Magnetic moment oscillations in a domain wall[1]

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The problem of small oscillations of the magnetic moments in a 180° domain wall of the Bloch type is solved. The intrinsic magnetic field of the oscillating moments is taken into account exactly. The problem is reduced to a set of integral equations which can be solved exactly in the long- or short-wave limits. Three modes of surface oscillations exist, one of which begins at the anisotropy frequency. It is shown that reflection of intradomain spin waves from the domain wall produces co-moving surface waves along with the reflected wave. A rigorous solution confirms the existence, against a background of a continuous spectrum, of resonance levels corresponding to surface magnetostatic oscillations [7]. The solution can be used to calculate the damping of the oscillations.

Interest in spin-wave spectra in magnets having a domain structure is due to a number of causes. Resonance effects in the presence of domains are much more varied than in saturated magnets. Experiment has shown, in particular, the existence of absorption bands consisting of sets of resonance lines[1]. The shapes and positions of the lines depend on the domain structure and on the applied magnetic field. It is difficult to interpret such a spectrum on the basis of the available theoretical data.

On the other hand, it is known that many important features of NMR in ferromagnets, as well as galvanomagnetic and thermodynamic properties of magnets in a nonsaturating field, also depend significantly on the domain structure[2]. Homogeneous precession of the magnetic moments in the presence of a domain structure was investigated theoretically in[3]. The study of the spectra of inhomogeneous oscillations was initiated by Winter[4], who considered a solitary Bloch-type wall. The main difficulty in the investigation of inhomogeneous oscillations is how to account correctly for the intrinsic magnetic field of the oscillating moments. Winter[4] dispensed with attempts at a self-consistent allowance for the dipole-dipole-interaction energy, and introduced a demagnetizing-field model in which it is assumed that

$$W_{\text{dip}}=2\pi M_y^2,\quad h_x=h_z=0,\quad h_y=-4\pi M_y,$$

OZ is the anisotropy axis, and the OY axis is perpendicular to the plane of the wall. According to[4], there are two oscillation modes in the system. One constitutes volume (intradomain) spin waves that are spatially modulated under the influence of the wall. The other, low-frequency mode describes surface oscillations propagating in the plane of the wall and localized near the wall. The magnetization oscillations in a magnet with a periodic domain structure was investigated in the Winter model[4] by Farztdinov and Turov[5]. The role of the external magnetic field in the same approximation was considered in[6].

The Winter model[4] yields exact results for one-dimensional oscillations that are homogeneous in the plane of the wall ($\mathbf{M}=\mathbf{M}(y)$). However, if the inhomogeneity of the oscillations in the XZ plane is small, it is necessary to take exact account of the dipole energy. It is therefore natural to attempt to go beyond the framework of the assumptions made by Winter[4]. Mints and the present author[7] did not take into account the exchange interaction, and assumed the domain wall to be a geometrical boundary. It turned out that there exists new surface-wave modes not observed by Winter[4]. However, those spectral modes whose formation receives an appreciable contribution from exchange interaction drop out of consideration in such an approach. Janak[8] started with an exact Hamiltonian but discarded a number of significant terms in the solution, without any justification, so that his results are at best hypothetical. We note finally, a recent article by Kurkin and Tankeev[9], where an attempt was made to obtain the results of earlier studies[4,7,8] within the framework of a unified calculation scheme.

Thus, in spite of numerous theoretical studies, the problem of determining the magnetic spectrum in the presence of a domain structure has not been rigorously solved. In this paper, the intrinsic magnetic field of the oscillating moments is taken into account exactly, i.e., the linearized Landau-Lifshitz equations and the magnetostatic equations are solved in a self-consistent manner. The problem can be reduced to a system of integral equations that admit of an analytic solution in the limiting cases of long and short waves. In the intermediate range of parameters, a numerical solution can be obtained if necessary.

FORMULATION OF PROBLEM. DERIVATION OF INTEGRAL EQUATIONS

Let the magnetic moments form at equilibrium a Bloch wall (The XZ plane)

$$M_z^0=M_s\cos\theta(y),\quad M_x^0=M_s\sin\theta(y),\quad M_y^0=0;$$
$$\cos\theta(y)=\operatorname{th}(y/\delta);\quad \sin\theta(y)=\operatorname{ch}^{-1}(y/\delta);$$

M_S is the saturation magnetization; $\delta=(\alpha/\beta)^{1/2}$ is the thickness of the domain wall; α and β are respectively the exchange constants and the anisotropy. We put

$$\mathbf{M}=\mathbf{M}^0(y)+\mathbf{m}(\mathbf{r},t),\quad |\mathbf{m}|\ll|\mathbf{M}^0|.$$

The magnetostatics equations and the linearized Landau-Lifshitz equations take the form

$$\begin{gathered}\mathbf{h}=-\nabla\psi,\quad \operatorname{div}(\mathbf{h}+4\pi\mathbf{m})=0,\\ \partial m_x/\partial t=-\gamma(m_yH_z^0-M_z^0H_y),\\ \partial m_y/\partial t=-\gamma[M_z^0H_x+m_zH_x^0-M_x^0H_z-m_xH_z^0],\\ m_xM_x^0+m_zM_z^0=0.\end{gathered}\tag{1}$$

In (1) we have put

$$H_z^0=\beta M_z^0+\alpha\Delta M_z^0,\quad H_x^0=\alpha\Delta M_x^0,\quad H_x=h_x+\alpha\Delta m_x,$$
$$H_y=h_y+\alpha\Delta m_y,\quad H_z=\beta m_z+h_z+\alpha\Delta m_z,$$

$\gamma>0$ is the gyromagnetic ratio.

