Quantum theory of Vavilov–Cerenkov radiation by an electron traveling in vacuum parallel to a dielectric surface

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The probabilities have been calculated for soft and hard (both spontaneous and induced) Vavilov–Cerenkov radiation by an electron traveling in vacuum parallel to a dielectric-vacuum boundary. The angular and spectral distributions of energetic Cerenkov photons have been found and estimates of the probabilities are given. Characteristic properties of hard Vavilov–Cerenkov radiation are noted, and the possibility of experimental observation of Cerenkov photons is discussed from a fundamental point of view.

1. In several earlier articles[1-4] one of us has theoretically predicted the possibility of energetic Vavilov-Cerenkov radiation as a two-quantum process in which on passage of a fast charged particle in a transparent medium two photons are produced, one soft with a refractive index $n(\omega) > 1$, and the other hard with $n(\omega) \leq 1$.

Let ω_0 be a frequency such that $n(\omega) < 1$ for all $\omega > \omega_0$. The frequency ω_0 lies in the ultraviolet region and corresponds to a photon wavelength $\lambda_0 \sim 10^{-5}$ cm. Ordinary one-quantum Vavilov-Cerenkov radiation becomes impossible for photon frequencies $\omega > \omega_0$. However, radiation of two photons, one soft with $\omega < \omega_0$ and the other hard with $\omega > \omega_0$, is kinematically possible, as was shown in refs. 1 and 2. This two-photon Vavilov-Cerenkov radiation, in contrast to the ordinary single-photon radiation, is essentially a quantum process (in ordinary Vavilov-Cerenkov radiation the quantum effects give only small corrections to the basic classical formula[5,6]).

We note several interesting features of the hard Vavilov-Cerenkov radiation:

a) It is a nontrivial example of a two-quantum process with participation of optical and hard photons;

b) hard Vavilov-Cerenkov radiation is related to bremsstrahlung: it can be considered as that form of bremsstrahlung in which the excess energy and momentum are transferred not to an individual nucleus but to collective excitation of the medium (photon with $n(\omega) > 1$);

c) it can arise, in contrast to bremsstrahlung, on passage of an electron outside the material at a sufficiently close distance (of the order the wavelength of the soft photon) from the material boundary;

d) hard Vavilov-Cerenkov radiation can be stimulated, in contrast to ordinary bremsstrahlung, by a flux of soft photons with frequency $\omega < \omega_0$;

e) stimulated hard Vavilov-Cerenkov radiation is similar to the inverse Compton effect; however, as a result of the participation of the medium it can not only be stimulated but also spontaneous.

Hard Vavilov-Cerenkov radiation has not been observed experimentally up to the present time, since the loss of energy by electrons in a condensed medium is due mainly to bremsstrahlung, which strongly masks hard Vavilov-Cerenkov radiation.

The purpose of the present work is to study hard Vavilov-Cerenkov radiation under conditions in which the hard radiation is present in pure form, i.e., in the absence of energy loss by bremsstrahlung, transition radiation, and so forth. This becomes possible on passage of fast electrons in vacuum near a dielectric boundary. In this case the energy loss of the electron is due to ordinary soft Vavilov-Cerenkov radiation and hard Vavilov-Cerenkov radiation (with $\omega > \omega_0$).

2. We will assume that in its initial state the electron is moving parallel to a plane dielectric boundary. In order to be able to speak of passage of an electron at a distance d from the boundary, it is necessary to take the wave function of the initial state of the electron in the form of a wave packet of width b, this wave packet being written in the form ($d \gg b$):

$$\Psi_1(X) = G(z)\, u(p_1)\, e^{ip_1X}/\sqrt{V_0}, \quad X = (\mathbf{r}, it), \tag{2.1}$$

where $p_1 = (\mathbf{p}_1, iE_1)$, $\mathbf{p}_1$ and E_1 are the average momentum and energy of the electron in the initial state; $V_0 = bS$ is the normalization volume; S is the area of the boundary surface; $u(p_1)$ is a bispinor normalized by the condition $(u^{+}, u) = 1$; $a(p_z)$ is the weight of the different eigenstates

$$G(z) = \int a(p_z)\, e^{ip_z z}\, dp_z, \quad \int |G(z)|^2 dz = b. \tag{2.2}$$

Since the motion of electron in the coordinates x and y is unlimited (the problem is uniform in these coordinates), we can assume that $\Delta p_{1x} = \Delta p_{1y} = 0$. In addition, with no loss of generality we can assume that $p_{1y} = 0$. The values of p_{1z} have been discarded in the interval $\Delta p_{1z} \sim 1/b$, in order to be able to speak of the parallelism of the electron trajectory to the boundary plane, and it is necessary that the inequalities $p_{1x} \gg |p_{1z}|$ or $\lambda \ll b$ be satisfied, where $\lambda = \hbar/p_1$ is the DeBroglie wavelength of the electron.

