Radio-frequency magnetic susceptibility and collective resonance of magnons in parallel pumping

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The response of parametrically excited magnons to a weak radio-frequency field parallel to the magnetization is investigated. The dispersion of the magnetic susceptibility in the 10^5–10^6 Hz frequency range is measured. Resonant absorption is observed at a certain frequency whose position depends on the experimental conditions, viz., supercriticality of the pumping field and magnitude of the applied constant magnetic field. The results are explained within the framework of the magnon collective oscillation model. Theoretical calculations that confirm this interpretation are presented.

1. INTRODUCTION

It was observed recently that the most important features of the beyond-threshold behavior of spin waves (magnons) under parallel pumping are determined by the efficiency of the interaction of the magnons with one another, in accordance with the law

$$\omega_k + \omega_{-k} = \omega_{k'} + \omega_{-k'} = \omega_p, \qquad (1)$$

where ω_p is the frequency of the pump field and $\omega_{\pm k}$ are the frequencies of the magnons with wave vectors $\pm\mathbf{k}$. The interaction (1) is the basis of the so-called S theory [1], with the aid of which it was possible to make noticeable progress in the study of parametric excitation of magnons and to obtain satisfactory agreement with experiment [2]. At the present time, the S theory can explain not only the level of excitation of the magnons, their distribution in **k** space, and the stationary nonlinear susceptibilities χ' and χ'', but also more subtle nonstationary characteristics of the system, such as self-oscillations of the magnetization and the response to a weak signal.

The nonstationary behavior of a parametric system of magnons can be conveniently described in terms of its elementary-excitation spectrum. These excitations (collective modes) are due to the interaction (1) between the different pairs of the magnons, and constitute "second sound" against the background of the stationary state of the system. The only important difference from ordinary second sound is that the amplitudes and phases of the magnon pairs take part simultaneously in these oscillations. The spectrum of the collective modes was obtained earlier [3] and a study was made of its stability to small perturbations. It was shown that an unstable situation corresponds to the regime of self-oscillations of the magnetizations, which are none other than unstable collective modes. This point of view has presently found experimental confirmation. [4]

In those cases when the collective modes are stable, they can be excited by an external field of corresponding frequency. The collective oscillations of the magnons are directly connected with oscillations of the longitudinal component of the magnetization M_z, so that to build up these oscillations it is necessary to use an alternating magnetic field parallel to M_z. Our first attempts at direct observation of this effect were not successful, apparently because of the small resonant susceptibility of the collective oscillations and the insufficient sensitivity of the employed detection method. More successful in this respect turned out to be another formulation of the experiment, in which the oscillations were excited by combination resonance between the pump and a weak microwave field that was close to it in frequency [5]. The calculations have confirmed that the weak-signal susceptibility should have resonant singularities at frequencies $\omega_p \pm \Omega_0$, where Ω_0 is the natural frequency of the zeroth (homogeneous) oscillation mode. A rather intense absorption peak was observed in experiment at the frequency $\omega_p + \Omega_0$[1)]. [5]

These results have confirmed indirectly the real existence of collective oscillations of parametrically excited magnons, and have made it possible to explain the main parameters of the oscillations, namely the resonant frequency and the damping. Unfortunately, the accuracy of these measurements was quite low, especially in cases when the Q of the oscillations was of the order of or less than the Q of the pump resonator.

In this paper we report results of experiments on direct excitation of collective oscillations of parametric magnons under the influence of a radio frequency (RF) field with frequency $\Omega \approx \Omega_0$. The measurements were performed in the frequency range 10^5—2×10^6 Hz with the aid of a NMR technique specially intended for the measurement of low radio-frequency susceptibilities. We measured the resonant frequency and the waveform and half-width of the resonance curves of the real (χ') and imaginary (χ'') parts of the magnetic susceptibility as functions of the pump power and of the external magnetic field.

