Contribution to the theory of superluminescence in a dispersive medium with molecular relaxation

M. V. Bogdanova, T. M. Il'inova, and R. V. Khokhlov

Moscow State University
(Submitted June 16, 1974)
Zh. Eksp. Teor. Fiz. 68, 469–478 (February 1975)

Results are presented of a theoretical investigation of superluminescence in molecular media (organic dyes, CO_2 molecules). The calculations are based on a three-level model in the given pulsed-pump field approximation, and under the condition that the produced superluminescence does not influence the motion of the level populations. Account is taken of molecular relaxation processes. The influnece of group retardation effects on the formation of the superluminescence pulses is studied. It is shown that when dispersion and transverse relaxation act simultaneously, superluminescence pulses of stationary form with exponential gain are produced. The amplitude profiles of the stationary superluminescence pulses are investigated.

1. INTRODUCTION

The problem of obtaining ultrashort pulses at fixed frequencies has by now been technically solved. These pulses are obtained for its accomplishment, using either solid state lasers[1,2] and mode-locked dye lasers[3-5], or multistage amplifiers.[6-7]

Superluminescence of organic dyes pumped by powerful ultrashort pulse generators is a sufficiently effective method tuning radiation frequencies[8-10] and also serves as one of the mechanisms of shortening the duration of the pulse.[11] The results on the experimental realization of superluminescence in dyes have been discussed in the literature. "Front-back" asymmetry of the radiation has been observed for the longitudinal variant of pumping.[8,11] The spectra and the angular structure of superluminescence have been studied.[9,11] The possibility has been noted of a regime of stationary mode amplification.[12]

For correct interpretation of the experimental data, it is necessary to take into account not only processes of energy relaxation for the level populations (the longitudinal T_1 and the vibrational T_V relaxation times), but also phenomena connected with the presence of a "phase memory" of the system (the transverse relaxation time T_2) If the length of the cell exceeds the group length, then account of the effects of group retardation of the waves is also necessary. Such a consideration can lead to useful information both on the properties of the superluminescence radiation and on the internal characteristics of the material (for example, it can serve as a method of measurement of short relaxation times).

In the present work, results are given of a theoretical study of superluminescence in a three-level medium with account of the factors enumerated above. The analysis can be used for study of superluminescence in dyes and in other molecular media (for example, CO_2 molecules). The basic results of the work were reported at the VII All-Union Conference on Coherent and Nonlinear Optics.[13]

2. FUNDAMENTAL EQUATIONS

The study of superluminescence in organic dyes is carried out on the basis of a simplified three-level energy model.[14] Level 1 belongs to the ground singlet electron state S_0, levels 2 and 3 are vibrational sublevels of the electronic state S_1.

This model allows us to take into account processes of vibrational relation, which becomes necessary in the regime of ultrashort pulses, the length of which is comparable with the vibrational relaxation time T_V. In dyes, $T_V \sim 10^{-11}$–10^{-12} sec.

We shall describe the three-level medium by the quantum mechanical equations for the density matrix with longitudinal and transverse relaxations.[14] The field, which is a superposition of the pumping wave of frequency $\omega_p = \omega_{31}$, and the superluminescence of frequency $\omega_c = \omega_{21}$ we shall assume to be classical and to satisfy the wave equation. The study of the contracted equations, obtained by the method of slowly varying amplitudes, is carried out in the approximation of a given pumping field for which the pumping amplitude $E_p(z, t)$ is a function only of the running time $\tau = t - z/u_p$ (u_p is the pump group velocity).

We shall also assume that the arising superluminescence does not affect the level population motion. Further, we neglect nonstationary effects connected with the finiteness of the time T_2^p of damping of the polarization at the pumping frequency, i.e., we set $T_2^p \ll \tau_p$ (τ_p is the pump pulse length).

Under the assumptions made, the differences in the level populations are functions of the running time τ: $N_{21}(z, t) = N_{21}(\tau)$, $N_{31}(z, t) = N_{31}(\tau)$. The forms of these pulses are found when solving the equations given in[12]. The superluminescence amplitude $E_c(z, t)$ and the polarization $P_c(z, t)$ at the frequency $\omega_c = \omega_{21}$ satisfy the following equations:

$$\frac{1}{u_c}\frac{\partial E_c}{\partial t} \pm \frac{\partial E_c}{\partial z} = -i\frac{2\pi\omega_c}{u_c}P_c, \qquad (1)$$

$$\frac{\partial P_c}{\partial t} + \frac{P_c}{T_2^c} = \frac{i}{\hbar}d_c^2 N_{21}(\tau)E_c + \frac{P_{\text{noise}}}{T_2^c}, \qquad (2)$$

where T_2^c is the polarization damping time at the frequency of superluminescence, and d_c^2 is the square of the modulus of the dipole matrix element. The sign (+) corresponds to superluminescence which propagates together with the pump in the direction of the z axis ("forward" superluminescence), the sign (−) pertains to superluminescence in the opposite direction ("back" superluminescence).

