Electron-nuclear magnetic resonance in a thin ferromagnetic film

V. A. Ignatchenko and V. I. Tsifrinovich

L. V. Kirenskiĭ Institute of Physics, Siberian Branch, USSR Academy of Sciences
(Submitted July 23, 1974)
Zh. Eksp. Teor. Fiz. 68, 672–676 (February 1975)

This study deals with the susceptibility of the electron-nuclear magnetic system of a ferromagnetic film in the region of strong ferromagnetic-resonance (FMR) and NMR interaction. It is shown that the behavior of the susceptibility peaks does not agree with the behavior of natural frequencies of the system even qualitatively. The electron-nuclear magnetic resonance spectrum possesses a fine structure, the study of which permits one to measure the characteristics of the nuclear magnetic system of a ferromagnet.

It has already been demonstrated[1,2] that a thin magnetic film magnetized in its own plane is a suitable object for the observation of electron-nuclear magnetic resonance (ENMR). In such a film it is possible to create conditions for matching the ferromagnetic resonance (FMR) and NMR frequencies and to obtain a resonance based on coupled electron-nuclear oscillations. The separation between the intrinsic frequencies of such oscillations is determined by the frequency of interaction

$$\omega_q=\gamma_e(4\pi AM\mu)^{1/2}, \tag{1}$$

where γ_e is the electron gyromagnetic ratio, A is the dimensionless constant of the hyperfine interaction, and M and μ are the electron and nuclear magnetization respectively. The effect of electron-nuclear interaction was first observed experimentally by Pogorelov and Kotov[3]. In the region of strong interaction, they obtained a characteristic kink on the plot of the field dependence of the FMR. At the point of intersection of the unperturbed FMR and NMR frequencies, they observed a zero shift of the resonance signal instead of the maximum shift of $\omega(H)$ by an amount ω_q. Later, in the work of Portis[4], the theory of this phenomenon was further developed: the frequencies of the coupled oscillations were calculated with allowance for the relaxation Γ_e in the electron magnetic system (the nuclear relaxation Γ_n was not taken into account, because $\Gamma_n \ll \Gamma_e$). Portis obtained $\omega(H)$ curves with a kink in the region of strong interaction at $\omega_q \approx \Gamma_e$. One of these curves agrees qualitatively with the experimental results[3], but no quantitative agreement was obtained.

The ENMR is too complicated a phenomenon for the experiment to be interpreted successfully by means of the formulas for the natural frequencies of the system. One must know the susceptibility of the system χ to compare correctly the experimental results with the theory. Indeed, as we will demonstrate below, the positions of energy-absorption peaks in the region of strong interaction differ considerably from the positions of the natural frequencies of the system.

It is worth noting that Botvinko and Ivanova[5] have already calculated the susceptibility of such a system by the method of temperature Green's functions. However, in their work they did not investigate the situation corresponding to the real relation between the parameters of the system. This is the task of the present study. Since we are not concerned with quantum effects, we use the method of classic equations of motion for the nuclear and electron magnetization.

The phenomenological Hamiltonian of the electron-nuclear magnetic system is

$$\mathscr{H}=-{}^1/_2\beta(\mathbf{Mn})^2-(\mathbf{M}+\boldsymbol{\mu})(\mathbf{H}+\mathbf{h})+{}^1/_2(\mathbf{M}+\boldsymbol{\mu})\hat{N}(\mathbf{M}+\boldsymbol{\mu})+A\boldsymbol{\mu}\mathbf{M}, \tag{2}$$

where β is the anisotropy constant, $\hat{N}$ is the tensor of the demagnetizing coefficients, **H** is the constant magnetic field, and **h** is the alternating magnetic field. The equations of motion of the system are the Landau-Lifshitz and Bloch equations respectively

$$\dot{\mathbf{M}}=\gamma_e\left[\mathbf{M}\times\frac{\partial\mathscr{H}}{\partial\mathbf{M}}\right]+\frac{\xi}{M}[\mathbf{M}\times\dot{\mathbf{M}}],$$
$$\dot{\boldsymbol{\mu}}=-\gamma_n\left[\boldsymbol{\mu}\times\frac{\partial\mathscr{H}}{\partial\boldsymbol{\mu}}\right]-\mathbf{k}\frac{\mu_z-\mu}{T_1}-\mathbf{i}\frac{\mu_x}{T_2}-\mathbf{j}\frac{\mu_y}{T_2}. \tag{3}$$

We consider the following case: a thin magnetic film is magnetized in its own plane along the Z axis, which is parallel to H; the anisotropy axis is parallel to X; the Y axis is perpendicular to the plane of the film; the alternating field h is homogeneous in space and is polarized along the X axis.