2. EXPERIMENTAL SETUP

A block diagram of the setup is shown in Fig. 1. It includes a microwave unit for parallel pumping of the spin waves and a unit for the measurement of the radio-frequency magnetic susceptibility. The pump generator (magnetron) had a power rating ~15 W at 9400 MHz. The magnetron was modulated at 50 Hz by connecting it in series with a 650-V voltage source and the output resistance of a cathode follower, to the input of which were applied positive rectangular pulses of 500 μsec duration. A small spherical sample of single-crystal $Y_3Fe_5O_{12}$ (YIG) was placed at the center of the rectangular resonator (TE_{102} mode). Alongside the sample were small Helmholtz coils (measuring 2 × 3 mm) wound of copper wire of 0.06 mm diameter. The coil axis was oriented along the constant magnetic field. The coils were part of

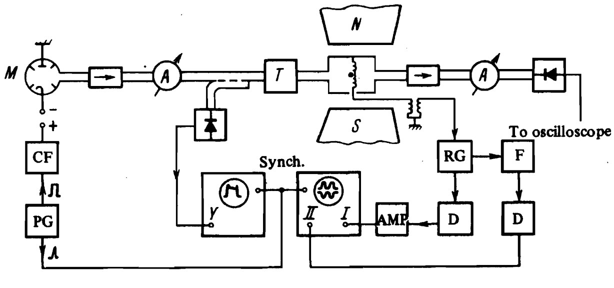

FIG. 1. Experimental setup: M—magnetron, CF—cathode follower, PG—rectangular-pulse generator, A—attenuator, T—impedance transformer, RG—radio-frequency generator, F—frequency meter, D—detector, AMP—tuned amplifier

the tank circuit of a radio-frequency oscillator, which was the standard E11-2 magnetic-induction measuring oscillator.

The measurements of the RF susceptibility are based on the NMR procedure, which uses the reaction of the weak-signal generator to the sample-induced perturbations of the active and reactive components of the tank-circuit impedance. The obtained signal was amplified with a tuned amplifier and fed to beam I of the oscilloscope. The oscillator frequency shift was monitored with a special frequency meter (heterodyne voltmeter), the output signal of which was fed to beam II of the oscilloscope.

To ensure a more or less uniform sensitivity of the system in a wide range of frequencies, we used transformer coupling between the measuring coil and the principal tank circuit of the RF oscillator. Tuning was effected by switching tank-circuit inductances in and out and by adjusting a variable capacitor.

For an independent measurement of the real and imaginary parts of the RF susceptibility, a variable standard resistor R_{st} and a standard capacitor C_{st} were introduced into the circuit and were connected in parallel to the measuring coil. R_{st} and C_{st} were connected into the circuit with the aid of a vibrator-inverter for a short time, approximately equal to the duration of the magnetron pulse, but with a certain relative shift. The responses of the RF generator to the sample and to the $R_{st} - C_{st}$ standard were then separated in time. Obviously, by suitable choice of R_{st} and C_{st} it is possible to produce an impedance equivalent to that of the sample, $z_{st} = z_{sample}$. A criterion for the satisfaction of this equality is equality of the amplitude and frequency of the oscillator when working successively in the two regimes.

The choice of R_{st} and C_{st} was effected in the following manner. C_{st} was first used to make the frequencies equal in both regimes, meaning equality of the signals at the frequency-meter output (beam II). Then, by varying R_{st}, the amplitudes of the oscillations in both regimes are made equal, meaning equality of the signals on beam I. Since the oscillation frequency depends to a certain degree on R_{st}, a slight unbalance in beam II takes place. It is therefore necessary to readjust C_{st}, and then again R_{st}, etc., until pairwise equality of the signals in beams I and II is reached. The values of R_{st} and C_{st} obtained as a result of this procedure make it possible to calculate the equivalent impedance introduced by the sample or, which is more convenient, the dimensionless values of the real (χ') and imaginary (χ'') parts of the magnetic susceptibility. The formulas for the connection between χ' and χ'' and the directly measured quantities take the following form:

$$\eta\chi''=\Omega(L/R_{st}+2C_{st}r), \tag{2}$$

$$\eta\chi'=\Omega^2LC_{st}-2r/R_{st}, \tag{3}$$

For L and r are the unperturbed values of the inductance and resistance of the measuring coil (in our case $L \cong 18\ \mu H$ and $r \cong 6\ \Omega$); η is the coil filling factor, is of the order of the ratio of the sample volume to the coil volume.