To take account of the spontaneous luminescence, random noise polarization $P_{noise}(z, \tau)$ has been introduced in the right side of Eq. (2). If the sources of spontaneous noise have dimensions much less than the wavelength, and are uniformly distributed along the cell, then the noise polarization δ is correlated with respect to z. One can also assume it to be δ-correlated with respect to τ if the pump duration $\tau_p \gg T_3^p$.

Then

$$\langle P_{noise}(z', \tau') P_{noise}(z, \tau)\rangle = g n_2(\tau)\delta(z'-z)\delta(\tau'-\tau), \qquad (3)$$

where g = const, and $n_2(\tau)$ is the population of the upper laser level.

Equations (1) and (2) with the noise polarizations (3) will be studied for a rectangular pumping pulse:

$$E_p(\tau) = \begin{cases} E_{p0}, & |\tau| \leq \tau_p/2, \\ 0, & |\tau| \geq \tau_p/2. \end{cases} \qquad (4)$$

Here, we shall take into consideration two mechanisms of nonstationarity: the nonstationarity associated with the finite damping time of the polarization T_2^c at the frequency $\omega_c = \omega_{21}$, and the "wave" nonstationarity, which arises in the presence of mismatch between the group velocities of the pumping waves and the superluminescence.

3. PROPERTIES OF SUPERLUMINESCENCE PULSES IN A MEDIUM WITH SHORT RELAXATION TIMES

If the duration τ_p of the pumping (4) significantly exceeds the times T_V of vibrational and T_1 of radiation relaxations, then the difference in the populations of the laser levels $N_{21}(\tau)$ duplicates the shape of the pumping pulse:[12]

$$N_{21}(\tau) = \begin{cases} -|N_{21}^0|, & |\tau| \geq \tau_p/2, \\ |N_{31}^0|\beta - |N_{21}^0|, & |\tau| \leq \tau_p/2, \end{cases} \qquad (5)$$

where $\beta = 2(1 + \hbar^2/2d_p^2 T_1 T_2^p E_{p0}^2)^{-1}$.

In this case, we can obtain more complete information from Eqs. (1) and (2) on the properties of the superluminescence pulse in a molecular medium.

3.1. Effects of Group Retardation

We first neglect the nonstationary phenomena connected with the finite time T_2^c, assuming $T_2^c \ll \tau_c$ (τ_c is the characteristic time for the change of the amplitude of the superluminescence). In this approximation it is easy to obtain a graphic picture of the formation of the superluminescence pulse in a dispersive medium. The effects of group retardation of the pump waves and superluminescence become important in cells of length $l > l_{v\mp} = \tau/|v_\mp|$. Here $v_\mp = 1/u_p \mp 1/u_c$ is the mismatch of the group velocities.

Solving the set of equations (1) and (2), we find that the averaged intensity of the superluminescence for δ-correlated noise polarization (3) is of the form

$$I_c^{(\mp)} = g\left(\frac{2\pi\omega_c}{u_c}\right)^2 \left\{ \int_{z_0}^{z} n_2(\eta_\mp - z'v_\mp) \exp\left(-2\sigma_c \int_{z_0}^{z'} N_{21}(\eta_\mp - z''v_\mp)\,dz''\right) dz' \right\}$$
$$\times \exp\left(2\sigma_c \int_{z_0}^{z} N_{21}(\eta_\mp - z'v_\mp)\,dz'\right), \qquad (6)$$

where $\eta_\pm = t \pm z/u_c$ is the traveling time of the superluminescence pulses, and $\sigma_c = 2\pi\omega_c d_c^2 T_2^c/\hbar u_c$ is the absorption cross section at the frequency of the superluminescence. The signs (−) and (+) refer respectively to the "forward" and "back" superluminescence intensity. The solution (6) is obtained at the zero boundary condition $I_c^{(\mp)}(z_0, \eta_\mp) = 0$, where z_0 is determined from physical considerations.