In this situation, the complex linear susceptibility of the system is determined by the expression

$$\chi_{xx}=\frac{M_x+\mu_x}{h}\approx\frac{1}{4\pi}\frac{(4\pi\gamma_eM)^2(\omega_n^2+2i\omega\Gamma_n-\omega^2)-\omega_n^2\omega_q^2-4\Gamma_e\Gamma_n\omega^2}{(\omega_e^2+2i\Gamma_e\omega-\omega^2)(\omega_n^2+2i\Gamma_n\omega-\omega^2)-\omega_n^2\omega_q^2}, \tag{4}$$

where $\omega_e = \gamma_e(4\pi M(H-\beta M))^{1/2}$, $\omega_n = \gamma_n AM$ are the frequencies of the unperturbed FMR and NMR, while $\Gamma_e = 2\pi\xi\gamma_e M$, and $\Gamma_n = T_2^{-1}$ are their relaxation parameters. Expression (4) implies satisfaction of the inequalities

$$\xi^2\ll1,\quad H\ll4\pi M,\quad 4\pi\ll A,\quad \Gamma_n^2\ll\omega_n^2,$$
$$\gamma_e\mu\ll\gamma_nM,\quad (H-\beta M)\ll\omega_n/\gamma_e\ll4\pi M. \tag{5}$$

In particular, μ_x was neglected in comparison with M_x, because $\mu_x/M_x \sim \mu\omega_n/M\Gamma_n$.

The electromagnetic-field energy absorbed per unit volume and per unit time is determined by the expression

$$P=\frac{1}{2}\omega\chi_{xx}''h^2=\frac{1}{4\pi}(4\pi\gamma_eM)^2h^2F, \tag{6}$$

$$F=\omega^2[\Gamma_e(\omega_n^2-\omega^2)^2+\Gamma_n(\omega_n^2\omega_q^2+4\omega^2\Gamma_n\Gamma_e)]\times\{[(\omega_e^2-\omega^2)(\omega_n^2-\omega^2)-\omega_n^2\omega_q^2-4\omega^2\Gamma_n\Gamma_e]^2+4\omega^2[\Gamma_e(\omega_n^2-\omega^2)+\Gamma_n(\omega_e^2-\omega^2)]^2\}^{-1}. \tag{7}$$

Far from the strong-interaction region, when the frequencies ω_e and ω_n are sufficiently separated, this equation describes the (dynamically) non-interacting FMR and NMR:

at $\omega \sim \omega_e \gg \omega_n$

$$F \approx \frac{\Gamma_e\omega^2}{(\omega_e^2-\omega^2)^2+4\Gamma_e^2\omega^2}, \tag{8}$$

at $\omega \sim \omega_n \ll \omega_e$

$$F \approx \frac{\Gamma_e\omega^2}{\omega_e^4}+\frac{\omega^2}{\omega_e^4}\frac{\Gamma_n\omega_n^2\omega_q^2}{(\omega_n^2-\omega^2)^2+4\Gamma_n^2\omega^2}. \tag{9}$$

Unlike the expression for the FMR, which is determined only by the parameters of the electron magnetic system, the exprèssion for the NMR is determined even outside the strong-interaction region by both the nuclear and the electronic parameters. This results from the distinguishing features of the NMR in ferromagnets, namely, both the excitation and observation of the NMR are effected via the electron magnetization.