3. MEASUREMENT RESULTS

We investigated YIG single crystals having a low parametric-excitation threshold; the spin-wave relaxation frequency γ_k, obtained from the threshold field h_c, was $\gamma_{k\to 0} \approx 0.35$ MHz for k = 0. The principal measurements were made on a spherical sample of 1.6 mm diameter, magnetized in the direction of the $\langle 100\rangle$ axis. Measurements in other crystallographic directions were as a rule hindered by self-oscillations of the magnetizations, which reached quite large amplitudes in a number of cases (for example at $M \parallel \langle 111\rangle$ or $\langle 110\rangle$).

Excitation of collective oscillations can be easily observed immediately beyond threshold ($h > h_c$) in the form of a voltage pulse on the tank circuit of the oscillator during the time of the pump action. This pulse appears only at an appropriate tuning of the oscillator, which depends on the excess of the pump power over the threshold and on the value of the constant magnetic field. At a fixed weak-signal generator frequency, it is possible to tune to resonance by varying the pump power or the magnetic field.

Figure 2 shows plots of $\chi''(\Omega)$ and $\chi'(\Omega)$ at different excesses above threshold; the resonant character of the reaction of the system to the weak signal is clearly seen. A similar behavior is exhibited by the plots of χ'' and χ' against the pump power, which are therefore not presented here.

Figure 3 shows the dependence of the square of the resonant frequency on the pump power. The quantity Ω^2_{res}, as will be seen below, is proportional to the "number" of the parametric magnons, which in turn depends linearly on the supercriticality h^2/h_c^2.

The width of the resonance curves (Fig. 2), measured at the level $\chi'' = \chi''_{res}/2$, is equal to $2\Delta\Omega = (0.45 \pm 0.05)$ MHz, and is independent, within the limits of the indicated accuracy, of the pump power. $\Delta\Omega$ is equal to the relaxation frequency γ_k (see formulas (12) and (13)). This circumstance makes it possible to use the observed

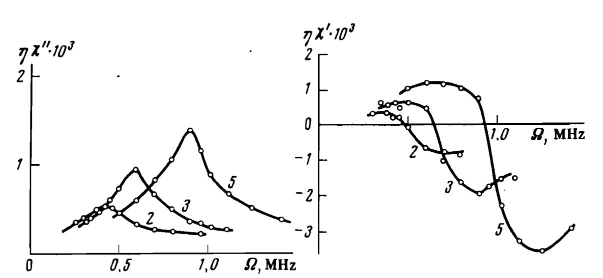

FIG. 2. Frequency dependences of the real (χ') and imaginary (χ'') parts of the susceptibility. The numbers at the curves represent the pump power in decibels above threshold, and η is the measuring-coil filling factor.

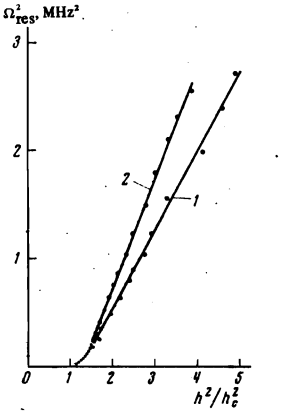

FIG. 3. Dependence of the resonant frequency on the supercriticality: 1) $H = H_c - 10$ Oe, 2) $H = H_c - 50$ Oe.

resonance as a new method for measuring the spin-wave relaxation frequency, namely by determining the width of their collective resonance. The accuracy with which γ_k is measured in this manner is much higher than when the threshold-field amplitude is measured, where the determination of the absolute value of γ_k calls for a determination of the pump power, of the coefficient of reflection from the resonator, and the resonator Q, and it is necessary generally speaking to take into account the influence of the adhesive, the sample holder, and other hard-to-control parameters.

4. THEORY OF COLLECTIVE RESONANCE

The procedure for calculating the reaction of parametrically excited magnons to a weak RF field consists in the following. We write down the equation of motion for the Fourier components of the magnetization in canonical form

$$\frac{\partial c_k}{\partial t} + \gamma_k c_k = -i \frac{\partial \mathscr{H}}{\partial c_k^*}. \quad (4)$$

In S theory [1], the Hamiltonian $\mathscr{H}$ of the system has a standard form

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_p + \mathscr{H}_{int}; \quad \mathscr{H}_0 = \sum_k \omega_k c_k c_k^*,$$

$$\mathscr{H}_p = \frac{1}{2} \sum_k (h_p V_k c_k^* c_{-k}^* + \text{c.c.}),$$

$$\mathscr{H}_{int} = \sum_{kk'} T_{kk'} c_k c_k^* c_{k'} c_{k'}^*$$

$$+ \frac{1}{2} \sum_{kk'} S_{kk'} c_k^* c_{-k}^* c_{k'} c_{-k'}, \quad (5)$$

where $\mathscr{H}_0$ is the kinetic energy of the magnons, $\mathscr{H}_p$ is the energy of interaction with the pump field, and $\mathscr{H}_{int}$ is the energy of the interaction of the pair of magnons.