The time of spreading of the wave packet (to a value of the order of its width) must be much greater than the time of the experiment t_e, i.e., the time of passage of the electron along the boundary must be much greater than the time of formation of the photons. We can assume that the lower limit for the formation time is of the order of $1/\omega$, since the uncertainty of the frequency of the transition from the initial state to the final state must be much less than the frequency of the photon (the softer of the two photons).

A spreading of the packet to a value of the order of its width occurs[7] in a time $\sim b^2E_1$. Thus, it is necessary that the inequalities $1/\omega \ll t_e \ll b^2E_1$ be satisfied. Hence $b \gg 1\sqrt{E_1\omega}$, i.e., $b > 10^{-8}$ cm for $E_1 \gtrsim 10m$, $\omega = 2\pi/\lambda \sim 10^5\ \text{cm}^{-1}$. For $b \sim 10^{-6}$ cm $\ll d \sim 10^{-5}$ cm this condition is satisfied. The time in the system of units chosen for an ultrarelativistic electron coincides with the path which the electron travels along the boundary. For $b \approx 3 \times 10^{-6}$ cm and $E_1 \approx 10m$, this path is $t_e \simeq 1$ cm and increases with increase of b and E_1.

In what follows we will set (in view of the inequalities written above) $E_1^2 = p_{1x}^2 + m^2$, i.e., we will neglect the weak dependence of E_1 on p_{1z}. This neglect leads to a negligible error, since $\Delta E_1 \sim 1/b^2E_1 \sim 10^{-10}E_1$.

It is also necessary that in the time t_e the electrical image force does not succeed in attracting the electron into the dielectric. The path traversed by the electron along the boundary before its attraction into the dielectric is, for an ultrarelativistic electron,

$$x_0=\pi[(\varepsilon+1)md^3/(\varepsilon-1)2e^2]^{1/2}(E_1/m)^{1/2}.$$

For $\epsilon \approx 2$, $d \sim 10^{-5}$ cm, and $E_1 \sim 10^4 m \sim 5$ GeV, this path is sufficiently great, $x_0 = 10$ cm.

Generally speaking, the final state of the electron should also be described by a wave packet, but since the proposed measuring device localizes the electron in the final state in a region much larger than the size of the wave packet, we will write the final-state wave function in the form of a plane wave:

$$\Psi_2(X)=u(p_2)e^{ip_2X}/\sqrt{V}, \quad p_2=(\mathbf{p}_2, iE_2). \tag{2.3}$$

3. We will calculate the probability of soft single-photon Vavilov-Cerenkov radiation for an electron traveling in vacuum parallel to the separation boundary plane.

The main difference of the problem with a separation boundary from the problem of an unlimited medium is the violation of the law of conservation of the z component of momentum, since part of the momentum is transferred to the separation boundary.

The probability of radiation of a photon of momentum **q**, frequency ω, and polarization l is written in the form

$$dw=|S_{fi}^{(1)}|^2 V^2 d\mathbf{p}_2 d\mathbf{q}/(2\pi)^6, \quad \hbar=c=1, \tag{3.1}$$

where

$$S_{fi}^{(1)}=-e\int d^4X(N(\bar{\Psi}_2(X)\hat{A}_M(X)\Psi_1(X)))_{fi}$$

is the matrix element of the scattering matrix for the transition between the initial state i and the final state f, $\hat{A}_M(X)$ is the electromagnetic-field operator in the presence of a plane boundary of separation of two media, which is a superposition of three waves—incident, reflected, and transmitted. The explicit form of this operator is determined in an article by Garibyan.[8]