The shapes of the superluminescence pulses in the case of normal dispersion are shown in Fig. 1. As the pulse propagates in the medium, a broadening of the pulse occurs: the points of the leading edges $x_i^- = -\frac{1}{2} - z_i/l_{\nu^-}$ ("forward" superluminescence) and $x_i^+ = \frac{1}{2} + (l - z_i)/l_{\nu^+}$ ("back" superluminescence) travel with the velocity $u_c > u_p$, while the wavefronts travel with velocity u_p in the pump direction. Cessation of the amplification takes place over the group lengths (Fig. 2, c). In the case considered, the asymmetry of the superluminescence radiation with respect to energy is clearly manifest. Depending on the length of the cell, the ratio of the energies of the "forward" and "back" radiation is determined by the formulas

$$\gamma = \{|\nu_-|[4[\exp(2\Gamma_0 l_{\nu-}) - 1] - [\exp(4\Gamma_0 l_{\nu-}) - 1] \times \exp(-2\Gamma_0 l)] - 4\Gamma_0\tau_p\}\{\nu_+[4[\exp(2\Gamma_0 l_{\nu+}) - 1] - [\exp(4\Gamma_0 l_{\nu+}) - 1]\exp(-2\Gamma_0 l)] - 4\Gamma_0\tau_p\}^{-1},$$
$$l > l_{\nu+},\ l_{\nu-},$$

$$\gamma = 4\Gamma_0\tau_p[\exp(2\Gamma_0 l) - 1]\{\nu_+[4[\exp(2\Gamma_0 l_{\nu+}) - 1] - [\exp(4\Gamma_0 l_{\nu+}) - 1]\exp(-2\Gamma_0 l)] - 4\Gamma_0\tau_p\}^{-1},$$
$$l_{\nu+} < l < l_{\nu-}. \qquad (7)$$

Here $\Gamma_0 = n_0\sigma_c$ is the stationary amplification coefficient in the case of unmodulated pumping.

For very small cells of length $l < l_{\nu^-}, l_{\nu^+}$, group effects do not develop and the "front-back" asymmetry disappears ($\gamma = 1$).

We note that at large amplification ($\Gamma_0 l_{\nu\mp} \gg 1$) the Eqs. (7) simplify and take the form

$$\gamma \approx \begin{cases} (l_{\nu+}/l_{\nu-})\exp[2\Gamma_0(l_{\nu-} - l_{\nu+})], & l > l_{\nu+}, l_{\nu-}, \qquad (8) \\ \Gamma_0 l_{\nu+}\exp[2\Gamma_0(l - l_{\nu+})], & l_{\nu+} < l < l_{\nu-}. \qquad (9) \end{cases}$$

Equation (8) coincides with the asymmetry coefficient for stimulated Raman scattering (SRS) given in[15].

The value of the asymmetry coefficient γ, calculated from (9) for the corresponding values of the parameters, agrees with the experimental data.[11]

Simultaneous account of the mismatches of the group velocities and of processes of molecular relaxation of the medium introduces significant difficulties in the solution of the set of equations (1) and (2). Therefore, before proceeding to an investigation of the joint effects of both nonstationarity mechanisms, we consider the effect of a finite time T_2^c: on the formation of the "forward" superluminescence pulses in the case in which group effects can be neglected.

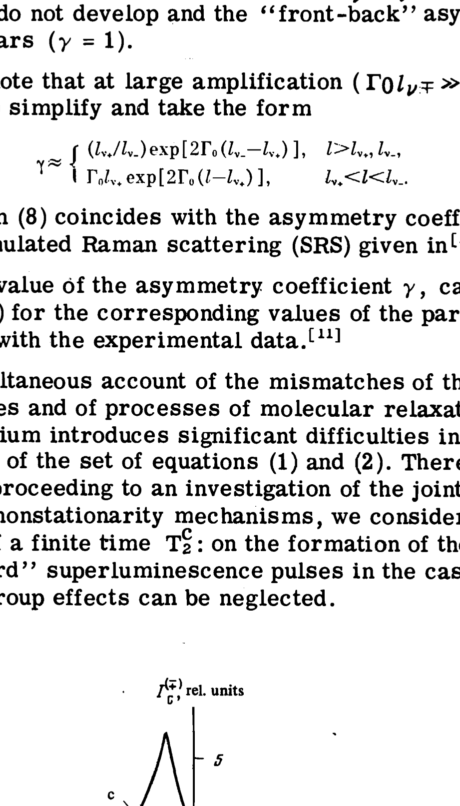


FIG. 1. Pulses of "forward" superluminescence ($\nu_- > 0$, intensity reduced by a factor of 200; $\Gamma_0 = 1\ cm^{-1}$, $l/l_{\nu-} = 2$, $l/l_{\nu+} = 40$, $x = \tau/\tau_p$): a) $z < l_{\nu-}$; b) $l_{\nu-} < z < l$; c) $z = l$; and "backward" superluminescence; d) $z > l - l_{\nu+}$; e) $z < l - l_{\nu+}$; f) $z = 0$.