The last relation in (1) is the linearized first integral of the Landau-Lifshitz system, M^2 = const, and can be naturally employed in place of one of the equations. It can be satisfied by putting

$$m_x=\pm m_\parallel\cos\theta(y),\quad m_z=\mp m_\parallel\sin\theta(y).$$

the plus and minus signs correspond to the possibility of two polarizations of the oscillation at a fixed frequency. We choose, for the sake of argument, the upper sign. We assume that all the quantities are proportional to $\exp[i(k_\parallel\cdot\rho-\omega t)]$ (ρ are the coordinates in the plane of the wall) and confine ourselves for simplicity to the case $k_x = k$ and $k_z = 0$. This limitation is not fundamental, but simplifies the calculations greatly. Introducing the dimensionless coordinate $\xi = y/\delta$ and putting

$$\omega_a=\beta\gamma M_s,\quad \omega_M=4\pi\gamma M_s,\quad \eta=k_\parallel\delta,$$
$$a=4\pi m_y,\quad b=4\pi m_\parallel,\quad \psi=\delta\varphi,$$

we arrive at the following initial equations:

$$\begin{aligned}
&i\omega a=\omega_a(L^-L^+-\eta^2)b-i\eta\omega_M\varphi\,\mathrm{th}\,\xi,\\
&i\omega b=-\omega_a(L^-L^+-\eta^2)a+\omega_M\varphi',\\
&\varphi''-\eta^2\varphi=i\eta b\,\mathrm{th}\,\xi+a'.
\end{aligned}\tag{2}$$

The primes denote differentiation with respect to ξ.

The differential operators $L^\pm$ are equal to

$$L^\pm=d/d\xi\pm\mathrm{th}\,\xi,\quad L^-L^+=d^2/d\xi^2-\cos 2\theta(\xi).$$

The "effective local anisotropy"[4], which maintains the wall in a definite equilibrium position, is not taken into account by us, i.e., we assume the frequency of the homogeneous wall oscillations to be equal to zero.

The system (2) depends parametrically on η. The case of one-dimensional oscillations $\eta = 0$, as noted in the Introduction, is quite special. Indeed, at $\eta = 0$ we have $\varphi' = a$ and the first two equations yield the solutions[4]

$$\begin{aligned}
&a,b\sim\frac{(\mathrm{th}\,\xi-ik_0)\exp(ik_0\xi)}{2\pi(k_0^2+1)}\equiv\chi_{k_0},\\
&\omega>[\omega_a(\omega_a+\omega_M)]^{1/2},\\
&a=0,\quad b\sim 1/2\,\mathrm{ch}\,\xi\equiv\chi_0,\quad \omega=0,
\end{aligned}\tag{3}$$

$k_0^2 > 0$ is the root of the equation

$$\omega^2-\omega_a\omega_M(k^2+1)-\omega_a^2(k^2+1)^2=0.$$

The continuous-spectrum functions χ_k describe volume oscillations. The discrete-spectrum function χ_0 yields in our case simply the displacement of the wall position.

We note that at $\eta = 0$ the functions χ_k and χ_k^* contain only the wave incident on the domain wall. Yet even the solution of a second-order equation should in the general case contain both an incident and a reflected wave. The explanation lies in the fact that the system of functions χ_k and χ_0 satisfies not only (2) but also the simple first-order equation

$$L^+\chi_k=e^{ik\xi}/2\pi,\quad L^+\chi_0=0.$$

Thus, at $\eta = 0$ the system (2) is reduced, via the natural physical requirement that the solutions be finite and continuous, to a first-order equation. It is obvious that at $\eta \neq 0$ this reduction is impossible and the solution becomes more complicated. What changes, in particular, is its asymptotic behavior. We note also that even at $|\eta| \ll 1$ the system (2) cannot be solved by iteration by putting $\varphi' = a + O(\eta)$. This procedure again lowers the order of the system of equations, which leads to an incorrect asymptotic behavior of the solutions and as a consequence yields an incorrect spectrum of the oscillations. For a correct solution of the problem it is necessary to separate (in our representation or another) the asymptotic behavior of the solutions. We proceed in the following manner:

We introduce instead of the unknown function $a(\xi)$ a new function $u(\xi) = a(\xi) - i\eta b(\xi)$. We multiply the second equation of the system (2) by $i\eta$, add it to the first, and use the resultant equation instead of the first equation of (2). We introduce next the Green's function $G(\xi-\mu)$ of the equation $\varphi'' - \eta^2\varphi = 0$:

$$\begin{aligned}
&G=-\frac{1}{2\pi}\int\frac{e^{iq(\xi-\mu)}dq}{q^2+\eta^2}=-\frac{1}{2\rho}e^{-\rho|\xi-\mu|},\\
&G''-\eta^2G=\delta(\xi-\mu).
\end{aligned}\tag{4}$$

We designate from now on $\rho = |\eta|$. Determining φ from the third equation of the system (2) and substituting in the two others, we obtain

$$\begin{aligned}
&i\omega u=\omega_a(1+\eta^2)(L^-L^+-\eta^2)b-i\eta\omega_a(L^-L^+-\eta^2)u+2\eta\omega b\\
&+i\eta\omega_M\int d\mu\,L_\xi^-G\frac{du(\mu)}{d\mu}-\eta^2\omega_M\int d\mu\,L_\xi^-GL_\mu^+b(\mu),\\
&i\omega b=-\omega_a(L^-L^+-\eta^2)u-i\eta\omega_a(L^-L^+-\eta^2)b\\
&+\omega_Mu+i\eta\omega_M\int\frac{dG}{d\xi}L_\mu^+b(\mu)\,d\mu+\eta^2\omega_M\int G(\xi-\mu)u(\mu)\,d\mu.
\end{aligned}\tag{5}$$

The subscript ξ or μ of the operators $L^\pm$ identifies the variable on which the operator acts. Here and throughout, unless otherwise indicated, the integration limits are $-\infty$ and ∞.