If the system is acted upon, besides the pump $h_p = h \exp(-i\omega_p t)$, also by weak homogeneous alternating magnetic field $h_z = h_\Omega \exp(-i\Omega t)$ parallel to the magnetization, then it is necessary to take into account in the Hamiltonian (5) an additional term

$$\mathscr{H}_z = \frac{g}{2} \sum_k (h_z c_k c_k^* + \text{c.c.}), \quad (6)$$

corresponding to the Zeeman energy $-g \Delta M_z h_z$. Substituting (5) and (6) in (4) and averaging over all the directions of **k** (we are interested only in the homogeneous oscillation mode), we obtain

$$[d/dt + \gamma_k + i(\omega_k - {}^1\!/_2 \omega_p + 2T_0 cc^*)] c + i(hV_0 + S_0 c^2) c^* = -g \operatorname{Re}(h_z c). \quad (7)$$

Here $c = N^{1/2} e^{i\psi/2}$, $N \equiv \Sigma c_k c_k^*$ is the integral amplitude of the packet, while V_0, S_0, and T_0 are the Hamiltonian coefficients averaged over the packet (formulas for these coefficients are given in [3]).

We seek the solution of (7) in the form

$$c = c_0 + c_\Omega e^{i\Omega t} + c_{-\Omega} e^{-i\Omega t}, \quad |c_{\pm\Omega}| \ll c_0.$$

The stationary solution of (7) is well known, it corresponds to (see [1])

$$c_0 = N_0^{1/2} e^{i\psi_0/2}; \quad N_0 = [(hV_0)^2 - \gamma_k^2]^{1/2}/S_0, \quad (8)$$

$$\cos \psi_0 = -S_0 N_0 / hV_0; \quad \omega_k + 2T_0 c_0 c_0^* = \omega_p/2.$$

For the amplitudes $c_{\pm\Omega}$ of the induced oscillations we obtain from (7) and (8) the expressions

$$c_\Omega = -\frac{g}{2} h_\Omega \frac{(\Omega - i\gamma_k - SN_0) c_0 + h_p V_0 c_0^*}{\Omega^2 - \Omega_0^2 - 2i\gamma_k \Omega}, \quad (9)$$

$$c_{-\Omega} = \frac{g}{2} h_\Omega \frac{(\Omega + i\gamma_k - SN_0) c_0 - h_p V_0 c_0^*}{\Omega^2 - \Omega_0^2 + 2i\gamma_k \Omega};$$

$$\Omega_0 = 2[S_0(2T_0 + S_0)]^{1/2} N_0. \quad (10)$$

The RF-field energy absorbed by the sample per unit time is

$$P_\Omega = -\partial \mathscr{H}_z / \partial t = \Omega h_\Omega g \operatorname{Im}(c_0 c_\Omega^* + c_0^* c_{-\Omega}),$$

and the corresponding RF susceptibility $\chi_\Omega = \chi'_\Omega + i\chi''_\Omega$ is determined by the formula

$$\chi_\Omega = 2g(c_0 c_\Omega^* + c_0^* c_{-\Omega}) / h_\Omega. \quad (11)$$

Substituting here (9) and separating the real and imaginary parts, we obtain

$$\chi_\Omega'' = \frac{4\gamma_k \Omega g^2 S_0 N_0^2}{(\Omega^2 - \Omega_0^2)^2 + 4\gamma_k^2 \Omega^2}, \quad (12)$$

$$\chi_\Omega' = -\frac{2g^2(\Omega^2 - \Omega_0^2) S_0 N_0^2}{(\Omega^2 - \Omega_0^2)^2 + 4\gamma_k^2 \Omega^2}. \quad (13)$$