3.2. "Forward" Superluminescence under Conditions of Group Synchronism in Media with Finite Time $T_2^c \lesssim \tau_c$

We shall consider cells of small length $l < l_{\nu^-}$. Here it is not necessary to take into account the difference in the group velocities, and we set $u_c = u_p = u$. Solution of Eqs. (1) and (2) are found by the Riemann method. For zero boundary conditions, we obtain the following expression for the averaged intensity of the superluminescence:

$$I_c = g\left(\frac{2\pi\omega_c}{T_2^c u}\right)^2 \int_0^\tau n_2(\tau')\exp[2(\tau-\tau')/T_2^c]d\tau' \int_0^z R^2(z,z',\tau,\tau')dz'. \quad (10)$$

Here $n_2(\tau)$ is the population of the upper laser level, which differs from zero at $0 \le \tau \le \tau_p$. (In this subsection, the start of the pump pulse is shifted to the coordinate origin for convenience in calculations.) $R(\xi)$ is the Riemann function.

In the region $0 \le \tau \le \tau_p$,

$$R(z,z',\tau,\tau') = I_0(\sqrt{4\Gamma_0(\beta-1)(z'-z)(\tau'-\tau)/T_2^c}). \quad (11)$$

Here $I_0(\xi)$ is the modified Bessel function.

We now investigate the behavior of the superluminescence pulse (10) for high amplification $\Gamma_0 z \gg 1$, $\beta = 2$. If the condition $T_2^c/\tau_p < 1/\Gamma_0 z$ is satisfied here, then the superluminescence intensity takes the form

$$I_c = \frac{I_0\tau_p}{\sqrt{2\tau T_2^c}}\{1-\Phi(\sqrt{2\Gamma_0 z}-\sqrt{2\tau/T_2^c})\}\exp(2\Gamma_0 z), \quad 0\le\tau\le\Gamma_0 z T_2^c,$$

$$I_c = \frac{I_0\tau_p}{\sqrt{2\Gamma_0 z}\,T_2^c}\{1-\Phi(\sqrt{2\Gamma_0 z}-\sqrt{2\tau/T_2^c})\}\exp(2\Gamma_0 z), \quad \Gamma_0 z T_2^c\le\tau\le\tau_p, \quad (12)$$

$$I_c = \frac{I_0\tau_p}{\sqrt{2\Gamma_0 z}\,T_2^c}\{\Phi(\sqrt{2\tau/T_2^c}-\sqrt{2\Gamma_0 z})-\Phi(\sqrt{2(\tau-\tau_p)/T_2^c}-\sqrt{2\Gamma_0 z})\}\exp(2\Gamma_0 z), \quad \tau\ge\tau_p.$$

Here $I_0 = g n_2(2\pi\omega_c)^2/8\sqrt{\pi}u^2\tau_p\Gamma_0$ and $\Phi(\xi)$ is the error integral.

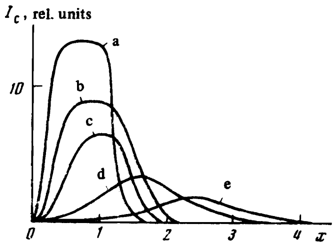
For the case $T_2^c/\tau_p \ge 1/\Gamma_0 z$, we get formulas similar to (12). The growth of the intensity (12) with coordinate z takes place more slowly than $\exp(2\Gamma_0 z)$. This fact was noted previously in[15] for nonstationary SRS.

As follows from Eqs. (12), the shape of the superluminescence pulse depends on the ratio T_2^c/τ_p (Fig. 2). For small T_2^c/τ_p, when the processes of damping of macroscopic polarization take place rapidly, the superluminescence pulse has an almost rectangular shape with steeply rising fronts (Fig. 2a, b). Upon increase in the ratio T_2^c/τ_p, a smoothing of the fronts takes place and the shift of the maximum in the direction of the trailing edge of the inversion pulse $N_{21}(\tau)$ takes place. For large values of $T_2^c/\tau_p > 1/\Gamma_0 z$, the superluminescence radiation can turn out to be localized in the region $\tau > \tau_p$ (Fig. 2e).

