Corrections to scaling laws in the theory of interacting pomerons

A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan

(Submitted October 29, 1974)
Zh. Eksp. Teor. Fiz. 68, 817–822 (March 1975)

Corrections are calculated to the scale-invariant solution [1] of the form $1/\ln^{\alpha}s$ of the equations of the theory of interacting pomerons. Corrections of the order $1/\xi$ to σ^{tot} appear on account of the difference between the effective coupling constant from its asymptotic constant value which it attains as $\xi \to \infty$. The remaining corrections $\sim 1/\xi^{\alpha_i}$, with $\alpha_i > 1$ are due to the contribution of incompletely enhanced diagrams, in particular, diagrams containing the vertices of absorption and emission of two pomerons by the particle and the vertex of direct four-pomeron interaction. The indices α_i of some of the most important terms have been obtained. The article does not contain a general method for the calculation of the α_i.

In a previous paper[1,2] we have investigated the scale-invariant solution[3] of the system of equations which describes the field of interacting pomerons. It was shown that this solution can be obtained naturally by means of the Wilson ϵ-expansion[4], which allows one to obtain the physical parameters (indices) which appear and to construct the unknown functions which determine the solution[1)]. The theoretical model which was considered is valid only in the region of superhigh energies, for which the parameter $\xi = \ln(s/\mu^2)$ is very large, i.e., for $\xi \gtrsim 1/r^2$. Here $r \approx 1/12$ is the three-pomeron interaction coupling constant which is now known experimentally[5]. The region of validity of this solution can be enlarged in the direction of lower energies if one finds for it corrections of the order $1/\xi^{\alpha_i}$ due to terms which were not taken into account for $\xi \gg 1$; here $\alpha_i > 0$ are certain powers. This problem is discussed and partially solved below.

The solution obtained in[1] takes into account only the main part of the contribution of the so-called "enhanced" reggeon diagrams (Fig. 1,a) and does not take into consideration the contribution from the incompletely enhanced ones (Fig. 1, b, c).

Therefore the corrections to this solution can be of two types:

a) Correction terms of the order $\omega^{\epsilon/2} \sim 1/\xi^{\epsilon/2}$, $\epsilon = 2$ appear from the contribution of the enhanced diagrams. They are related to the fact that for small ω/r^2 the effective value of the square of the three-pomeron vertex

$$\lambda_\varepsilon(\omega) = \frac{\Gamma^2\beta^3}{R^{4-\varepsilon}\omega^{\varepsilon/2}} \approx \frac{r^2\omega^{-\varepsilon/2}}{1+{}^3/_2 r^2(\omega^{-\varepsilon/2}-1)/\varepsilon} \tag{1}$$

differs only little from its limiting value $\lambda_\alpha(\omega) \approx 2\epsilon/3$ which it has for $\omega/r^2 = 0$.

b) Corrections of the order $\omega^{\alpha_i} \sim 1/\xi^{\alpha_i}$, where α_i are indices, appear also from the contribution of the incompletely enhanced diagrams of Fig. 1, b, c, which contain the vertices N_n describing the simultaneous emission or absorption of n pomerons by the particle, or the vertex $\delta\Gamma_{nm}$ describing the direct transition of n pomerons into m. (The "enhanced" diagrams of this type Γ_{nm}, constructed out of three-pomeron vertices, have already been taken into account in the contribution of the enhanced diagrams. For example, the vertex Γ_{13} is surrounded by the dashed loop on one of the diagrams in Fig. 1, a.)