Near the strong-interaction region, expression (7) was analyzed by numerical methods. The following was assumed:

$$\omega_n=4\pi\cdot10^8\ \text{sec}^{-1}\quad \omega_q=1.5\cdot10^8\ \text{sec}^{-1}$$
$$\Gamma_e=9\cdot10^8\ \text{sec}^{-1}\quad \Gamma_n=5.8\cdot10^6\ \text{sec}^{-1} \tag{10}$$

The value of ω_n corresponds to the frequency of the NMR in Co; the value of Γ_e was taken from FMR experiments in permalloy films in the microwave band; the value of Γ_n was taken from NMR experiments in single-domain cobalt particles[6]. The value of ω_q was calculated from formula (1) with the nuclear magnetization μ determined by Langevin's formula[7] for a film containing 40% Co at a temperature of 300°K.

Figure 1(a, b, c) shows plots of $F(\omega)$ for various values of ω_e. Figure 1a corresponds to the case when frequencies ω_e and ω_n are separated to a considerable extent: $\omega_n/2\pi$ = 200 MHz and $\omega_e/2\pi$ = 600 MHz. The $F(\omega)$ curve describes in this case two non-interacting signals, a strong FMR signal and a much weaker NMR signal. Figure 1b corresponds to the case $\omega_e/2\pi$ = 250 MHz, and Fig. 1c corresponds to $\omega_e/2\pi$ = 200 MHz. It is obvious from these figures that in the strong-interaction region the ENMR signal should have a rather peculiar shape, with a narrow peak at the point $\omega = \omega_n$ against the background of a wide peak. This results from the fact that the numerator or the function F contains the "resonance" term $(\omega_n^2 - \omega^2)^2$.

It is possible to perform a more detailed analysis in the most interesting case $\omega_e = \omega_n$ when the values of ω are close to ω_n. In this case

$$F \approx \frac{4\Gamma_e(\omega_n-\omega)^2+\Gamma_n(\omega_q^2+4\Gamma_n\Gamma_e)}{[4(\omega_n-\omega)^2-\omega_q^2-4\Gamma_n\Gamma_e]^2+16\Gamma_e^2(\omega_n-\omega)^2}. \tag{11}$$

If the numerator of this expression were not to contain the "resonance" term $(\omega_n - \omega)^2$, the function F would represent at $2\Gamma_e^2 > \omega_q^2 + 4\Gamma_n\Gamma_e$ one wide resonance peak with the maximum at the point $\omega = \omega_n$, in accord with the positions of the natural frequencies of the system[4]. The width of this resonance peak is $\Delta\omega_1 \approx 2\Gamma_e$. However, in a narrow vicinity of ω_n, the "inverse resonance" of the numerator, which is much narrower than the resonance of the denominator, is superimposed on this picture. As a result of this superposition, the function has three extrema: a minimum at the point $\omega = \omega_n$ and two maxima at the points

$$\omega\approx\omega_n\pm{}^1/_2[\omega_q^2(\omega_q^2+4\Gamma_n\Gamma_e)]^{1/4}. \tag{12}$$

The values of the function at the minimum and maximum points are determined by the expressions

$$F_{min}\approx\Gamma_n/(\omega_q^2+4\Gamma_n\Gamma_e),\quad F_{max}\approx1/4\Gamma_e. \tag{13}$$

The depth of the "inverse resonance" peak is

$$F_{max}-F_{min}=\omega_q^2/4\Gamma_e(\omega_q^2+4\Gamma_e\Gamma_n), \tag{14}$$

and the "line width" (midway between F_{max} and F_{min}) is

$$\Delta\omega_2=(\omega_q^2+4\Gamma_n\Gamma_e)/2\Gamma_e. \tag{15}$$

Thus, the absorption of energy in ENMR is described by a complicated function with markedly different characteristic scales ($\Delta\omega_2 \ll \Delta\omega_1$) and this should be kept in mind when the theory and experimental findings are compared.

Using apparatus with high resolution, the experimenter can trace the two maxima and find that in the strong-interaction region the corresponding frequencies are separated even more than the natural frequencies calculated in the absence of damping[1,2] (solid curves in Fig. 2), even though $\omega_q \ll \Gamma_e$ in this case.

Apparatus that does not permit the experimenter to resolve the "inverse resonance" over the whole strong-interaction interval does allow one to trace the broad FMR maximum. The frequency of which at $\omega_q \ll \Gamma_e$ is practically unperturbed, which is also true of the natural frequency calculated by Portis[4] (dot-dash curve in Fig. 2).