Thus the "wave" nonstationarity for $T_2^c \ll \tau_p$ leads to a cessation of amplification over the group lengths and shifts the superluminescence pulses to the leading edge of the pulses $N_{21}(\tau)$ if $\nu_- > 0$. In the opposite case, in a medium with finite time $T_2^c \lesssim \tau_c$ and $\nu_- = 0$, only a decrease in the rate of the amplification takes place, along with a shift of the pulses in the opposite direction.

The simultaneous action of both nonstationarity mechanisms can lead (in the case of normal dispersion) to the formation of stationary mode pulses of superluminescence, which have exponential amplification.

FIG. 2. "Forward" superluminescence pulses in a medium with finite time T_2^c under conditions of group synchronism at the length $z = 10/\Gamma_0 (x = \tau/\tau_p)$: a) $T_2^c/\tau_p = 0.03$; b) $T_2^c/\tau_p = 0.05$; c) $T_2^c/\tau_p = 0.07$; d) $T_2^c/\tau_p = 0.12$; e) $T_2^c/\tau_p = 0.2$.



Consideration of these pulses will be given in the next section.

3.3. Regime of Stationary Mode Amplification

The general solution of the set of equations (1) and (2) for a superluminescence pulse propagating "forward" in a medium with normal dispersion and finite time T_2^c, for zero boundary conditions, is of the form

$$I_c = g\left(\frac{2\pi\omega_c}{T_2^c u_c}\right)^2 \int_{-\infty}^{\eta}\exp\left[\frac{2(\eta'-\eta)}{T_2^c}\right]d\eta' \int_0^z n_2(\eta'-\nu z')R^2(z,z',\eta,\eta')dz', \quad (13)$$

where $\eta = t - x/u_c$ and $R(\xi)$ is the Riemann function.

In the case in which $N_{21}(\tau)$ has a rectangular shape (5), the population of the laser level $n_2(\tau)$ is different from zero for $-\tau_p/2 + \nu z \le \eta \le \tau_p/2 + \nu z$. In this region the following relation is valid for the Riemann function

$$R(z,z',\eta,\eta') = I_0(\sqrt{4\Gamma_0(\beta-1)(\eta'-\eta)(z'-z)/T_2^c}). \quad (14)$$

Approximate calculation of the integrals in (13) for high amplification $\Gamma_0 z \gg 1$ gives: at distances z exceeding the group length l_ν, the solution (13) takes the form

$$I_c \approx A_c^2(\tau)\exp[4\sqrt{\Gamma_0\nu(\beta-1)/T_2^c}\,z]. \quad (15)$$

Thus, at certain distances from the entrance to the cell $z > l_\nu$, a regime is established in which the intensity profile has a stationary shape $A_c^2(\tau)$, and the amplification increases linearly with increase in the coordinate z. Similar stationary regimes were obtained for SRS and for parametric amplification.[17]

We now study the stationary amplitude profile. We shall seek a solution of the set of equations (1) and (2) for $z > l_\nu$ in the form

$$E_c(z,t) = A_c(\tau)\exp(\Gamma_M z). \quad (16)$$

Neglecting the noise sources, we obtain an equation for the amplitude of the stationary superluminescence pulse:

$$\frac{d^2A_c}{d\tau^2} + \left(\frac{1}{T_2^c}-\frac{\Gamma_M}{\nu}\right)\frac{dA_c}{d\tau} + \frac{1}{\nu T_2^c}[N_{21}(\tau)\sigma_c - \Gamma_M]A_c = 0. \quad (17)$$

We note that an equation of similar type was obtained in[16] for the envelope of stationary Stokes pulses in SRS.

In the case of rectangular pumping pulse (4), which creates the inversion $N_{21}(\tau)$ in the form (5), the solution of Eq. (17) is of the form

$$A_c\left(x=\frac{\tau}{\tau_p}\right) = C\exp\left[\frac{p}{4}+\frac{p-a}{2}x\right], \quad x\le-\frac{1}{2},$$

$$A_c\left(x=\frac{\tau}{\tau_p}\right) = \frac{-C\sin S(x-1/2)}{\sin S}\exp\left(-\frac{a}{2}x\right), \quad |x|\le\frac{1}{2}, \quad (18)$$

$$A_c\left(x=\frac{\tau}{\tau_p}\right) = 0, \quad x\ge\frac{1}{2}.$$

Here $p/2 = \sqrt{\Gamma_0\tau_p^2\beta/\nu T_2^c - S^2}$, $a/2 = \tau_p/T_2^c - \sqrt{\Gamma_0\tau_p^2(\beta-1)/\nu T_2^c - S^2}$, C = const, the parameter S

is found by solving the equation $\sin S = S/n$, $n = \tau_p \sqrt{\Gamma_0\beta/\nu T_2^c}$. It is found that S can take on a discrete set of values S_k (for $S/n \ll 1$, we have $S_1 \approx \pi(1 - 1/n)$. This set of values of S_k corresponds to the growth-rate values

$$\Gamma_M^{(k)} = 2\sqrt{\Gamma_0\nu(\beta-1)/T_2^c - S_k^2\nu^2/\tau_p^2} - \nu/T_2^c. \quad (19)$$

The mode growth rate $\Gamma_M^{(k)}$ is always less than the stationary growth rate $\Gamma_0 = n_0\sigma_c$.