1. Let us find the corrections of the first kind, which

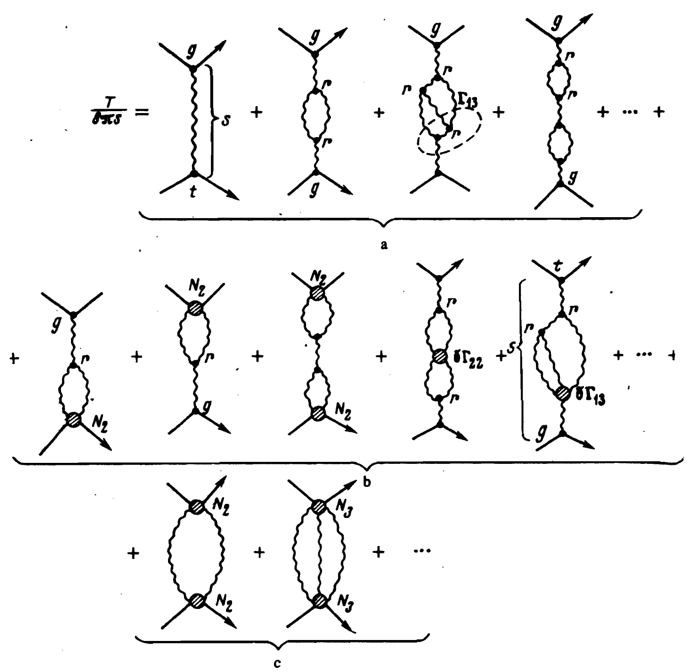

FIG. 1

are of order ω/r^2. An expansion of the function (1) into a series of powers of $\omega^{\epsilon/2}$ yields

$$\lambda_\varepsilon(\omega) = r_0^2 - r_0^2(r_0^2/r^2(4)-1)\,\omega^{\varepsilon/2},$$

where $r_0^2 = 2\epsilon/3$ is the stable root of the Gell-Mann–Low equation, i.e.,

$$\frac{d\lambda_\varepsilon(l)}{dl} = F(\lambda_\varepsilon) = 0, \quad l = \ln\frac{1}{\omega},$$

$$\frac{1}{\lambda_\varepsilon}F(\lambda_\varepsilon) = \frac{\varepsilon}{2} + \frac{2\Gamma'}{\Gamma} + \frac{3\beta'}{\beta} - \left(2-\frac{\varepsilon}{2}\right)\frac{(R^2)'}{R^2} = \frac{\varepsilon}{2} - \frac{3}{4}\lambda_\varepsilon,$$

and $r = r(d)$ is the three-pomeron coupling constant. This constant may be considered having an arbitrary dependence on the dimension $d = 4 - \epsilon$ of space and it appears in $\lambda_\epsilon(\omega)$ as $\epsilon \to 0$, i.e., for $d = 4$.

The quantity $r^2(4)$ differs from the physical four-pomeron coupling constant $r^2 = r^2(2)$. Denoting as in[1],

$$r^2(2)/r^2(4) = v^2 \tag{2}$$

and neglecting in $\lambda_\epsilon(\omega)$ the term $\sim\omega^{\epsilon/2}$ (compared to terms $\sim\omega^{\epsilon/2}/r^2(4)$; $r(4) \le 1$), we obtain for $\epsilon = 2$

$$\lambda_\varepsilon(\omega) \approx r_0^2 - r_0^4 v^2 \omega/r^2.$$

For the functions

$$\beta(l)\approx(\lambda/r^2)^{-1/6},\quad R^2(l)\approx(\lambda/r^2)^{-1/12},\quad \Gamma=(\lambda/r^2)^{2/3}$$

the taking into account of the second term in $\lambda_\epsilon(\omega)$ leads to small corrections to the values (3.5) in [1]:

$$\begin{aligned} Z_0^{-1}G(\omega,0)&=Z_0^{-1}\beta(l)\approx\omega^{-1-\eta}\left(1+\frac{2v^2}{9}\frac{\omega}{r^2}\right)+O(\varepsilon^2,\omega^\varepsilon),\\ R^2(l)&=R_0^2\omega^{\nu-1}\left(1+\frac{v^2}{9}\frac{\omega}{r^2}\right)+O(\varepsilon^2,\omega^\varepsilon),\\ \Gamma(l)&=Z^{-3/2}R_0^{d/2}\omega^\gamma\left(\frac{2\varepsilon}{3}\right)^{1/2}\left(1-\frac{8v^2}{9}\frac{\omega}{r^2}\right)+O(\varepsilon^2,\omega^\varepsilon). \end{aligned}\tag{3}$$

