat{u}_-(\omega)], \tag{36}$$
$$\int\frac{d\omega}{2}\cdot\frac{1}{2}\,\mathrm{Tr}\,\sigma_y[\hat{u}_+(\omega)\varphi^{(0)}(\omega-\sigma_z\Phi)+\varphi^{(0)}(\omega-\sigma_z\Phi)\hat{u}_-(\omega)] \tag{37}$$

are calculated using the expansion of $\tanh(\omega/2T)$ in partial fractions:

$$\frac{1}{2}\left(1+\mathrm{th}\frac{\omega}{2T}\right)=T\sum_{\omega_n}\frac{\exp(-i\omega_n 0)}{\omega-i\omega_n},\qquad \omega_n=\pi T(2n+1),\quad n=0,\pm1,\pm2,\ldots \tag{38}$$

and taking the analytic properties of the functions $\hat{u}_\pm(\omega)$ into account.

In the stationary case, Eqs. (26) and (27) are algebraic equations for the quantities $\hat{u}_\pm(\omega)$ and their gradients ($\hat{\omega}^{(0)}\to\omega$). Since the quantities Δ, $\mathbf{p}_s$ and the gradients are small near T_c (the potential Φ drops out of these equations), we obtain, expanding (26) and (27) in reciprocal powers of the frequency $\omega\sim T_c$,

$$\hat{u}=u+\sigma_x v+\sigma_y w,\quad u^2-v^2-w^2=1,\quad u\approx 1+{}^1/_2(\Delta/\omega)^2,$$
$$v\approx\frac{\Delta}{\omega}+\frac{1}{2}\left(\frac{\Delta}{\omega}\right)^3+\frac{i_s D}{2\omega^2}(\nabla^2-4p_s^2)\Delta,\quad w\approx-\frac{i_s D}{\omega^2}(\Delta\nabla\mathbf{p}_s+2\mathbf{p}_s\nabla\Delta). \tag{39}$$

Substituting (38) and (39) into (36) and (37), we find that in the first nonvanishing approximation the integrals (36) and (37) are respectively equal to

$$\pi T\sum_{\omega_n}\left[\frac{\Delta}{|\omega_n|}-\frac{(\Delta^2+2\Phi^2)\Delta}{2|\omega_n|^3}+\frac{1}{2\omega_n^2}D(\nabla^2-4p_s^2)\Delta\right], \tag{40}$$
$$\frac{\pi}{4T_c}[\Delta\Phi-D(\Delta\nabla\mathbf{p}_s+2\mathbf{p}_s\nabla\Delta)]. \tag{41}$$

The contribution of the unperturbed distribution function to the current (30) is calculated analogously. In this case, "nonanalytic" terms $\sim\hat{u}_+\hat{u}_-$ appear in the last term, proportional to the electric field, in formula (30). Confining ourselves in this term to the first nonvanishing approximation in the parameter $\eta=((T_c-T)/T_c)^{1/2}$, we obtain the expression

$$e\nu_F D\left(\frac{\pi}{2T_c}\Delta^2\mathbf{p}_s-\nabla\Phi\right). \tag{42}$$

As regards the contribution of the unperturbed distribution function to the electron-density change (31), it is proportional to $\nu_F(\Delta/T_c)^2\Phi$ and is small compared with the contribution of the nonequilibrium correction $\hat{\varphi}'$ to the distribution function (cf. formulas (4) and (5)).

The matrix equation (28) for the distribution function $\hat{\varphi}$ formally contains (in correspondence with the expansion in the Pauli matrices $\hat{1}$, σ_x, σ_y, σ_z) four equations for the two-component function $\hat{\varphi}=\varphi+\sigma_z\varphi_z$. However, since Eq. (28) is invariant under the operation of multiplication by $\hat{u}_+\sigma_z$ on the left and $\sigma_z u_-$ on the right, the corresponding system turns out to be degenerate and we can confine ourselves to just two equations from this system. A special role is played by the "projections" of

Eq. (28) on the matrices $\hat{1}$ and σ_z. A straightforward analysis shows that the "projection" of Eq. (28) (in the form in which it is written) on the matrix σ_z is effectively the detailed-balance equation for the energy, while the "projection" on the unit matrix contains the detailed-balance equation for the energy, while the "projection" on the unit matrix contains the detailed-balance of the particle number. In view of the basic assumption that the heat transfer is "ideal," we can neglect the first equation and the corresponding corrections to the function $\varphi^{(0)}$ and consider only the second equation[5]. Bearing these remarks in mind and substituting the expansion (35) into Eq. (28) with $\hat{\varphi}' = \sigma_z\varphi_z$, we obtain

$$(u_+ + u_-)\frac{\partial\varphi_z}{\partial t} - 2\Delta\frac{v_+ - v_-}{i}\varphi_z - D\nabla((1+u_+u_- + v_+v_- + w_+w_-)\nabla\varphi_z) = -\frac{d\varphi^{(0)}}{\partial\omega}\left[2\Delta\frac{v_+ - v_-}{i}\Phi + D\nabla((1+u_+u_- + v_+v_- + w_+w_-)\nabla\Phi)\right]. \quad (43)$$