The mode amplification regime has a threshold even in the absence of linear nonresonance losses in the medium. The pump threshold power density is determined by the condition

$$P_{thr}^{(k)} = P_{sat}\frac{\beta_{thr}^{(k)}}{2-\beta_{thr}^{(k)}}, \quad (20)$$

where

$$P_{sat} = \frac{\hbar^2 c}{8\pi d_p^2 T_2 \nu T_1}, \quad \beta_{thr}^{(k)} = 1 + \frac{S_k^2\nu T_2^c}{\Gamma_0\tau_p^2} + \frac{\nu}{4\Gamma_0 T_2^c}.$$

For large values of the pump power density $P \gg P_{sat}$, the difference in the populations of the working levels N_{21} is maximal:

$$(N_{21})_{max} = n_1^0 - n_2^0 + 2(n_2^0 - n_3^0), \quad (21)$$

and the increment tends toward its limiting value $\Gamma_M^{(k)}(\beta = 2)$ (Fig. 3).

The threshold of the mode regime (20) depends on the concentration of active particles n_0 (Fig. 3). There exists a limiting concentration

$$n_0^{thr} = \frac{1}{\sigma_c}\left[\frac{\nu}{4T_2^c} + \frac{\nu T_2^c}{\tau_p^2} S_k^2\right], \quad (22)$$

below which the mode amplification regime does not develop, since in this case the threshold power density (20) becomes infinitely large.

When the pumping threshold (20) is reached, a stationary superluminescence pulse is formed, which is localized near the leading edge of the pump (Fig. 4a). As the pump power increases, the crest of the superluminescence pulse shifts toward the trailing edge of the pump (Fig. 4b, c) until inversion saturation sets in (21). The shape of the superluminescence pulse remains unchanged with further amplification (Fig. 4e).

The stationary superluminescence pulses have the described shape if it is possible to neglect the processes of relaxation with times T_1 and T_V. Account of these processes becomes necessary in a regime of picosecond pulses.

4. SUPERLUMINESCENCE FOR PICOSECOND PUMPING. STATIONARY MODE PULSES

If the duration of the pumping pulse τ_p is comparable with the vibrational relaxation time T_V and smaller than the longitudinal relation time T_1 (for dyes, $T_V \sim 10^{-11}$–10^{-12} sec, $T_1 \sim 10^{-8}$–10^{-9} sec), then the inversion pulse N_{21} can no longer be regarded as rectangular. In this case ($T_V \le \tau_p \ll T_1$) the following expressions are valid for the population differences $N_{21}(\tau)$:[12]

$$\begin{aligned} N_{21}(\tau) &= -|N_{21}^0|, \quad \tau \le -\tau_p/2, \\ N_{21}(\tau) &= |N_{31}^0|\beta - N_{21}^0, \quad |\tau| \le \tau_p/2, \\ N_{21}(\tau) &= N_{31}^0\beta\exp\left(-\frac{\tau-\tau_p/2}{T}\right) - |N_{21}^0|, \quad \tau \ge \frac{\tau_p}{2}, \end{aligned} \quad (23)$$

Here β is the same parameter as in Eqs. (5), T is the characteristic damping time of the trailing edge of the inversion and is determined by the times T_V and T_1.

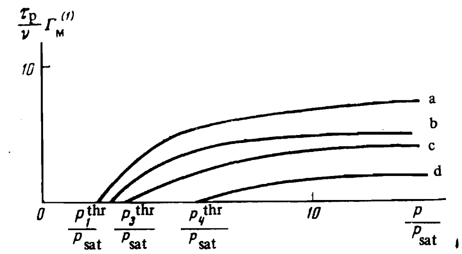


FIG. 3. Dependence of the growth rate in the fundamental mode of superluminsecence on the pumping power density for various concentrations ($T_2^c = 10^{-12}$ sec; $\nu = 10^{-12}$ sec/cm, $\tau_p = 10^{-11}$ sec): a) $n_0 = 1/\sigma_c$; b) $n_0 = 0.8/\sigma_c$; c) $n_0 = 0.7/\sigma_c$; d) $n_0 = 0.5/\sigma_c$.