Going over to the ξ-representation we obtain $\sigma^{\mathrm{tot}}(\xi)$:

$$\sigma^{tot}(\xi)\approx\frac{8\pi g^2}{Z_0}G(\xi,0)=\frac{8\pi g^2\xi^\eta}{\Gamma(1+\eta)}\left(1+\frac{2\eta}{9}\frac{v^2}{r^2\xi}\right),\tag{4}$$

where the constants $\eta\approx 1/6$, $\nu-1\approx 1/12$, $\gamma\approx 2/3$ and Z_0, R_0^2 have values which have been determined before (cf. [1]).

As we have seen in [1], the optimal value of the ratio $v^2(2)$ of the constants can be obtained from a condition of smooth matching at $\omega > r^2$ of the power-law solutions for $G(\omega, 0)$ and Γ in the strong coupling region $\omega < r^2$ with the perturbation theory series which converge for $\omega > r^2$. This condition yields [1] $v^2 \approx 0.11 \pm 0.01$. Therefore the correction terms in (3) and (4) are small already for attainable energies, when $\xi = \ln(s/\mu^2)$ has the order of magnitude 10 and for $1/r^2\xi \approx 15$, if [5] $1/r^2 \approx 150$.

We show how one can calculate the corrections from the incompletely enhanced diagrams and determine the main ones.

The general definition of pomeron asymptotics of an amplitude of the form (1.1), (1.2) of [1], taking all diagrams into account, is

$$\frac{T(\xi,k^2)}{8\pi s}=\int e^{ik\rho}i\langle 0|T_\xi\left[V(\rho,\xi)V^+(0,0)\exp\left(i\int\mathscr{L}_1\frac{d^2\rho'}{2\pi}d\xi'\right)\right]|0\rangle\frac{d^2\rho}{2\pi},\tag{5}$$

where T_ξ is the ξ-ordering operator, V is the operator describing the absorption of a pomeron by the particle:

$$V=V_0+\sum_n\delta V_n,\quad V_0=g_0\psi(\rho,\xi),\quad \delta V_n=N_n{}^0\psi^n(\rho,\xi),\tag{6}$$

and $\mathscr{L}_1$ is the pomeron interaction operator:

$$\begin{aligned} \mathscr{L}_1&=\mathscr{L}'+\sum_{n+m\geqslant 3}\delta\mathscr{L}_{nm},\quad \mathscr{L}'={}^1/_2 ir(\psi^2\psi^++\psi\psi^{+2}),\\ \delta\mathscr{L}_{nm}&=\frac{i^{n+m}\lambda_{nm}}{n!m!}(\psi^n\psi^{+m}+\psi^m\psi^{+n}); \end{aligned}\tag{7}$$

Here $\delta\mathscr{L}_{nm}$ denote higher order terms in the expansion of the Lagrangian in powers of the pomeron field $\psi(\rho, \xi)$ (cf. (1.3) in [1]). It is easy to verify that an expansion of the exponential in (5) in powers of the constants r and λ_{nm}, taking into account the terms δV_n in (5), (6), reproduces the contribution of all the pomeron diagrams of Fig. 1 to the scattering amplitude.