We now expand $u(\xi)$ and $b(\xi)$ in the complete orthonormal system of functions χ_k and χ_0, first used by Janak[8]:

$$u(\xi)=u_0\chi_0(\xi)+\int u_k\chi_k(\xi)\,dk,\quad b(\xi)=b_0\chi_0+\int b_k\chi_k dk.\tag{6}$$

We multiply (5) by $2\pi(k^2+1)\chi_k^*(\xi)$ and integrate with respect to ξ. Using the condition for the orthogonality of the functions χ_k and χ_0[8] we obtain

$$\begin{aligned}
&T(k)b_k=-i\left\{\alpha(k)u_k-\frac{i\pi\eta\omega_Mk(k^2+1)}{2\,\mathrm{ch}\,(k\pi/2)}u_0\right.\\
&\left.+\frac{\eta\omega_Mk(k^2+1)}{2}⨍\frac{u_p\,dp}{(p^2+1)\,\mathrm{sh}[\pi(p-k)/2]}\right\},\\
&\frac{i\alpha(k)b_k}{k^2+\eta^2}=[\omega_Mk^2+\omega_a(k^2+\eta^2)(k^2+\eta^2+1)]\frac{u_k}{k^2+\eta^2}\\
&+\frac{i\eta\omega_M}{2}⨍\frac{pb_pdp}{(p^2+\eta^2)\,\mathrm{sh}[\pi(p-k)/2]}+\eta^2\frac{\omega_M}{2}\int\frac{u_p[S(k)-S(p)]dp}{(p^2+1)\,\mathrm{sh}[\pi(p-k)/2]}\\
&-i\pi\eta^2\frac{\omega_MS(k)}{2\,\mathrm{ch}\,(k\pi/2)}u_0.
\end{aligned}\tag{7}$$

Multiplying (5) by $2\chi_0(\xi)$ and integrating, we obtain a second pair of equations:

$$\begin{aligned}
&i(\omega-\eta^3\omega_a)u_0=[2\eta\omega-\eta^2(1+\eta^2)\omega_a]b_0,\\
&i(\omega-\eta^3\omega_a)b_0=[\omega_M(1-J_0)+\eta^2\omega_a]u_0+\frac{\eta\omega_M}{2}\int\frac{pb_pdp}{(p^2+\eta^2)\,\mathrm{ch}(\pi p/2)}\\
&+i\eta^2\frac{\omega_M}{2}\int\frac{u_pS(p)\,dp}{(p^2+1)\,\mathrm{ch}(\pi p/2)},
\end{aligned}\tag{8}$$

is an integral in the sense of the principal value. In (7) and (8) we put

$$\begin{aligned}
&\alpha(k)=\omega(k^2+\eta^2)-\eta[\omega_Mk^2+\omega_a(k^2+\eta^2)(k^2+\eta^2+1)],\\
&T(k)=\omega_a(1+\eta^2)(k^2+\eta^2)(k^2+\eta^2+1)+\eta^2\omega_M(k^2+1)-2\eta\omega(k^2+\eta^2),\\
&J_0=\frac{\pi}{4}\eta^2\int\frac{dq}{(q^2+\eta^2)\,\mathrm{ch}^2(q\pi/2)};\\
&S(k)=\frac{k}{k^2+\eta^2}+\frac{1}{2}⨍\frac{dq\,\mathrm{cth}[\pi(q-k)/2]}{q^2+\eta^2},
\end{aligned}\tag{9}$$

$$\fint \frac{dq\,\mathrm{cth}[\pi(q-k)/2]}{q^2+\mu^2} = N(k,\mu) = -\frac{2}{\mu}\left[\frac{k}{k^2+\mu^2} + \Phi(k,\mu)\right],$$

$$\Phi(k,\mu) = \frac{i}{2}\left[\psi\left(1+\frac{\mu-ik}{2}\right) - \psi\left(1+\frac{\mu+ik}{2}\right)\right], \quad \mathrm{Re}\,\mu > 0.$$

$\psi(z) = d \ln \Gamma(z)/dz$ is the Euler ψ function.

If $\mathrm{Re}\,\mu < 0$, then μ is replaced by $-\mu$. In the derivation we took into account the relation

$$\fint \frac{dq}{q^2+\mu^2} \mathrm{sh}^{-1}\left[\frac{\pi}{2}(q-k)\right] \mathrm{sh}^{-1}\left[\frac{\pi}{2}(p-q)\right]$$
$$= \frac{2}{\pi} \iint e^{-iky+ip\xi} G(y-\xi,\mu)\,\mathrm{th}\, y\,\mathrm{th}\,\xi\, dy\, d\xi = -\frac{4\delta(p-k)}{k^2+\mu^2} + \frac{N(k,\mu)-N(p,\mu)}{\mathrm{sh}[\pi(p-k)/2]}. \tag{10}$$

Finally, we determine b_k and b_0 from the first equations of (7) and (8) and substitute them in the second equations. Using (10), we obtain

$$\frac{D(k)}{T(k)} u_k - \int \frac{u_p[F(p)-F(k)]\,dp}{(p^2+1)\,\mathrm{sh}[\pi(p-k)/2]} = \frac{i\pi F(k)}{\mathrm{ch}(k\pi/2)} u_0, \tag{11}$$

$$u_0(\omega^2-\eta^4\omega_a^2) + [2\eta\omega - \eta^2(1+\eta^2)\omega_a]\cdot[\omega_M(1-J)u_0 - Q] = 0. \tag{12}$$

In (11) and (12) we used the notation

$$D(k) = (k^2+\eta^2)[\omega^2 - \omega_a\omega_M(k^2+\eta^2+1) - \omega_a^2(k^2+\eta^2+1)^2],$$
$$Q = i\int \frac{u_p F(p)\,dp}{p^2+1}, \tag{13}$$
$$J = \frac{\pi}{4}\eta^2 \int \frac{dp[\omega_M(p^2+1) + \omega_a(1+\eta^2)(1+p^2+\eta^2) - 2\eta\omega]}{T(p)\,\mathrm{ch}^2(\pi p/2)}$$

$$F(k) = \frac{\eta\omega_M k}{2T(k)}[(\omega-\eta\omega_M)(k^2+1) + 2\eta^2\omega - \eta\omega_a(k^2+\eta^2+1)(k^2+\eta^2+2)]$$
$$-\frac{\eta^2\omega_M}{4} \fint \frac{dq}{T(q)} \mathrm{cth}\frac{\pi}{2}(q-k)[\omega_M(q^2+1) + \omega_a(1+\eta^2)(1+q^2+\eta^2) - 2\eta\omega],$$

Formulas (11) and (12) are the basic equations of our problem and are an exact consequence of the initial differential equations (2). The character of the solutions of (11) is determined by the behavior of the coefficient $D(k)$. $D(k) = 0$ is the dispersion equation of the spin waves inside the domain, far from the wall. If $\omega^2 > \omega_0^2 = \omega_a^2(1+\eta^2)^2 + \omega_a\omega_M(1+\eta^2)$, then $D(k) = 0$ has a positive root $k^2 = k_0^2 > 0$. In this frequency region, volume spin waves can propagate in the domain. Then (11) has a solution $u_k = \mathrm{const}\,\delta(k \pm k_0) + u_k^{\mathrm{scat}}$. If $\eta = 0$, then $u_k^{\mathrm{scat}} = 0$ and $u \sim \chi k_0$, i.e., it coincides with Winter's solution[4]. On the other hand, if $\eta \neq 0$, then the integral term in (11) describes the scattering of spin waves on going through the domain wall. The coefficients u_0 and b_0 are then determined by the amplitude of the wave incident from the domain on the wall.