After integration we obtain

$$dw=\frac{e^2}{(2\pi)^3}\frac{|QI|^2}{2\omega b}\delta(E_1-E_2-\omega)\delta(\mathbf{p}_{1\rho}-\mathbf{p}_{2\rho}-\boldsymbol{\tau})d\mathbf{p}_2 d\mathbf{q},$$

$$d\mathbf{p}_2=d\mathbf{p}_{2\rho}dp_{2z}, \quad d\mathbf{q}=q^2dq\sin\theta d\theta d\Phi,$$

θ is the angle between $\mathbf{p}_1$ and **q**, Φ is the angle between the projection of **q** on the yz plane and the negative z direction;

$$\boldsymbol{\tau}=(q_x, q_y), \quad \boldsymbol{\rho}=(x, y);$$

$$Q=\bar{u}(p_2)\beta^{(l)}e_{2m}^{(l)}u(p_1)/\sqrt{g^{(l)}}, \quad m=1, 2, 3, 4,$$

$$I=\exp\{-id(p_{2z}-\lambda_2)\}[\exp\{-ib(p_{2z}-\lambda_2)\}-1]/i(\lambda_2-p_{2z}),$$

$$g^{(l)}=1+|\alpha^{(l)}|^2+|\beta^{(l)}|^2,$$

$$\alpha^{(1)}=(\lambda_1\varepsilon_2-\lambda_2\varepsilon_1)/(\lambda_1\varepsilon_2+\lambda_2\varepsilon_1), \quad \alpha^{(2)}=(\lambda_1-\lambda_2)/(\lambda_1+\lambda_2),$$

$$\beta^{(1)}=2\lambda_1\varepsilon_2/(\lambda_1\varepsilon_2+\lambda_2\varepsilon_1), \quad \beta^{(2)}=2\lambda_1\sqrt{\varepsilon_2/\varepsilon_1}/(\lambda_1+\lambda_2),$$

$\epsilon_1 = \epsilon(z < 0)$ is the dielectric permittivity of the medium, $\epsilon_2 = 1(z > 0)$ is the dielectric permittivity of vacuum, $\lambda_1(\lambda_2)$ is the projection of the photon wave vector on the z axis in the incident (transmitted) wave.

One δ function is removed by integration over $dp_{2\rho}$, and the second by integration over dp_{2z}.

For reality of p_{2z} it is necessary that the inequality $p_{2z}^2 \geq 0$ be satisfied. It is satisfied only for angles which satisfy the condition

$$\cos\theta \geqslant \cos\theta_0, \quad \cos\theta_0=\frac{1}{\beta n}+\frac{\omega(n^2\sin^2\Phi-1)}{2p_1 n}. \tag{3.2}$$

For these angles λ_2 is pure imaginary. The imaginary value of λ_2 means that there is no radiation in the vacuum (the field in the vacuum is exponentially damped), while reality of λ_1 shows that Cerenkov radiation of a photon occurs in the dielectric.

Taking into account the imaginary value of λ_2, the probability of radiation is written in the form

$$dw=\frac{e^2}{(2\pi)^3}\frac{4E_2\sin^2(bp_{2z_0}/2)}{(p_{2z_0}{}^2+\lambda^2)p_{2z_0}}e^{-2d\lambda}|Q|^2 d\mathbf{q}, \quad \lambda=|\lambda_2|. \tag{3.3}$$

It is evident from (3.3) that there is a sharp dependence on $\cos\theta$ of the factor

$$\frac{\sin^2(bp_{2z_0}/2)}{(p_{2z_0}{}^2+\lambda^2)p_{2z_0}}, \quad p_{2z_0}=[2E_1\omega(\beta n\cos\theta-1)-\lambda^2]^{1/2},$$

which differs substantially from zero for angles sufficiently close to θ_0, and therefore in integration the remaining factors are taken out from under the integral sign for $\theta = \theta_0$, and the integration over $d(\cos\theta)$ reduces to integration over dp_{2z_0} with allowance for the fact that the region of angles giving the main contribution to the integral is determined from the condition

$$\cos\theta_0 \leqslant \cos\theta \leqslant \cos\theta_0+\omega(n^2\sin^2\Phi-1)/2p_1 n.$$