As was noted in [1], the scale invariance of the theory in the region of small $\omega^\nu \sim k^2 \sim r^2$ means the invariance of the n-point Green's functions with respect to the transformation

$$\psi(\rho,\xi)\longrightarrow\xi^{-\Delta}\psi(\rho/\xi^{\nu/2},1)$$

where

$$\Delta=\nu d/2-\eta/2=1-\varepsilon/4+O(\varepsilon^2).\tag{8}$$

Here the field ψ has the dimension Δ and the n-point Green's function

$$G_n=\langle 0|T_\xi(\psi(\rho_1,\xi_1)\psi(\rho_2,\xi_2)\ldots\psi(\rho_n,\xi_n))|0\rangle$$

has the dimension $\approx\xi^{-n\Delta}$. The transition to the ω,k-representation yields for $G(\omega, k^2) \sim G_2$ and $\Gamma \sim G_3$ scale-invariant values of the type (3.8) in [1] with

$$\eta=\nu d/2-2\Delta,\quad \gamma=1+\nu d/2-3\Delta.$$

The operators $\delta V_n = N_n^{(0)}\psi^n$ and $\delta\mathscr{L}_{nm}$, the contribution of which in (5) leads to the incompletely enhanced diagrams, also have as $\xi\to\infty$ the dimension[2]:

$$\delta V_n\sim\xi^{-\Delta_{n,0}},\quad \delta\mathscr{L}_{nm}\sim\xi^{-\Delta_{n,m}}.\tag{9}$$

The numbers $\Delta_{n,0}$ and $\Delta_{n,m}$ can be determined by means of perturbation theory and the renormalization group method. They are positive and increase with n and m.

We denote by $T^{(1)}/8\pi s \sim \omega G(\omega, k^2) \sim \omega^{-\eta} \sim \xi^\eta$ the contribution from one or all the enhanced diagrams (they are all of the same order for $\xi\to\infty$) and let $T^{(n)}/8\pi s$ and $T^{(nm)}/8\pi s$ denote the contributions of the incompletely enhanced diagrams, containing respectively one vertex N_n^0 or λ_{nm}.

It follows from the definition (5) that the ratio $T^{(n)}/T^{(1)}$ is directly determined in order of magnitude by the ratio of the operators δV_n and the fundamental operator $V_0 = g\psi$

$$T^{(n)}/T^{(1)}\sim\delta V_n/\psi\sim\xi^{\Delta-\Delta_{n,0}}.\tag{10}$$

In the same manner one can see from (5) that the relative contribution of the diagrams containing unenhanced vertices λ_{nm} describing the direct transition of n pomerons into m is determined also by the quantity $\int\delta\mathscr{L}_{nm}d\rho d\xi$

$$T^{(n,m)}/T^{(1)}\sim\int\delta\mathscr{L}_{nm}d\rho\, d\xi\sim\xi^{1+\nu d/2-\Delta_{n,m}}.\tag{11}$$

As ξ increases all these ratios decrease, since in general $\Delta_{n,0}$ is always larger than Δ, and Δ_{nm} is larger than $1 + \nu d/2$. Before calculating some of these numbers we explain how these estimates can be obtained directly in the method of reggeon diagrams. The operators δV_n and $\delta\mathscr{L}_{nm}$ in (6), (7) lead to the appearance of incompletely enhanced diagrams in Fig. 1, b, containing "dressed" vertices N_n describing the direct transition of n pomerons into a system particle-antiparticle and the vertices $\delta\Gamma_{nm}$ describing a direct transition of n pomerons into m. The quantities $N_n^{(0)}$ and λ_{nm} are "bare" values of these vertices. In place of these vertices the completely enhanced diagrams contain the "enhanced" vertices $(N_n)_{enh}$ and Γ_{nm} of the same type, which are the contribution of "tree-like" diagrams of Fig. 2 and also the contribution from a large number of more complicated "enhanced" diagrams. Their order of magnitude is

$$\begin{aligned} (N_n)_{yc}&\sim g_0(G\Gamma)^{n-1}\sim\omega^{-(n-1)\Delta},\\ \Gamma_{nm}&\sim\Gamma(G\Gamma)^{n+m-3}\sim\omega^{-(n+m-3)\Delta+\gamma}=\omega^{-(n+m)\Delta+1+\nu d/2}, \end{aligned}\tag{12}$$