At $\omega^2 < \omega_0^2$, there are no volume oscillations in the domain. Then (11) has only an "induced" solution of the surface-oscillation type, in which $u_k \sim u_0$. In this case (12) is the dispersion equation of the surface oscillations.

An analytic solution of (11) can be obtained only in the limiting cases of long waves ($\rho = |\eta| \ll 1$) and short waves ($\rho \gg 1$). We present here, for future use, the expressions for $T(k)$ and $F(k)$ in these limiting cases. Let $T(k) = \omega_a(1+\eta^2)(k^2+\delta_1^2)(k^2+\delta_2^2)$, where $\delta_{1,2}^2$ are the roots of the equation $T(k^2) = 0$ taken with the negative sign. For the sake of argument we put $\delta_1^2 > \delta_2^2$. Putting $\sigma = \mathrm{sign}\,\eta$, we have at $|\eta| \ll 1$

$$\delta_1^2 \approx 1 + \eta^2 - 2\eta\omega/\omega_a, \quad \delta_2^2 = \eta^2\delta^2,$$
$$\delta^2 \approx 1 + \frac{\omega_M}{\omega_a} + 2\eta\frac{\omega\omega_M}{\omega_a^2} + 2\eta^2\frac{\omega_M}{\omega_a^3}(2\omega^2 - \omega_a^2),$$

$$F(k) \approx \rho\frac{\omega_M}{2}\left\{\frac{(\gamma+\delta)k}{k^2+\eta^2\delta^2} + \delta\Phi(k)\right\}, \tag{14}$$
$$\Phi(k) \equiv \Phi(k,0) = \frac{\pi}{2}\mathrm{cth}\frac{\pi}{2}k - \frac{1}{k},$$
$$\gamma \approx \frac{\omega\sigma}{\omega_a} - \frac{\rho}{\omega_a^2}(\omega_a\omega_M + 2\omega_a^2 - 2\omega^2)\left(1 + 2\eta\frac{\omega}{\omega_a}\right).$$

At $\rho \gg 1$ we need only the value of $F(k)$ in the region $k^2 \ll \eta^2$:

$$F(k) \approx -\omega_M k(k^2+1)/6\eta^2. \tag{15}$$

We now proceed to solve the problem.

SPECTRUM OF SURFACE OSCILLATIONS

We consider first the long-wave oscillations. The integral equation (11), with allowance for (14), takes the form

$$D(k)u_k + \frac{\rho(1+\eta^2)}{2}\omega_a\omega_M(\gamma+\delta)(k^2+\delta_1^2)\int \frac{u_p(p-k)(kp-\eta^2\delta^2)\,dp}{(p^2+1)(p^2+\eta^2\delta^2)\,\mathrm{sh}[\pi(p-k)/2]}$$
$$-\rho\frac{\omega_a\omega_M\delta}{2}(k^2+1)(k^2+\eta^2\delta^2)\int \frac{u_p[\Phi(p)-\Phi(k)]\,dp}{(p^2+1)\,\mathrm{sh}[\pi(p-k)/2]} \tag{16}$$
$$= \frac{i\pi\rho\omega_a\omega_M(k^2+1)u_0}{2\,\mathrm{ch}(k\pi/2)}[(\gamma+\delta)k + \delta k^2\Phi(k)],$$

As seen from (16), u_k is an odd function of k. In spite of the fact that the integrals in (16) are preceded by the parameter $\rho \ll 1$, it is impossible to solve (16) by iteration, since the integrand in the first of the integrals of (16) is large at small $p^2 \sim \eta^2$ and the integration makes a contribution $\sim\rho^{-1}$. However, the behavior of u_k at small $|k| \ll 1$ can be easily obtained, inasmuch as in this case the main contribution to the integrals is made by small $|p| \ll 1$. Recognizing that at $k^2 \ll 1$ we have $D(k) \approx (\omega^2 - \omega_0^2)(k^2+\eta^2)$ and $\omega_0^2 \approx \omega_a(\omega_a + \omega_M)$, we obtain

$$u_k \approx \frac{i\pi\rho A k u_0}{2(\omega^2-\omega_0^2)(k^2+\eta^2)}, \quad A \approx \frac{\omega_M(\omega^2-\omega_0^2)}{\omega\sigma - \omega_a}. \tag{17}$$

Turning to the dispersion equation (12), we note that the integral Q which enters in it is also determined by the values of u_p at $|p| \ll 1$.