Thus, there is a smearing of the Cerenkov cone by an amount of the order of the quantum correction (see ref. 9), in contrast to the unbounded medium, where the photon-emission angle is fixed and with allowance for the quantum correction is determined by the condition

$$\cos\theta_0=\frac{1}{\beta n}+\frac{\omega(n^2-1)}{2p_1 n}.$$

After averaging over the electron polarization in the initial state and summing over the electron and photon polarizations in the final state we obtain

$$dw=\frac{2e^2}{(2\pi)^2}\frac{\lambda_1{}^2(\lambda^2\varepsilon+\tau_y{}^2)}{\omega^2\varepsilon(\lambda_1{}^2+\lambda^2\varepsilon^2)}e^{-2d\lambda}d\Phi\, d\omega.$$

Substituting here the classical condition $\cos\theta_0 = 1/\beta n$, we obtain an expression for the probability of soft Vavilov-Cerenkov radiation which agrees with the formula obtained classically.[9]

4. We will calculate the probability of two-photon hard spontaneous Vavilov-Cerenkov radiation in which one of the two photons is hard ($\omega > \omega_0$):

$$dw=|S_{fi}^{(2)}|^2 V^2 d\mathbf{p}_2 d\mathbf{q}\, d\mathbf{k}/(2\pi)^9,$$

$$S_{fi}^{(2)}=e^2\int(\bar{\Psi}_2(X_2)[\hat{A}^*(X_2)K(X_2-X_1)\hat{A}_M(X_1) + \hat{A}_M{}^*(X_2)K(X_2-X_1)\hat{A}(X_1)]\Psi_1(X_1)d^4X_1 d^4X_2)_{fi}, \tag{4.1}$$

$\hat{A}(X)$ is the electromagnetic-field operator of the hard photons (see ref. 10), K(X) is the electron propagator (see ref. 10).

Omitting the further calculations, which are similar to the calculations in the single-photon case, we obtain

$$dw=\frac{e^4}{2(2\pi)^5}\frac{E_2}{E_1 k\beta}e^{-2d\lambda}|\bar{u}(p_2)Qu(p_1)|^2\,d\omega\,d\Phi\,dk, \tag{4.2}$$

$$Q=\frac{A^{(l)}}{\sqrt{g^{(l)}}}\left(e_j\cdot\frac{i\hat{f}_1-m}{f_1^2+m^2}\hat{e}_{2m}^{*(l)}+\hat{e}_{2m}^{*(l)}\frac{i\hat{f}_2-m}{f_2^2+m^2}e_j\cdot\right)$$

$$f_1=p_2+k,\quad f_2=p_2+q,\quad \cos\theta=\cos\theta_0=\frac{1}{\beta n}+\frac{k}{\omega}\frac{(1-\beta\cos\vartheta)}{(1-k/E_1)\beta n}, \tag{4.3}$$

θ is the angle between $\mathbf{p}_1$ and $\mathbf{q}$, ϑ is the angle between $\mathbf{p}_1$ and $\mathbf{k}$. Here as in the single-photon case, radiation of soft Cerenkov photons occurs only in the dielectric.

The condition (4.3) is satisfied to terms of order $\sim\omega/E_1 \ll 1$, i.e., we can assume with a higher degree of accuracy that the condition for appearance of hard Vavilov-Cerenkov radiation in an unbounded medium (4.3) is satisfied.

Let us consider the ultrarelativistic case in which

$$\beta=1-\alpha^2/2,\quad \alpha=m/E_1\ll 1.$$

The probability of radiation of a hard photon is appreciably different from zero when $\vartheta \ll 1$, $1-\beta\cos\vartheta=(\alpha^2+\vartheta^2)/2$, and the condition (4.3) takes the form

$$\cos\theta_0=\frac{1}{n}+\frac{r}{1-r}\frac{\alpha^2+\vartheta^2}{2\alpha\nu n},\quad r=\frac{k}{E_1},\quad \nu=\frac{\omega}{m}. \tag{4.4}$$

Carrying out the summation over polarizations in (4.2), and retaining in the cumbersome expression obtained only terms of lowest order in α^2, r^2, and $\nu\alpha$, and assuming $E_1 \gg 10^3 m\sqrt{r}$, we finally obtain

$$dw=\frac{me^4}{2(2\pi)^4}\frac{[1+(1-r)^2]}{r}\frac{\alpha^2}{(\alpha^2+\vartheta^2)^2}\frac{\lambda_1^2(\lambda^2\varepsilon+\tau_\nu^2)}{\omega^2(\lambda_1^2+\lambda^2\varepsilon^2)}\cdot e^{-2d\lambda}\nu^2(n^2-1)\,d\nu\,d\Phi\,d\vartheta^2\,dr. \tag{4.5}$$