Finally, a case is possible when the apparatus cannot resolve the "inverse resonance" at $\omega_e = \omega_n$ (Fig. 1c) and only partially resolves it (as an asymmetry-induced

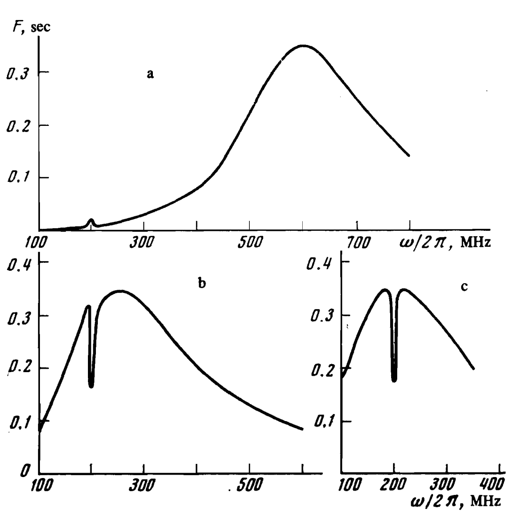

FIG. 1. Electromagnetic field energy absorption as a function of the field frequency: a)for $\omega_n/2\pi$ = 200 MHz and $\omega_e/2\pi$ = 600 MHz; b) for $\omega_e/2\pi$ = 250 MHz; c) for $\omega_e/2\pi$ = 200 MHz.

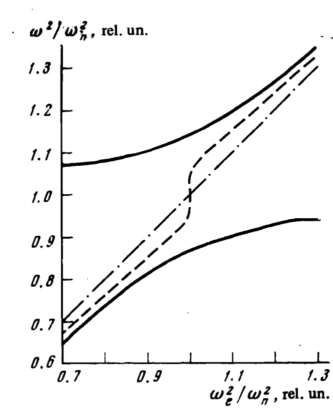

FIG. 2. Squares the frequencies corresponding to the energy absorption maxima vs. ω_e^2.

shift from the maximum) at $\omega_e \neq \omega_n$ (Fig. 1b). In this case we obtain a curve similar to the dashed curve of Fig. 2. It may be that the experiment[3] corresponds to such a case.

It is easy to demonstrate that the presence of the "inverse resonance" (or a transparency window) against the background of a wide absorption peak is a general property of two interacting oscillators with markedly different characteristics. In our notation, this difference corresponds to satisfaction of the two inequalities

$$\Gamma_n \ll \omega_q \ll \Gamma_e, \quad P_n \ll P_e, \tag{16}$$

where P_n and P_e are the powers absorbed at the resonance peaks by the first and second oscillators in the absence of interaction between them.

As is evident from formulas (12)–(15), a special study of the fine structure of ENMR—the "inverse resonance"—with apparatus having a sufficiently high resolution may enable us to determine the important properties of the nuclear magnetic system of a ferromagnet. It may also be that the narrow line and the high intensity of "inverse resonance" can be used in radio-frequency devices.

[1] V. A. Ignatchenko and Yu. A. Kudenko, Izv. Akad. Nauk SSSR, ser. fiz. 30, 77 (1966).

[2] V. A. Ignatchenko and Yu. A. Kudenko, Radiospektr. tverdogo tela (sbornik materialov konferentsii, Krasnoyarsk, 1964) (Radio-frequency Spectroscopy of Solids. Collection of Materials from the Krasnoyarsk Conference, 1964), Atomizdat, 1967, p. 181.

[3] A. N. Pogorelyi and V. V. Kotov, ZhETF Pis. Red. 14, 305 (1971) [JETP Lett. 14, 202 (1971)].

[4] A. M. Portis, AIP Conf. Proc. (USA) 10, pt. I, 120 (1972).

[5] M. N. Botvinko and M. A. Ivanov, Fiz. Tverd. Tela 15, 1704 (1973) [Sov. Phys.-Solid State 15, 1142 (1973)].

[6] A. C. Gossard, A. M. Portis, M. Rubinstein, and R. H. Lindquist, Phys. Rev. 138, A1415 (1965).

[7] A. Loesche, Nuclear Induction (Russ. transl.), IIL, 1963.

Translated by Larisa Doctorow
74