Substituting u_p from (17), we obtain

$$Q = i\int \frac{u_p F(p)\,dp}{p^2+1} \approx -\frac{\pi^2\rho\omega_M(\gamma+\delta)}{4(\omega^2-\omega_0^2)(\delta+1)} A u_0. \tag{18}$$

It is seen from (12) that the term with Q can be neglected if $\rho \ll 1$, $|\omega| \ll \omega_a$, and $|\omega \ll \omega_M$. However, at $\omega \approx \sigma\omega_a$ we have $A \to \infty$, and the contribution of this term to the dispersion equation becomes significant. To obtain a unified dispersion equation that is suitable also in the region $\omega \approx \sigma\omega_a$, it is necessary to determine the denominator of the coefficient A with greater accuracy. To this end, in turn, it is necessary to know the function u_k in the entire range of its variation, and not only at small k. We shall seek the solution of (16) in the form

$$u_k = \frac{i\pi\rho(k^2+1)u_0}{2D(k)\,\mathrm{ch}(k\pi/2)} W_k,$$
$$W_k = Ak + Bk^2\Phi(k) + \rho f(k) + \ldots \tag{19}$$

We substitute (19) in (16) and neglect $\rho f(k)$ under the integral sign, assuming that the expansion of $f(k)$ at small k begins with terms $\sim k^3$. We calculate the remaining integrals, using (10) and the relations

$$\fint \frac{q\,dq}{q^2+\mu^2}\mathrm{ch}^{-1}q\frac{\pi}{2}\,\mathrm{sh}^{-1}\frac{\pi}{2}(q-k) = \frac{2}{\mathrm{ch}(\pi k/2)}\left\{\frac{\mu}{k^2+\mu^2} + \psi\left(\frac{1+\mu}{2}\right) - \Lambda(k,\mu)\right\},$$
$$P(k,\alpha) = \int \frac{(\alpha-p)(p-k)}{p^2+\mu^2}\mathrm{sh}^{-1}\frac{\pi}{2}(p-k)\,\mathrm{sh}^{-1}\frac{\pi}{2}(\alpha-p)\,dp$$

$$=2\,\mathrm{sh}^{-1}\frac{\pi}{2}(\alpha-k)\left\{\frac{(\alpha-k)(1-\mu)}{\mu}+\frac{(\mu^2-\alpha k)}{\mu}[\Phi(\alpha,\mu)-\Phi(k,\mu)]+(\alpha+k)[\Lambda(\alpha,\mu)-\Lambda(k,\mu)]\right\},$$

$$\int\frac{p\Phi(p)(p-k)}{p^2+\mu^2}\mathrm{sh}^{-1}\frac{\pi}{2}p\,\mathrm{sh}^{-1}\frac{\pi}{2}(p-k)\,dp=\left[\frac{\partial P(k,\alpha)}{\partial\alpha}\right]_{\alpha=0}, \tag{20}$$

$$\Lambda(k,\mu)={}^1/_2\left[\psi\left(1+\frac{\mu-ik}{2}\right)+\psi\left(1+\frac{\mu+ik}{2}\right)\right],\quad \mathrm{Re}\,\mu>0.$$

Comparing the coefficients of $k^2\Phi(k)$ in the principal order in ρ, we express B in terms of A. Further, gathering, after integration, all the terms that contain the small parameter ρ and are proportional to k^2 and to the higher powers of k at $|k| \ll 1$, we determined $f(k)$. Stipulating that the coefficients of the terms linear in k vanish, we obtain A accurate to the terms linear in ρ in the denominator. The corrections $\sim\eta^2$ in the denominator can be obtained by taking into account $\rho f(k)$ under the integral sign; we do not present the result because it is extremely unwieldy. We have ultimately

$$B\approx-\frac{\omega_a\omega_M(\gamma+\delta)}{1+\delta}a_\rho A+\omega_a\omega_M\delta,$$

$$A\approx\omega_M(\omega^2-\omega_0^2)[\omega\sigma-\omega_a-\rho\omega_M R+O(\eta^2)]^{-1}, \tag{21}$$

$$R=\frac{\omega\sigma+\omega_0}{\omega_a+\omega_0}+(\omega_0^2-\omega^2)\sum_{i=1,2}\frac{a_i}{k_i}\left[1+\frac{k_i^2}{2}\psi'\left(1+\frac{k_i}{2}\right)\right],$$

$\psi' = d\psi(z)/dz$ is the derivative of the Euler ψ function with respect to its argument. The quantities a_i and k_i in (19) are determined by the expansion of $[D(k)]^{-1}$:

$$[D(k)]^{-1}=\frac{a_\rho}{k^2+\eta^2}+\sum_{i=1,2}\frac{a_i}{k^2+k_i^2},$$

$$a_\rho=(\omega^2-\omega_a^2-\omega_a\omega_M)^{-1}\approx(\omega^2-\omega_0^2)^{-1},$$

$$a_{1,2}=\mp\frac{1}{\omega_a^2(k_1^2-k_2^2)(k_{1,2}^2-\eta^2)}, \tag{22}$$

$$k_{1,2}^2=1+\eta^2+\frac{\omega_M}{2\omega_a}\pm\frac{1}{2|\omega_a|}(\omega_M^2+4\omega^2)^{1/2},$$

$$k_2^2=-k_0^2\quad\text{at}\quad\omega^2>\omega_0^2.$$

Substituting Q and A in (12), we obtain the dispersion equation for the surface waves at $\rho \ll 1$:

$$\omega^2+2\rho\omega_M\omega\sigma-\eta^2\omega_a\omega_M+\eta^2\frac{\pi^2\omega_M^2\omega\sigma(\omega\sigma+\omega_0)}{2(\omega_0+\omega_a)\{\omega\sigma-\omega_a-\rho\omega_M R-O(\eta^2)\}}=0. \tag{23}$$

Equation (23) determines the low-frequency ($|\omega| \ll \omega_a, \omega_M$) modes and one low-frequency mode. The dispersion of the low-frequency oscillations is linear[2)]:

$$\omega_{\mathrm{I,\,II}}\approx\rho\{-\sigma\omega_M\pm(\omega_M^2+\omega_a\omega_M)^{1/2}\}. \tag{24}$$

The high-frequency mode begins with the frequency $\omega = \sigma\omega_a$ and also has a linear dispersion on the initial section:

$$\omega_{\mathrm{III}}\approx\sigma\{\omega_a+\rho R\omega_M+O(\eta^2)\}. \tag{25}$$

The coefficient R in (25) is of the order of unity if $\omega_M/\omega_a = \lambda_0 \sim 1$. At $\lambda_0 \gg 1$ and $\lambda_0 \ll 1$, only the term with a_2 remains in the sum and we obtain $R \approx \pi^2/4$ at $\lambda_0 \gg 1$ and $R \approx (\lambda_0/2)^{-1/2}$ at $\lambda_0 \ll 1$.