We shall carry out the subsequent calculations for the simplest case of current values of the order of the critical current j_c. Unlike the current j_{c2}, which is of the order of $\eta^2 = (T_c - T)/T_c$ [1], the critical current $j_c \sim \eta^3$. From formulas (29), (40) and (42) it can be seen that in this case Δ, $p_s \sim \eta$. We make the assumption, justified below, that, as in the vicinity of j_{c2} [1], $\Phi \sim \eta$. Then, according to formula (42), the gradients should be of order η^2, in contrast to the situation near j_{c2}, where the characteristic distances have the "natural" order $\xi(T) \sim \sqrt{\xi_0 l}/\eta$.

The estimates presented make it possible to establish that the nonequilibrium correction φ_z to the distribution function makes a small contribution to Eq. (29) for Δ, and in the latter it is sufficient to confine ourselves to the expression (40) for the integral (36), thus omitting the small gradients. Hence, carrying out well-known transformations [11], we obtain

$$\left[\frac{7}{8}\zeta(3)\frac{\Delta^2 + 2\Phi^2}{(\pi T_c)^2} + \frac{\pi}{2T_c}Dp_s^2 - \frac{T_c - T}{T_c}\right]\Delta = 0. \quad (44)$$

As can be seen from Eq. (43), for $\nabla^2\Phi \neq 0$ the characteristic frequencies in the function $\varphi_z(\omega)$ are of the order of T_c. Therefore, in the principal approximation the contribution of this correction to the expression (30) for the current reduces to the single term $e\nu_F D\int d\omega\nabla\varphi_z$. Combining this term with the expression (42), we have

$$j = e\nu_F D\left(\frac{\pi}{2T_c}\Delta^2 p_s - \nabla\Phi + \int d\omega\,\nabla\varphi_z\right) \quad (45)$$

and, analogously, for the electron-density change (31)

$$\delta N = -\nu_F\int d\omega\,\varphi_z. \quad (46)$$

In this same approximation it is sufficient to confine ourselves to the following terms in Eq. (43):

$$\left(\frac{\partial}{\partial t} - \Delta\frac{v_+ - v_-}{i} - D\nabla^2\right)\varphi_z = -\frac{\partial\varphi^{(0)}}{\partial\omega}\left(\Delta\frac{v_+ - v_-}{i} + D\nabla^2\right)\Phi. \quad (47)$$

Combining the expression (41) with the contribution of the nonequilibrium correction φ_z to the reality condition for the parameter Δ (cf. (34) and (37)), we obtain

$$\frac{\pi}{2T_c}[\Delta\Phi - D(\Delta\nabla p_s + 2p_s\nabla\Delta)] + \int d\omega\frac{v_+ - v_-}{i}\varphi_z = 0. \quad (48)$$

If we now require the electrical-neutrality condition $\delta N = 0$ (46) to be fulfilled, it follows from (45) that

$$j = e\nu_F D\left(\frac{\pi}{2T_c}\Delta^2 p_s - \nabla\Phi\right). \quad (49)$$

On the other hand, integrating over the frequency in Eq. (47) gives, when the electrical-neutrality condition is taken into account,

$$\Delta\int d\omega\frac{v_+ - v_-}{i}\varphi_z = D\nabla^2\Phi - \frac{\pi}{2T_c}\Delta^2\Phi.$$

By substituting this expression into the reality condition (48) for the parameter Δ it is not difficult to see that this condition expresses, in fact, the continuity of the total current (49): $\nabla\cdot j = 0$.

Thus, in the case under consideration the required macroscopic equations are the simple local relations (44) and (49). The situation with the electrical-neutrality condition $\delta N = 0$ (46) is more complicated. Substituting into this the formal steady-state solution of Eq. (47) and calculating the difference $v_+ - v_-$ at high frequencies by means of the formulas (39), we obtain

$$\int d\omega\frac{\partial\varphi^{(0)}}{\partial\omega}\left(D\nabla^2 - \frac{4Dp_s^2\Delta^2}{\omega^2} - \frac{\partial}{\partial t}\right)^{-1}\left(D\nabla^2 - \frac{4Dp_s^2\Delta^2}{\omega^2}\right)\Phi = 0, \quad \partial\Phi/\partial t \to 0. \quad (50)$$

The latter equation confirms the assumption made above about the order of magnitude of the gradients: $\nabla \sim \eta^2$. Unfortunately, for $\Delta \neq 0$ Eq. (50) is so complicated that it is necessary to approximate it qualitatively by some simpler equation. The simplest such approximation is the equation

$$\nabla^2\Phi - a^2\Phi = 0, \quad (51)$$

where a is a certain constant, being a functional of the quantities p_s and Δ having the order of magnitude $a \sim p_s\Delta/T_c$.