The maximum value of χ''_Ω corresponds to a resonant frequency Ω_{res} which, generally speaking, does not coincide with Ω_0 and is equal to

$$\Omega_{res}^2 = {}^1\!/_3 [\Omega_0^2 - 2\gamma_k^2 + 2(\Omega_0^4 - \gamma_k^2 \Omega_0^2 + \gamma_k^4)^{1/2}]. \quad (14)$$

With good accuracy we have

$$\Omega_{res}^2 \approx \Omega_0^2 - \gamma_k^2 \quad \text{and} \quad \Omega_0 \gg 2\gamma_k. \quad (15)$$

In this approximation, the half-width of the collective-resonance curve, measured at the level $\chi'' = \chi''_{res}/2$, is equal to the magnon relaxation frequency, $\Delta\Omega = \gamma_k$.

We present the values of the susceptibilities at resonance:

$$\chi''_{res} \approx \frac{g^2 (\Omega_0^2 - \gamma_k^2)^{1/2} S_0 N_0^2}{\gamma_k (\Omega_0^2 - {}^3\!/_4 \gamma_k^2)}. \quad (16)$$

At large values above critical, $\Omega^2 \gg \gamma_k^2$, formula (16) becomes simpler. Expanding the dependences of Ω_0 and N_0 on h^2/h_c^2 in accordance with (8) and (10), we obtain

$$\chi''_{res} = \frac{g^2}{2[S_0(2T_0 + S_0)]^{1/2}} \left(\frac{h^2}{h_c^2} - 1 \right)^{1/2}. \quad (17)$$

We find similarly from (13) that at $\Omega^2 \gg \gamma_k^2$ the real part of the susceptibility at resonance does not depend on the excess above critical value, and is equal to

$$\chi'_{res} = g^2 / 8(2T_0 + S_0). \quad (18)$$

5. DISCUSSION OF RESULTS

It follows from (8), (10), and (15) that the frequency of the homogeneous collective resonance of magnons is determined in the case of parallel pumping by the formula

$$\Omega_{res}^2=\gamma_k^2\frac{4(2T_0+S_0)}{S_0}\left(\frac{h^2}{h_c^2}-1\right)-\gamma_k^2, \quad (19)$$

where $h^2/h_c^2 \equiv (hV_0/\gamma_k)^2$ is the excess above critical value beyond threshold. The coefficients S_0 and T_0 in this formula were calculated earlier [3] as functions of the pump frequency ω_p and of the principal parameters of the sample, viz., the demagnetizing factor N_z, the magnetization, and the anisotropy constants. We emphasize that S_0 and T_0 do not depend on the wave number $|\mathbf{k}|$ and consequently on the external magnetic field H. The numerical values of S_0 and T_0 in the standard experimental situation (YIG sphere at room temperature, orientation $\mathbf{M} \parallel \langle 100\rangle$, frequency ω_p = 9.4 GHz) are equal to

$$S_0=2\pi g^2\cdot 0.52, \quad T_0=2\pi g^2\cdot 0.28, \quad T_0/S_0=0.54$$

(g is the magnetomechanical ratio). Using these values, we obtain from (19)

$$\Omega_{res}^2=8{,}3\gamma_k^2(h^2/h_c^2-1)-\gamma_k^2. \quad (20)$$

Thus, the square of the resonant frequency depends linearly on the excess above the critical value, and the slope of this dependence is determined only by the damping γ_k.

Let us compare formula (20) with the experimental data (Fig. 3). The experimental plots of $\Omega_{res}^2(h^2/h_c^2)$ indeed follow a linear law, with the exception of the region of lower frequencies $\Omega_{res}^2 < 0.2$ MHz², where the simple formula (15) is no longer applicable and it is necessary to use the exact expression (14). The slopes of lines 1 and 2 on Fig. 3 can be calculated with good accuracy from formula (20). Thus, for line 1 ($H = H_c - 10$ Oe) we have $2\gamma_k = 0.45$ MHz, which yields $d\Omega_{res}^2/d(h^2/h_c^2) = 8.3\gamma_k^2 = 0.42$ MHz², and the corresponding experimental value is 0.7 ± 0.1 MHz². With decreasing magnetic field, the wave number of the spin waves increases, and with them the damping γ_k. At $H = H_c - 50$ Oe (curve 2) we have $2\gamma_k = 0.6$ MHz and the slope of (20) is equal to 0.75 MHz², which agrees with the experimental value 1.0 ± 0.1 MHz² (Fig. 3).