We write out also the solutions corresponding to the surface oscillations (24) and (25):

$$u=u_0\chi_0(\xi)+\frac{i\rho u_0}{4}\int\frac{(\mathrm{th}\,\xi-ik)e^{ik\xi}}{D(k)\,\mathrm{ch}(k\pi/2)}\{Ak+Bk^2\Phi(k)+\ldots\}dk,$$

$$b=b_0\chi_0(\xi)+\frac{\sigma\rho u_0}{4}\int\frac{(\mathrm{th}\,\xi-ik)e^{ik\xi}}{(k^2+1)D(k)\,\mathrm{ch}(k\pi/2)}\left\{Ak+\frac{\omega\sigma}{\omega_a}Bk^2\Phi(k)+\frac{\omega_M}{\omega_a}k^3[\omega_0^2+\omega_a^2(k^2+1)]+\ldots\right\}dk, \tag{26}$$

$$u=a-i\eta b,\quad u_0\approx-i\rho(2\omega\sigma-\rho\omega_a)b_0/\omega.$$

As seen from (26), in the low-frequency region there are excited, in the main, oscillations in the plane of the wall ($|a| \sim \rho|b|$). The contribution of the integral term at $|\xi| \lesssim 1$ is small. However, the integrals in (26) contain a long-range part determined by the pole $k = \pm i\rho$, to that at $|\xi| \gg 1$ the magnetization is determined by the integral term

$$b\approx i\eta\frac{\pi}{4}u_0\frac{\omega_M}{\omega_a}e^{-\rho|\xi|}.$$

At higher frequencies (the mode ω_{III}), the form of the oscillation becomes more complicated, since the integral term in the region of the wall is of the same order as the term outside the integral sign. Indeed, as seen from (21) and (23), $B - A \sim \rho A$ and $A \approx 2\omega_a^2/\pi^2\eta^2$. Thus,

$$b\approx b_0\chi_0(\xi)-\frac{ib_0\omega_a^2}{2\pi}\int\frac{(\mathrm{th}\,\xi-ik)e^{ik\xi}}{(k^2+1)D(k)\,\mathrm{sh}(\pi k/2)}k^2\,dk.$$

Asymptotically at $|\xi| \gg 1$ we have

$$b\approx-\frac{b_0\omega_a}{\pi\omega_M}e^{-\rho|\xi|}.$$

We turn now to an investigation of the short-wave surface oscillations ($|\eta| \gg 1$). Inasmuch as the right-hand side of (11) is proportional to $[\cosh^{-1}(k\pi/2)]$, it suffices to know the kernel of Eq. (11) and the function u_k itself in the region $k^2 \ll \eta^2$ only. Using $F(k)$ from (15) and estimating the integral J from (13), ($J \approx 1 - {}^1\!/_3\eta^2$), we transform (11) and (12) into

$$(k^2+\Delta^2)u_k+\frac{\lambda_0}{12}\int\frac{u_p[k(k^2+1)-p(p^2+1)]dp}{(p^2+1)\,\mathrm{sh}[\pi(p-k)/2]}=\frac{i\pi u_0\lambda_0 k(k^2+1)}{12\,\mathrm{ch}(k\pi/2)},$$

$$u_0\left(\Delta^2-1-\frac{\lambda_0}{3}\right)=-\frac{i\lambda_0}{12}\int pu_p\,dp, \tag{27}$$

$$\Delta^2=\frac{\omega_a^2\eta^4-\omega^2}{2\eta^2\omega_a^2}+1+\frac{\lambda_0}{2},\quad \lambda_0=\frac{\omega_M}{\omega_a}.$$

The equation $\Delta = 0$ determines the boundary of the volume-oscillation spectrum.

If $\lambda_0 \ll 1$, then the first equation of (27) is solved by iteration, and from the second we obtain $\Delta^2 = 1 + \lambda_0/3 + O(\lambda_0^2)$ or

$$\omega=\pm(\eta^2\omega_a+\omega_M/6). \tag{28}$$

The result means that a small gap $\Delta\omega \approx \omega_a + \omega_M/3$ exists between the frequencies of the short-wave volume and surface oscillations. This fact was already noted by Kurkin and Tankeev[9], but is proved rigorously here for the first time.

In the opposite limiting case $\lambda_0 \gg 1$, Eq. (27) cannot be solved. It can only be stated that $\Delta^2 \sim \lambda_0$, and that at $\eta^2 \gg \lambda_0$ the equation for the surface-oscillation frequencies is analogous to (28), but with different coefficients of ω_M. The "gap" between the frequencies of the volume and surface oscillations is of the same order as between the different surface-oscillation modes. We note finally that at $\eta^2 \sim 1$ all the terms of (12), including the integral Q, are of the same order. The oscillation spectrum in the region $|\eta| \sim 1$ can therefore be obtained only by numerically integrating (11) and (12).

We now compare our results with those of[8,9]. We neglect in (8) the integral terms and equate to zero the determinant of the remaining system of equations for u_0 and b_0. The expression obtained in this manner for ω_{sur} coincides with formula (24) of[9], provided we put in the latter $k_z = 0$ and $\omega_a' = 0$, and recognize that $2(1 - J_0) = J_2$ (ω_a' is the frequency of the effective local anisotropy," and J_2 is an integral introduced in[8,9].

Of course, this procedure for finding the surface-oscillation spectrum is incorrect in the general case. In particular, at $|\eta| \gtrsim 1$ all the terms of (8) are of the same order, so that there is not sufficient justification for the conclusions drawn in[8,9] concerning the character of the spectrum in this region of η. At $|\eta| \ll 1$ and $\omega \ll \omega_a$, ω_M, however, the integral terms in (8) make a small contribution, and the corresponding expansion given in[8,9] for ω_{sur} coincides with our formula (24) (the mode ω_I). The results of these papers for the surface oscillations are correct, as already noted, also in the region $\eta^2 \gg 1$, provided that $\omega_M \ll \omega_a$. At the same time, the high-frequency oscillation mode ω_{III}, the decisive contribution to which is made by the dipole interaction, cannot be obtained at all without taking the integral terms into account.