3. STRUCTURE OF THE RESISTIVE STATE AND THE VOLT-AMPERE CHARACTERISTICS

Equating the current (49) to the specified external current, we write Eqs. (44), (49) and (51) in the one-dimensional case in dimensionless variables:

$$\Delta \to \Delta_0\Delta, \quad \Phi \to \frac{\Delta_0}{\gamma 2}\Phi, \quad p_s \to \sqrt{\frac{2(T_c - T)}{\pi D}}\,p_s,$$
$$x \to \frac{T_c}{\Delta_0}\sqrt{\frac{D}{\pi(T_c - T)}}\,x, \quad j \to \frac{e\nu_F\Delta_0^2}{T_c}\sqrt{\frac{\pi D(T_c - T)}{2}}, \quad (52)$$

where

$$\Delta_0 = \pi T_c\sqrt{\frac{8(T_c - T)}{7\zeta(3)T_c}}.$$

We have ($\Delta \neq 0$)

$$\Delta^2 + p_s^2 + \Phi^2 = 1, \quad j = \Delta^2 p_s - d\Phi/dx, \quad (53)$$

$$d^2\Phi/dx^2 - \lambda^2\Phi = 0 \quad (\lambda \sim 1). \quad (54)$$

We emphasize that the two equations (53), unlike the approximating equation (54), are asymptotically exact. Important consequences stem from them. The first of these equations, together with the formula $j_s = \Delta^2 p_s$ for the superconducting current, determines a family of "pair-breaking" curves, depending on the local value of the potential Φ (Fig. 1). The maximum value of the superconducting current equals $j_c = 2/3\sqrt{3}$. From the second equation (53) it can now be seen that for $j > j_c$ the potential Φ should increase without limit along the channel, and this leads to the "suppression" of the order parameter Δ in accordance with the first equation (53). Thus, to describe a resistive current state with nonzero (on the average) superconducting order along the channel, it is necessary to seek singular solutions of Eqs. (53) and (54), containing discontinuities of the potential Φ at the points at which $\Delta = 0$ [1] (but with conservation of the continuity of the electric field). These discontinuities result from discontinuities in the phase of the complex order parameter and are related to the vortex singularities in the theory of the mixed state of type-II super-

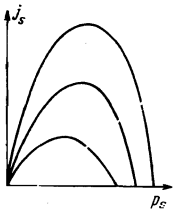

FIG. 1

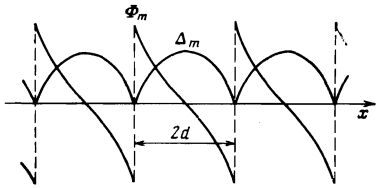

FIG. 2

conductors [15]6). As a whole, the structure of the resistive state is a microscopic phase separation with superconducting and normal regions alternating along the channel [1,16] (see Fig. 2, in which the behavior of the quantities Φ and Δ is qualitatively illustrated). It should be noted that, unlike the layered structure itself, the discontinuities of Φ are a purely mathematical construction and, like the analogous vortex singularities, should not be observed experimentally.

The integration of Eqs. (53) and (54) is elementary to perform. The corresponding results, of course, have only a qualitative character. It is sufficient to consider one period of the structure, and, taking the symmetry into account, to consider the interval from the point $\Delta = 0$ to the point Δ_m within the period (see Fig. 2). It follows from Eqs. (53) and (54) that

$$E=-d\Phi/dx, \quad E^2=E_0^2+\lambda^2\Phi^2, \quad j=\Delta_m^2\sqrt{1-\Delta_m^2}+E_0. \tag{55}$$

On the other hand, denoting the maximum value of the potential at the point $\Delta = 0$ by Φ_m, we obtain from (53) and (55)

$$j^2=E_0^2+\lambda^2\Phi_m^2. \tag{56}$$

Using (55) and (56) we determine the period 2d of the structure:

$$d=\int_0^{\Phi_m} d\Phi\,(j^2-\lambda^2\Phi_m^2+\lambda^2\Phi^2)^{-1/2}=\frac{1}{2\lambda}\ln\frac{j+\lambda\Phi_m}{j-\lambda\Phi_m}. \tag{57}$$