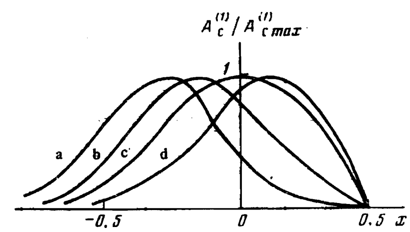


FIG. 4. Amplitude profiles of the fundamental mode of superluminescence for various values of the pump power ($\Gamma_0 = 1.5$ cm^{-1}, $T_2^c = 10^{-12}$ sec, $\nu = 10^{-12}$ sec/cm, $\tau_p = 10^{-11}$ sec, $x = \tau/\tau_p$): a) $\beta = \beta_{thr} = 1.23$; b) $\beta = 1.50$; c) $\beta = 1.73$; d) $\beta = 2.0$.

Solution of Eq. (17) for the population difference of the form (23) leads to stationary superluminescence modes. The excitation threshold, increment and threshold concentration for the fundamental mode are the same as in the case considered of a rectangular inversion pulse (5). The presence of an exponential trailing edge in (23) changes the shape of the superluminescence pulse. The most significant changes affect the trailing edge of the amplitude profile. For the fundamental mode at $\tau \ge \tau_p/2$, we have

$$A_c^{(1)}\left(x = \frac{\tau}{\tau_p}\right) = \begin{cases} C\sin(Y - Y_1)\exp(-\tfrac{1}{2}ax), & \tfrac{1}{2} \le x \le \tfrac{1}{2} + T\tau_p^{-1}\ln\beta, \\ 0, & x \ge \tfrac{1}{2} + T\tau_p^{-1}\ln\beta, \end{cases} \quad (24)$$

where

$$C = \text{const}, \quad Y = 2T\sqrt{\Gamma_0\beta/\nu T_2^c}\exp[-\tau_p(x - \tfrac{1}{2})/2T],$$
$$Y_1 = 2T\sqrt{\Gamma_0/\nu T_2^c}, \quad a/2 = \tau_p/T_2^c - \sqrt{\Gamma_0\tau_p^2(\beta-1)/\nu T_2^c - \pi^2}.$$

The shape of the trailing edge (24) is determined by the values of the parameters T/τ_p, T_2^c/τ_p, and β. We consider the case of strong pumping ($P \gg P_{sat}$, $\beta = 2$). If

$$T_2^c/\tau_p \ge 0.5/\pi\sqrt{0.36\tau_p^2/T^2 - 1}, \quad T/\tau_p < 0.6, \quad (25)$$

then the shape of the trailing edge (24) can change, depending on the concentration of active particles n_0. For small values of the concentration $n_0^{thr} < n_0 < n_0^{crit} = 0.18\pi\hbar u_c\nu/T^2\omega_c d_c^2$, the trailing edge falls off smoothly to zero (Fig. 5a); for large $n_0 > n_0^{crit}$, it oscillates. The number of oscillations (number of minima and maxima at $\tau \ge \tau_p/2$) is equal to k if the parameters of the system satisfy the relation

$$-\tfrac{1}{2} + k \le 1.17 d_c T\sqrt{n_0\omega_c}/\sqrt{\pi\hbar u_c\nu} \le \tfrac{1}{2} + k, \quad k = 1, 2, \ldots \quad (26)$$

If the same ratios T_2^c/τ_p and T/τ_p do not satisfy the requirements (25) simultaneously, then the trailing edge (24) oscillates at all concentrations (Fig. 5b, c).

The relations (24)–(26) can be used for an estimate of the times T and T_2^c.

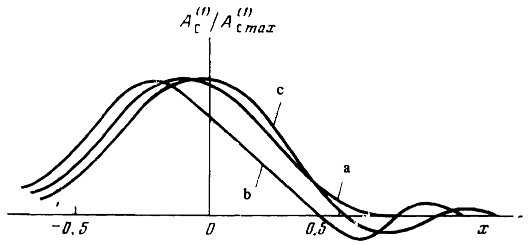


FIG. 5. Pulse shapes of the fundamental mode of superluminescence for pumping power density $P > P_{sat}$ ($\beta = 2$) for various values of the time T ($T_2^c = 10^{-12}$ sec, $\tau_p = 10^{-11}$ sec, $\nu = 10^{-12}$ sec/cm; $x = \tau/\tau_p$): a) T = 2 × 10^{-12} sec; b) T = 8 × 10^{-12} sec; c) T = 9.5 × 10^{-12} sec.