where $\Delta = \eta + 1 - \gamma$ is the dimension of the operator ψ

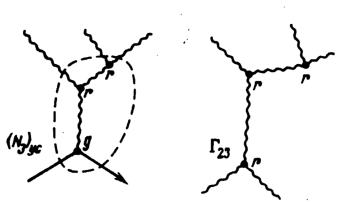

FIG. 2

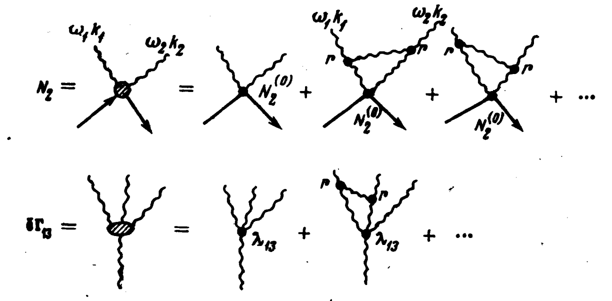

FIG. 3

(and $3\Delta + \gamma = 1 + d/2$). Since replacing the vertices N_n and $\delta\Gamma^{(nm)}$ respectively by $(N^{(n)})_{enh}$ and Γ_{nm} has the result that an incompletely enhanced diagram goes over into an enhanced diagram, the ratios of the contribution of the diagrams is directly the ratio of the vertices:

$$\frac{T^{(n)}}{T^{(1)}} \sim \frac{N^{(n)}}{(N^{(n)})_{yc}} \sim \xi^{\Delta-\Delta_{n,0}}, \qquad \frac{T^{(n,m)}}{T^{(1)}} \sim \xi^{1+\nu d/2-\Delta_{nm}} \sim \frac{\delta\Gamma_{nm}}{\Gamma_{nm}}.$$

Taking into account that $\omega \sim 1/\xi$, we obtain hence

$$N^{(n)} \sim \omega^{\Delta_{n,0}-n\Delta}, \quad \delta\Gamma_{nm} \sim \omega^{\Delta_{nm}-(n+m)\Delta}. \tag{13}$$

Thus, the equations (9) are equivalent to the almost obvious assertion that the dependence of the vertices N_n and Γ_{nm} on $\omega \sim 1/\xi$ has a power-law character as $\omega \sim 1/\omega \to 0$.

Let us determine some of the numbers $\Delta_{n,m}$. In order to compute the index $\Delta_{2,0}$ we construct the vertex $N^{(2)}$ which is analogous to the three-pomeron vertex Γ in [1]. Determining according to the diagrams of Fig. 3 the corrections of order $r^2 l$ to N_2 and using the renormalization group method in four-space we obtain

$$N_2 \approx N_2^0 (1 - {}^1/_2 r^2 l) \to N_2^0 (1 + {}^3/_4 r^2 l)^{-2/3}.$$

Similar to analogous formulas for Γ in [1]. Going over to a space of dimension $d = 4 - \epsilon$ by replacing l by $2(\omega^{-\epsilon/2} - 1)/\epsilon$, [1], we obtain, for $\omega \to 0$

$$N_2 \approx N_2^0 (r^2/r_1^2)^{-2/3} \omega^{\epsilon/3}.$$

Comparing this value with (12), we obtain

$$\Delta_{2,0} = 2\Delta + \varepsilon/3 + O(\varepsilon^2) \approx {}^5/_3.$$

This means that the corrections (10) from the semi-enhanced diagrams (the first two on the left in Fig. 1, b) to the main contribution have the relative value

$$T^{(2)} \sim T^{(1)}/\xi^{\Delta_{2,0}-\Delta} = T^{(1)}/\xi^{\Delta+\varepsilon/3} \sim T^{(1)}/\xi^{2/3}$$

for $\epsilon = 2$. The correction $T^{(2)}$ from the unenhanced two-reggeon diagram of Fig. 1, c are very small indeed:

$$T^{(2)} \sim T^{(1)}/\xi^{2(\Delta_{2,0}-\Delta)} \sim T^{(1)}/\xi^{4/3}.$$

It is easy to calculate the index $\Delta_{1,2}$ if one takes into account the fact that the corresponding corrections are due to the perturbation $\Delta\mathscr{L}' = i\lambda_{12}(\psi^+\psi^2 + \psi^{+2}\psi)/2$ of exactly the same form as the fundamental interaction $\mathscr{L}'$ in (6). In other words, they appear from the replacement of the constant $r \to r_1 + \delta r$, where $\delta r = \lambda_{12}$. For $r = r_1$ all quantities have the values determined in [1], and under the substitution $r \to r_1 + \lambda_{12}$ there appear corrections of the order $\omega^{\epsilon/2}$, obtained above in Eqs. (1) and (2). Therefore

$$\int \Delta\mathscr{L}_{12}\, d\xi\, d^2\rho \sim \xi^{-\varepsilon/2} \sim 1/\xi,$$

i.e., according to (11) $\Delta_{12} = 1 + \nu d/2 + \epsilon/2$.

As the numbers n and m increase, the quantities $\Delta_{n,m}$ increase rapidly, in particular $\Delta_{n,m} > (n+m)\Delta$ (since for $\omega \to 0$ the vertices $N^{(n)}$ and $\delta\Gamma_{nm}$ decrease). Therefore the corrections from multipomeron interactions, in particular from the four-pomeron vertices λ_{13}, λ_{22}, are small and decrease rapidly as ξ increases.

Taking into account the main correction (4) from λ_{12} and from $N^{(2)}$, we obtain

$$\sigma^{tot}(\xi) = \frac{8\pi g^2}{\Gamma(1+\eta)} \xi_1^{\eta} \left[1 + \frac{v_1^2}{27 r^2 \xi_1} + \frac{C_{2,0}}{\xi^{2/3}} + \ldots\right], \tag{14}$$

where $C_{2,0}$ is an unknown constant, $r \approx 1/10$.

---

1) Some of the results of [1,2] have also been obtained recently by Abarbanel and Bronzan [6]. The authors are grateful to M. Baker who has made them aware of these results.

2) More precisley, linear combinations of operators of this type have a definite dimension. This makes the problem of determining the numbers Δ_{nm} more difficult and was not taken into account in [2], where the numbers $\Delta_{n,0}$ and $\Delta_{n,m}$ are determined incorrectly. For $\xi \to \infty$ the operators $\delta V_n, \mathscr{L}_{nm}$ are also proportional to some powers of the quantity ξ.

1. A. A. Migdal, A. M. Polyakov and K. A. Ter-Martirosyan, Zh. Eksp. Teor. Fiz. **67**, 848, 2009 (1974) [Sov. Phys.-JETP **40**, 420, 999 (1975)].
2. A. A. Migdal, A. M. Polyakov and K. A. Ter-Martirosyan, Preprint ITEP N 102, 1973; Preprint ITP, Chernogolovka, 1974.
3. V. N. Gribov and A. A. Migdal, Zh. Eksp. Teor. Fiz. **55**, 1498 (1968) [Sov. Phys.-JETP **28**, 784 (1969)].
4. K. G. Wilson, Phys. Rev. Lett. **28**, 548 (1972); Phys. Rev. **D7**, 2910 (1973).
5. A. B. Kaĭdalov, Yu. F. Pirogov, M. L. Ter-Isaakyan and V. A. Khoze, ZhETF Pis. Red. **17**, 626 (1973) [JETP Lett. **17**, 440 (1973); A. B. Kaĭdalov and K. A. Ter-Martirosyan, Preprint ITEP No. 96, 1973.
6. H. D. I. Abarbanel and J. B. Bronzan, Phys. Lett. **48B**, 345 (1974).

Translated by Meinhard E. Mayer
91