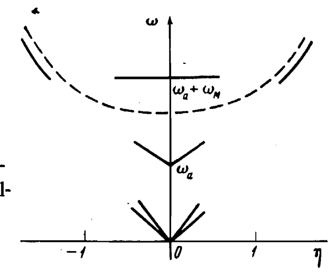
In concluding this section, we return to formulas (23)–(25), which describe the oscillation spectrum in the long-wave region, and discuss the symmetry of this spectrum. It is known[10] that in the presence of an external magnetic field (the role of which is assumed in our problem by the static magnetization $\mathbf{M}^0$), the time-reversal operation does not leave the equations invariant. The invariance of the equations (and accordingly the transformation of the spectrum into itself) takes place under the simultaneous substitutions $t \to -t$ and $H_0 \to -H_0$ (or $\omega \to -\omega$ and $\mathbf{M}^0 \to -\mathbf{M}^0$). On the other hand, if the distribution of the magnetization does not change, then simultaneously with the time reversal it is necessary to carry out a corresponding coordinate transformation. In our case the spectrum is not altered by a simultaneous substitution $\omega \to -\omega$ and $\eta \to -\eta$, in agreement with the principles of magnetic symmetry[10].

At the same time, the surface-oscillation spectrum (23) is not even in η ($\omega(\eta) \neq \omega(-\eta)$). The lack of symmetry of $\omega(\eta)$ can produce magnon energy and momentum flow along the wall following thermal excitation of the low-frequency states. But there should be no such states in the equilibrium state. The paradox in the resultant situation was pointed out to the author by M. I. Kaganov.

A discussion of this interesting and not quite clear question is beyond the scope of the present article. We note, however, the following circumstance: Assume that we have a system of alternating domains separated by walls. The question for the oscillation spectrum of such a system, in the wavelength region for which we can neglect the wall interaction ($\delta \ll \lambda \ll d$, where λ is the wavelength and d is the dimension of the domain), is simply the product of equations describing the oscillations of the moments in each wall. Since these equations differ only in the substitution $M_s \to -M_s$, their product is even in ω and η, and there should be no net transport in the system. A similar situation takes place in real domain structures. Thus, the spectrum of the surface-oscillation frequencies in the domain structure, in the wavelength range $\delta \ll \lambda \ll d$, takes the form (24), (25) with a factor $\pm\sigma$ instead of σ. In the short-wave region $|\eta| \gg 1$, the described difficulties, naturally, do not arise. A schematic form of the spectrum at $|\eta| \ll 1$ and $|\eta| \gg 1$ and $\omega > 0$ is shown in the figure.

VOLUME OSCILLATIONS. QUASISTATIONARY STATE IN CONTINUOUS SPECTRUM

We proceed to a study of the reflection and scattering of intradomain spin waves. Volume oscillations



Oscillation spectrum of a system of "noninteracting" domain walls. $k_z = 0$, $k_x\delta = \eta$. Dashed—boundary of the volume-wave spectrum. At $\omega = \omega_a + \omega_M$ we have a state-density singularity corresponding to a quasistationary surface oscillation.

exist in the frequency region $\omega^2 > \omega_0^2 = \omega_a^2(1+\eta^2)^2 + \omega_a\omega_M(1+\eta^2)$. At these frequencies the equation $D(k^2) = 0$ has a root $k^2 = k_0^2 > 0$. The quantity $\eta/(k_0^2 + \eta^2)^{1/2} = \sin\theta_0$ determines the incidence angle of the "modulated" spin wave χ_{k_0} on the wall, with $\eta = 0$ corresponding to normal incidence. We confine ourselves to the case $|\eta| \ll 1$ and $k_0^2 \gg \eta^2$, i.e., to almost normal incidence of the wave.

The initial equation is (16). The solution of (16) at $\omega^2 > \omega_0^2$ consists of an induced part proportional to u_0 in the form (19), and the solution of the homogeneous equation. The latter takes the form

$$u_k = C_0\delta(k \pm k_0) + \Delta u_k; \tag{29}$$

Δu_k describes the reflection and scattering of the intradomain wave χ_{k_0} with amplitude C_0. We assume for the sake of argument that $k_0 > 0$ and that the incident wave propagates in the direction of positive y; we seek Δu_k in the form

$$\begin{aligned}\Delta u_k = {} & \frac{(k^2+1)(k_0-k)}{D(k)}\,\mathrm{sh}^{-1}\frac{\pi}{2}(k_0-k)\,\{C_1k + C_2 + \varphi(k) + \ldots\} \\ & + \frac{C_0\rho\omega_a\omega_M\delta k^2(k^2+1)}{2(k_0^2+1)D(k)\,\mathrm{sh}[\pi(k_0-k)/2]}[\Phi(k_0) - \Phi(k)].\end{aligned} \tag{30}$$

The expansion of $\varphi(k)$ at $|k| \to 0$ begins with terms $\sim k^2$. In all the integrals containing $[D(k)]^{-1}$, the corresponding poles at $k = \pm k_0$ should be eliminated by adding to k_0 a small imaginary part ($k_0 \to k_0 + i\alpha$, $\alpha > 0$), which is set equal to zero in the final result. This circling rule corresponds to separation of wave scattered by the domain wall and diverging from it. We substitute (29) in the homogeneous equation (16), neglect the integral of $\varphi(k)$, which makes a small contribution, and carry out integration by using formulas (10) and (20). Comparing, in the principal order in ρ, terms containing k raised to the zeroth, first, and higher powers, we obtain

$$\begin{aligned} C_1 & \approx -C_0\rho\frac{\omega_M(\omega^2-\omega_0^2)}{2(\omega\sigma-\omega_a)k_0(k_0^2+1)}, \\ C_2 & \approx \frac{C_0}{2}\rho^3\left(\frac{\omega_0}{\omega_a}\right)^2\frac{\omega_M(\omega^2-\omega_0^2)[\omega\sigma-\omega_a-(\omega_0-\omega_a)k_0\Phi(k_0)]}{k_0^2(k_0^2+1)(\omega\sigma-\omega_a)(\omega\sigma-\omega_a-\omega_M)}; \\ \varphi(k) & \approx -\frac{2\omega_a\omega_M(\gamma+\delta)}{(\omega^2-\omega_0^2)(1+\delta)}C_1k\left\{(k_0+k)^2\sum_{m=1}^{\infty}\frac{1}{(k^2+4m^2)(k_0^2+4m^2)}\right. \\ & \qquad \left. -k_0^2\sum_{m=1}^{\infty}\frac{1}{4m^2(k_0^2+4m^2)}\right\}. \end{aligned} \tag{31}$$

The constant u_0, determined by substituting (19) and (29) in (12), turns out to be of the order of $\eta^2 C_0$.