We will carry out the integration over $d\Phi$ from $-\pi/2$ to $\pi/2$ and over the spectrum of soft photons $\Delta(r,\vartheta)$. The region of integration over $d\nu$ is determined from the condition $\cos\theta_0 \le 1$, which gives

$$f(\nu)=\frac{r}{1-r}(\alpha^2+\vartheta^2),\quad f(\nu)=2\alpha\nu[n(\nu)-1];$$

we will designate this region by $\Delta(r,\vartheta)$. The size of the region $\Delta(r,\vartheta)$ increases with increasing E_1 and decreasing ϑ and r. For $r\sim r_{min}=\omega_0/E_1$, $\vartheta \lesssim \alpha$, and $E_1 \gg m$, the region $\Delta r,\vartheta)$ is determined essentially by the condition $n(\omega)>1$; with increase of r, ϑ the region $\Delta(r,\vartheta)\to 0$.

The spectral and angular distributions of hard photons will appear as follows:

$$\frac{dw}{dr}=\frac{me^4}{2(2\pi)^4}\frac{[1+(1-r)^2]}{r}\int\limits_{\Delta(r,0)}\nu^2(n^2-1)\,d\nu\cdot \tag{4.6}$$

$$\times\int\limits_{-\pi/2}^{\pi/2}\frac{\lambda_1^2(\lambda^2\varepsilon+\tau_\nu^2)}{\omega^2(\lambda_1^2+\lambda^2\varepsilon^2)}e^{-2d\lambda}\,d\Phi, \tag{4.7}$$

$$\frac{dw}{d\Omega}=\frac{me^4}{2(2\pi)^4}\frac{\alpha^2}{(\alpha^2+\vartheta^2)}\ln\left(\frac{r_{max}}{r_{min}}\right)\int\limits_{\Delta(r_{min},\vartheta)}\nu^2(n^2-1)\,d\nu \tag{4.8}$$

$$\times\int\limits_{-\pi/2}^{\pi/2}\frac{\lambda_1^2(\lambda^2\varepsilon+\tau_\nu^2)}{\omega^2(\lambda_1^2+\lambda^2\varepsilon^2)}e^{-2d\lambda}\,d\Phi,$$

$$r_{max}=f(\nu)/[\alpha^2+\vartheta^2+f(\nu)],\quad \vartheta_{max}=[f(\nu)(1-r)/r-\alpha^2].$$

From these expressions it is evident that the angular distribution of hard photons has a sharp forward directivity: $\vartheta \lesssim \alpha$, and the radiation spectrum drops rather sharply with increase of r.

Estimates show that the energy loss by hard Vavilov-Cerenkov radiation for $\omega d<1$ and $E_1 \gtrsim m^2/2\omega_{max} \sim 10^4$–$10^5$m is of the order:

$$\left(-\frac{dE}{dx}\right)^{V-C}=E_1\int\limits_{r_{min}}^{r_{max}}r\left(\frac{dw}{dr}\right)dr\sim(0{,}1-1)\frac{E_1}{m}\left(\frac{eV}{cm}\right). \tag{4.9}$$

It was taken into account that for $E_1 \gtrsim m^2/2\omega_{max}$ hard photons with energy of the order E_1 can be radiated and $r_{max}\sim 1$. For the maximum frequency of the soft photons we took the value $\sim 10^6$ cm^{-1} corresponding to a wavelength $\sim 10^{-5}$ cm. This loss coincides in order of magnitude with the energy loss by hard Vavilov-Cerenkov radiation in an unbounded medium. However, in an unbounded medium the hard Cerenkov radiation is strongly masked by similar losses, mainly by bremsstrahlung, which are of the order

$$(-dE/dx)\sim e^6Z^2N_0k_{max}/m^2\sim 10^4E_1/m\ (\text{eV/cm}),$$

for $Z^2N_0 \sim 10^{24}$ cm^{-3}, where N_0 is the number of nuclei with charge eZ per unit volume (this estimate corresponds to light nuclei, which occur mainly in the composition of glasses). On passage of electrons outside the medium, there is no bremsstrahlung (more accurately, there remains the weak background of bremsstrahlung in the residual gas and from electrons which accidentally hit the dielectric surface). This makes possible observation of spontaneous Vavilov-Cerenkov radiation.