5. CONCLUSION

Using as an example a three-level model, a theoretical analysis has been carried out of the development of superluminescence in pumping with durations T_1, $T_V \ll \tau_p$, and $T_V \leq \tau_p \ll T_1$. The effect of group retardation of the pumping waves and superluminescence and the finiteness of the damping times of macroscopic polarization T_2^c are taken into account.

It is shown that the presence of group-velocity mismatch in the case in which neglect of the time T_2^c is possible leads to a cessation of amplification over the group lengths. Here "front-back" asymmetry of the superluminescence radiation is observed. Under conditions of group synchronism and finite values of T_2^c, the rate of amplification is somewhat reduced in comparison with the case of monochromatic pumping.

The simultaneous effect of molecular relaxation of the medium with time T_2^c and the effects of group retardation on the development of superluminescence is similar in its general outlines to the action of similar mechanisms of nonstationarity on the Stokes pulse in SRS[16].

Allowance for the motion of level populations leads to somewhat different features in the stationary mode regime of superluminescence. As a consequence of the saturation of the population inversion in a powerful pump field, it happens that the growth rate Γ_M does not increase linearly with the pump power, and asymptotically approaches its limiting value for the prescribed parameters of the system. This brings about the appearance of a concentration threshold of the mode regime.

The characteristic shape of the stationary pulse of the fundamental mode of superluminescence with an oscillating trailing edge corresponds to the specifics of the process of vibrational (T_V) and radiative (T_1) relaxations in the three-level medium, which can be of significant practical interest as a criterion for the estimate of the relaxation times.

1. A. J. de Maria, D. A. Stetser, and H. Heynau, Appl. Phys. Lett. 8, 174 (1966).
2. I. K. Krasyuk, P. P. Pashinin, and A. M. Prokhorov, ZhETF Pis. Red. 7, 117 (1968) [JETP Lett. 7, 89 (1968)].
3. W. H. Glenn, M. J. Brienza, and A. J. De Maria, Appl. Phys. Lett. 12, 54 (1968).
4. E. P. Ippen, C. V. Shank, and A. Dienes, Appl. Phys. Lett. 21, 348 (1972).
5. E. G. Arthurs, D. I. Bradly, and A. G. Roddie, Appl. Phys. Lett. 23, 88 (1973).
6. P. G. Kryukov and V. C. Letokhov, Usp. Fiz. Nauk 99, 169 (1969) [Sov. Phys.-Uspekhi 12, 641 (1970)].
7. A. J. De Maria, R. Gagosz, H. A. Heynau, A. W. Penney, and G. Wisner, J. Appl. Phys. 38, 2693 (1967).
8. M. E. Mack, Appl. Phys. Lett. 15, 166 (1969).
9. E. A. Tikhonov and M. T. Shpak, Ukr. Fiz. Zh. 15, 344 (1970).
10. C. Lin, T. K. Gustafson, and A. Dienes, Opt. Comm. 8, 210 (1973).
11. I. L. Klyukach, R. Yu. Orlov, and L. S. Telegin, Papers presented at the VII All-Union Conference on Coherent and Nonlinear Optics, Tashkent, 1974. Moscow State Univ. Press, p. 402.
12. T. M. Il'inova, I. L. Klyykach, and R. V. Khokhlov, Izv. Akad. Nauk SSSR, ser. fiz. 37, 2203 (1973).
13. M. V. Bogdanova, T. M. Is'inova, I. L. Klyukach, and R. V. Khokhlov, Papers presented at the VII All-Union Conference on Coherent and Nonlinear Optics, Tashkent, 1974. Moscow State Univ. Press, p. 400.
14. T. M. Il'inova, M. P. Il'inov, and R. V. Khokhlov, in: Quantovaya elektronika (Quantum Electronics) edited by N. G. Basov, No. 6, 43 (1971).
15. S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. 59, 485 (1970) [Sov. Phys.-JETP 32, 266 (1971)].
16. S. A. Akhmanov, K. N. Drabovich, A. P. Sukhorukov, and A. K. Shchednova, Zh. Eksp. Teor. Fiz. 62, 525 (1972) [Sov. Phys.-JETP 35, 279 (1972)].
17. A. P. Sukhorukov and A. K. Shchednova, in: Nelineǐnye protsessy v optike (Nonlinear Processes in Optics). Proceedings (Trudy) of the II Vavilov Conference on Nonlinear Optics, Novosibirsk, 1972, p. 17.

Translated by R. T. Beyer
55