Formulas (30) and (31) solve the problem of the incidence of a spin wave on the domain wall. In particular, the wave reflection coefficient is determined by the pole of Δu_k at the point $k = -k_0$. At $-\xi \gg 1$ we have

$$u_{\mathrm{ref}} \approx \frac{\exp(-ik_0\xi)}{k_0 - i}[(k+k_0)\Delta u_k]_{k=-k_0}.$$

The amplitude of the reflection coefficient is proportional to the small parameter ρ.

The poles of Δu_k at the points $k = \pm i\rho$ determine the amplitude of the co-moving surface wave produced when the intradomain-magnon is incident on the wall. Closing the integral with respect to k in the expansion (6) of $u(\xi)$, we obtain at $|\xi| \gg 1$

$$u_{\text{com}} \approx -\frac{i\rho C_0}{4} \frac{\omega_M e^{-\rho|\xi|}}{(\omega\sigma-\omega_a)(k_0^2+1)\operatorname{sh}(\pi k_0/2)}. \tag{32}$$

The co-moving wave propagates along the domain wall and decreases exponentially as it moves farther away from it. A wave of this type is not a natural oscillation of the system and is produced only in the presence of a source (incident magnon!).

We note for comparison that in the problem of the incidence of a spin wave on the interface between a magnet and vacuum (this problem was solved in a somewhat different formulation by Bulaevskiĭ [11]), to co-moving surface waves are produced upon reflection. One corresponds to (32), and the other is determined by the pole $k = \pm i|k_1|$. The modulus of the reflection coefficient is equal in this case to unity, and the phase differs from zero, i.e., total internal reflection takes place.

In our problem, owing to the smearing of the boundary, the corresponding part of the solution has a more complicated form, and therefore the second exponential co-moving wave is not formed.

We call further attention to the fact that at $\omega_{\text{res}} \approx \sigma(\omega_a + \omega_M)$ we have $C_2 \to \infty$. According to the results of[7] in the model of the geometric wall, it is precisely at this frequency that a surface oscillation of the Dimon-Eschbach magnetostatic wave exists. Our solution (30) and (31) becomes incorrect in a small vicinity of ω_{res}. Estimates show that as $\omega \to \omega_{\text{res}}$ the coefficients C_1 and C_2 increase, and with them also the amplitude of the reflected wave, while the amplitude of the transmitted wave decreases. This phenomenon can be naturally called the capture of a magnon into a bound state that exists at $\omega \approx \omega_{\text{res}}$. We shall not investigate the capture here, since the corresponding calculations are exceedingly cumbersome. We confine ourselves only to proving the existence at $\omega \approx \omega_{\text{res}}$ of a quasi-stationary state of the type of Dimon-Eschbach waves; we obtain the solution corresponding to this state and the level width that determines the decay time of the surface magnon.

To this end, we turn to Eq. (16), put $u_0 \equiv 0$, and seek for this equation a solution even in k in the form

$$u_k=\rho C\frac{(k^2+1)}{D(k)}\frac{k}{\operatorname{sh}(\pi k/2)}\{1+O(\rho)+\ldots\}. \tag{33}$$

Substituting (33) in (16) and calculating the integrals with the aid of (10) and (20), we verify that (16) will be satisfied in principal order in ρ if we put

$$\omega=\sigma(\omega_a+\omega_M)+O(\rho)+\ldots-i\Gamma,$$

$$\Gamma\approx\rho^3\frac{\pi^2}{4}\frac{\sigma\omega_M(\omega_a+\omega_M)\left[\frac{1}{2}\pi\operatorname{cth}(\pi k_0/2)+k_0^{-1}(1+\omega_M/\omega_a)^{1/2}\right]}{\omega_a[\omega_M^2+4(\omega_a+\omega_M)^2]^{1/2}k_0^2\operatorname{sh}^2(\pi k_0/2)}. \tag{34}$$

At $\omega < 0$ it is necessary to replace the momentum k_0 in Γ by $-k_0$.

Thus, the damping of the quasistationary state at $\rho \ll 1$ is proportional to the cube of a small parameter. It is easy to verify that the solution (33) indeed defines a surface magnon. In the coordinate representation we have

$$u(\xi)=\frac{\rho C}{2\pi}\int\frac{k(\operatorname{th}\xi-ik)e^{ik\xi}\,dk}{D(k)\operatorname{sh}(\pi k/2)}=-\frac{C}{2\pi}L_\xi^-\int\frac{d\mu}{\operatorname{ch}^2\mu}\Big\{\exp(-\rho|\xi-\mu|)$$
$$+\frac{\rho}{|k_1|}\exp(-|k_1||\xi-\mu|)+\frac{i\rho}{k_0}\exp ik_0|\xi-\mu|\Big\}. \tag{35}$$

At $|\xi| \gg 1$ we have

$$u(\xi)\approx C\operatorname{sign}\xi\left\{\exp(-\rho|\xi|)+\rho(ik_0-1)\frac{\pi}{\operatorname{sh}(\pi k_0/2)}\exp(ik_0|\xi|)\right\}. \tag{36}$$

The first term in (36) corresponds to the previously obtained surface wave[7]. The second describes a volume wave that diverges from the domain wall, i.e., the decay of a surface magnon, causing broadening of the level Γ. We note in conclusion that a generalization of the calculations for waves that propagate at an arbitrary angle to the anisotropy axis does not raise fundamental difficulties but the results turn out to be quite cumbersome.

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[1)]Some of the results were reported at the International Conference on Magnetism (Moscow, 22-28 August 1973).

[2)]Within the framework of classical oscillation theory, there are no grounds for discarding the negative frequencies, if only because the Fourier expansion of the solution of the initial-condition problem contains both positive and negative frequencies.

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111