Figure 4 shows the experimental and theoretical plots of Ω_{res} against the magnetic field at a fixed excess above critical value. To plot the curve, we used the known experimental $\gamma_k(H)$ dependence. The shapes of the curves in Fig. 4 are similar, indicating thus that the entire observed variation of Ω_{res} with changing H is due to the dependence of γ_k on H. In particular, it is confirmed that the coefficients S_0 and T_0 do not depend on the magnetic field.

We now calculate the resonant susceptibility χ''_{res}, using the values of S_0 and T_0 presented above. For example, for $h^2/h_c^2 = 2$ we obtain, according to (17), $\chi''_{res} \approx 0.1$; experiment (Fig. 2, curve 3) gives $\eta\chi''_{res} \approx (1 \pm 0.2) \times 10^{-3}$. The filling factor η is not known exactly, but an estimate based on the geometry of the coil with the sample yields $\eta \approx 0.01-0.02$, which explains the order of magnitude of χ''_{res}. As seen from Fig. 5, the theory accounts also for the dependence of the resonant susceptibility on the excess above critical value: in accord with (17), this dependence is linear. We emphasize that unlike the resonant frequency, the resonant susceptibility does not depend on the damping and consequently should not depend on the magnetic field. This is also seen from Fig. 5.

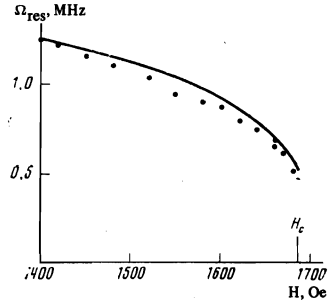

FIG. 4

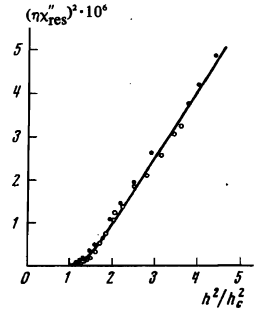

FIG. 5

FIG. 4. Field dependence of the collective-resonance frequency: points—experimental results, solid line—theoretical curve calculated from (19).

FIG. 5. Square of the resonant susceptibility vs. the excess above critical value: •—$H = H_c - 10$ Oe, ○—$H = H_c - 50$ Oe.

Thus, a simple theory that starts out directly from the Hamiltonian (5), without using any fitting parameters, provides a description that agrees with experiment for the RF resonance of magnons in parallel pumping. This increases the assurance that the physical concepts concerning the collective oscillations of magnons, used in the present paper and earlier [4,5], are correct. Parallel pumping in ferro- (or ferri-) magnets is not the only way of parametrically exciting magnons, and collective oscillations exist, of course, also in other situations, for example in nonlinear ferromagnetic resonance or in parametric excitation of antiferromagnets. Although oscillations where not yet observed experimentally in these cases, their properties can be predicted on the basis of the S theory. Of particular interest in this connection is the situation in antiferromagnets, which offers abundant possibilities, and where it is apparently possible to obtain higher magnon collective-oscillation frequencies than in the case of YIG.

[1)] Amplification of the weak microwave signal is possible, in principle, at the mirror frequency $\omega_p - \Omega_0$ (see also [4]).

1. V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **59**, 1200 (1970) [Sov. Phys.-JETP **32**, 656 (1971)].
2. V. V. Zautkin, V. E. Zakharov, V. S. L'vov, S. L. Musher, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **62**, 1782 (1972) [Sov. Phys.-JETP **35**, 926 (1972)].
3. V. S. L'vov, S. L. Musher, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **64**, 1074 (1973) [Sov. Phys.-JETP **37**, 546 (1973)].
4. V. V. Zautkin and S. S. Starobinets, Fiz. Tverd. Tela **16**, 678 (1974) [Sov. Phys.-Solid State **16**, 446 (1974)].
5. V. V. Zautkin, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. **63**, 182 (1972) [Sov. Phys.-JETP **36**, 96 (1973)].

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