The yield of hard Cerenkov photons can be estimated by means of Eq. (4.6) or (4.7). It is

$$\frac{dW}{dx}=\int\limits_{r_{min}}^{r_{max}}\frac{dw}{dr}dr\sim 10^{-7}-10^{-6}\ \text{phot/cm} \tag{4.10}$$

The yield of bremsstrahlung photons of comparable energy for $N_0^2Z \sim 10^{24}$ cm^{-3} would be $\sim 10^{-2}$ photons/cm.

Let us consider now stimulated hard Vavilov-Cerenkov radiation by an electron traveling in vacuum parallel to the dielectric boundary, under the influence of a beam of soft photons which all have the same momentum q_1 and frequency ω_1 and hit the dielectric boundary at a total-internal-reflection angle $\theta_1 \le \theta_0$. The probability of stimulated Vavilov-Cerenkov radiation is

$$dw=|S^{(2)}|^2\frac{(2\pi)^3}{2}N_1\delta(\mathbf{q}-\mathbf{q}_1)\,d\mathbf{q}\,d\mathbf{k}\,d\mathbf{p}_2, \tag{4.11}$$

where N_1 is the number of soft photons per unit volume.

Omitting the calculations, which are similar to the foregoing, and considering a moderately ultrarelativistic case in which $\alpha \sim \vartheta \ll 1$, $\nu/\alpha \ll 1$, we obtain for the spectral distribution of hard photons:

$$\frac{dw}{dr}=\frac{e^4}{8(2\pi)^2}\frac{N_1}{E_1\omega}\left[1+\left(1-\frac{r}{r_{max}}\right)^2\right]\left[1+\frac{n_1^2-1}{2\gamma_1^2}\sin^2\theta_1\right]$$

$$\times\frac{\lambda_1^2(\lambda^2\varepsilon+\tau_\nu^2)}{\omega_1(\lambda_1^2+\lambda^2\varepsilon^2)}e^{-2d\lambda},\quad r_{max}=\frac{2\nu_1\gamma_1}{\alpha},\quad \gamma_1=\beta n_1\cos\theta_1-1,$$

$$n_1=n(\omega_1).$$

In the case of induced radiation, condition (4.3) is written in the form

$$\cos\vartheta=\cos\vartheta_0=\frac{1}{\beta}-\frac{\omega_1}{k\beta}(\beta n_1\cos\theta_1-1)\frac{E_1-k}{E_1}.$$

Since θ_1 and ω_1 are fixed, the emission angle of the hard photon is uniquely related to its energy.

The spectrum (4.12) has an approximately rectangular shape—the probabilities for emission of any hard Cerenkov photon are approximately the same, in contrast to bremsstrahlung, for which the emission of low-energy photons is more probable. The maximum energy is achieved for $\vartheta = 0$, and the minimum for $\vartheta \sim 1$ and lies in the soft x-ray region.

The dependence of (4.12) on θ_1 is characteristic. When $\cos \vartheta_1 \to 1/\beta n_1$, the quantity $\gamma \to 0$ and the probability of hard radiation rises strongly, but simultaneously the energy of the soft photons decreases in accordance with the following formula:

$$k=2\omega_1\gamma_1/(\alpha^2+\theta^2+2\alpha\nu\gamma_1).$$

The probability of stimulated hard radiation in the problem with a dielectric-vacuum boundary is of the order

$$w \sim \frac{e^4 N_1}{\gamma_1^2 E_1^2 \omega_1} k_{max} \sim \frac{e^4 N_1}{\gamma_1 m^2}$$

and is comparable with the probability of stimulated radiation in an unbounded medium, but in contrast to the unbounded medium the strongly competing effect of bremsstrahlung will be absent. For $N_1 \sim 10^{18}$ photons/cm (this corresponds to the intensity of a beam of soft photons with energy $\hbar\omega \sim 2$ eV amounting to $\sim 10^{10}$ W/cm^2) and $\gamma_1 \sim 0.1$, the yield of hard photons will be $w \sim 10^{-4}$ photons/cm.

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