From this a formula is obtained for the potential drop per unit length of the channel:

$$V=\frac{\Phi_m}{d}=2\lambda\Phi_m\Big/\ln\frac{j+\lambda\Phi_m}{j-\lambda\Phi_m}. \tag{58}$$

The value of the parameter Δ_m (55) could be chosen from considerations of minimum entropy production, putting $\Delta_m = \sqrt{2/3}$. Then, according to (55) and (56), we have

$$(\lambda\Phi_m)^2=j^2-(j-j_c)^2. \tag{59}$$

However, since the functional $\lambda(\Delta, p_s)$ remains unknown, it is simpler to make immediate use of the fact that, according to the first equation (53), the quantity Φ_m is bounded ($\Phi_m < 1$) and, thus, the product $\lambda\Phi_m$ obeys Eq. (59) in only a limited range of values of the current j, and varies insignificantly in this range. Replacing $\lambda\Phi_m$ by a certain effective constant $\lambda\Phi_m = j_c'$, we obtain from (58) the approximate volt-ampere characteristics (in the usual units):

$$j=j_c'\,\mathrm{cth}\,(R_n j_c'/V), \tag{60}$$

where R_n is the normal resistance of the channel. We note that with this approximation the period (57) of the structure decreases at high currents like $d \sim 1/j$. In order of magnitude the period equals $(\xi_0 l)^{1/2}T_c/(T_c - T)$ $(10^{-2}(1/\xi_0)^{1/2}$ cm for $(T_c - T)/T_c \sim 10^{-2}$), and for $j \to j_c'$ we shall have $d \to \infty$.

Figure 3 presents a comparison of experimental volt-

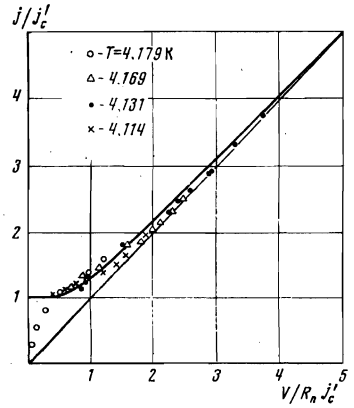

FIG. 3

ampere characteristics with formula (60). The greatest discrepancy occurs in the immediate vicinity of the current j_c', where a smooth variation of the differential resistance dV/dj is observed instead of the theoretical horizontal slope. It is precisely in this region that generation of electromagnetic oscillations in films is observed experimentally [17]. The origin of the oscillations can be discerned in the instability at sufficiently low currents of the solutions considered here. In fact, as can be seen from Eqs. (53), at currents close to j_c, to construct a solution $\Delta(x)$ that decreases in the region of the point $\Delta = \Delta_m$ on increase of the electric field, one uses unstable (decreasing) portions on the "pair-breaking" curves (Fig. 1). Because of this an oscillatory regime should develop, so that formula (60) remains qualitatively valid only in the average and at larger values of the current. The corresponding dynamical problem is rather complicated and its treatment goes outside the scope of the present paper. We merely note that, if we estimate the characteristic frequencies corresponding to the period of the structure, $\partial/\partial t \sim Dv^2 \sim T_c((T_c - T)/T_c)^2$, we obtain reasonable, experimentally-observed [17] orders of magnitude (10^8 sec^{-1} for $(T_c - T)/T_c \sim 10^{-2}$).

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1) In [6] the resistive current states are described by the complex solutions of the equilibrium Ginzburg-Landau equation [7] for the absolute value (!) of the superconducting order parameter.

2) This distribution is analogous to the distribution of charges near a chemical source of current, where it is determined by the chemical affinity of the electrons. In the present case the role of this "affinity" is played by the energy of condensation of the electron pairs. We note that this situation differs fundamentally from the Josephson case, in which accumulation of electric charges occurs on the plates of a capacitor.

3) In view of the difference in the orders of the isotropic and anisotropic terms, the exact expansion is performed in the parameter $\sqrt{\tau T_c}$.

4) Eqs. (3) and (28) coincide for $\hat{\Delta} = 0$, to within the Pauli matrix σ_z, the appearance of which in Eq. (28) is due to the choice of the special "electron-hole" representation for the density matrix [4].

5) This approximation is justified by practical estimates of the orders of magnitude of the quantities in both equations.

6) With such a choice of singularity, in the initial equations (44), (49) and (50) all the terms are continuous. To avoid misunderstanding we note that the "true" electric potential φ (cf. formulas (12)) remains continuous. In this case, strictly speaking, in the vicinity of the points $\Delta = 0$ it is necessary to write the electric field in the form (33), in order that the singularities $\delta(x-x_0)$ in $\partial p_s/\partial t$ and $\Delta\Phi$ cancel each other. Owing to the multiplication by Δ these singularities in p_s give no contribution in the other cases.

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