Adiabatic piezopolaron in a strong magnetic field

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The problem of a piezopolaron in an external magnetic field is considered in the adiabatic approximation. The magnetic field is assumed to be so strong that the magnetic length is much smaller than the size of the polaron state. This condition permits one to find the analytic form of the electron wave function and the deformation distribution. The polaron optical spectrum is obtained. The dispersion law for a polaron moving along the magnetic-field lines and the transverse mass are calculated.

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This paper deals with the spectrum of the piezopolaron in a strong magnetic field. The coupling constant is assumed to be so large that the adiabatic approximation is applicable and the phonon field can be regarded as classical. As a result, the problem reduces to a solution of a system of differential equations. The magnetic field is assumed to be so strong that the magnetic length is much shorter than the longitudinal dimension of the polaron state. This makes possible the use of the method developed in the theory of excitons in a strong magnetic field[1], and to reduce the problem to one-dimensional. The resultant equation, which describes the dependence of the electron wave function on the coordinate along the magnetic field, admits of an exact solution. It is possible as a result to calculate the wave function of the electron in the polaron well, and the spectrum.

In contrast to the spectrum of the free electron in a magnetic field, the spectrum of an electron bound in a polaron well is not degenerate in the projection of the angular momentum on the direction of the magnetic field. This fact should lead to a fine structure of the cyclotron resonance. The very existence of the polaron state leads to a threshold for the absorption of an electromagnetic wave polarized along the magnetic field.

The problem can be solved for the case of a polaron moving with a velocity lower than that of sound. This makes it possible to calculate the dispersion law for the motion of the polaron parallel to the magnetic field, and the angular frequency of this rotation around the magnetic field.

An isotropic model is studied, in which it is assumed that the piezoelectric fields are produced only by longitudinal strains, and the anisotropy of the elastic moduli, of the piezomoduli, and of the dielectric constant can be neglected.

The problem of the piezopolaron with strong coupling in a strong magnetic field was considered in[2], where a variational calculation method was used. The polaron binding energy obtained in the present paper agrees in order of magnitude with that obtained in[2], and the longitudinal mass differs by a large logarithmic factor.

1. BASIC EQUATIONS OF THE PROBLEM

The Lagrangian describing the isotropic model of a piezopolaron in a strong magnetic field **H** and a weak potential field U(**r**) is of the form[3]

$$\mathscr{L}=\int d^3r\left\{\frac{i\hbar}{2}\left(\Psi^*\frac{\partial\Psi}{\partial t}-\Psi\frac{\partial\Psi^*}{\partial t}\right)+\frac{\rho}{2}\left(\frac{\partial u}{\partial t}\right)^2-\frac{1}{2m^*}\left|\left[-i\hbar\frac{\partial}{\partial r_l}-\frac{e}{c}A_l(\mathbf{r})\right]\Psi\right|^2-\frac{4\pi\beta e}{\varepsilon}u|\Psi|^2-\frac{\lambda}{2}(\nabla u)^2-U(\mathbf{r})|\Psi|^2\right\}. \quad (1.1)$$

Here e and m* are the charge and effective mass of the electron, β is the piezomodulus, λ is the elastic modulus, ϵ is the dielectric constant, ρ is the density of the crystal, and u is the elastic displacement, which is assumed in the adiabatic approximation to be a c-number. The vector-potential gauge is assumed for the time being to be arbitrary but linear:

$$A_l(\mathbf{r})=\alpha_{lk}r_k,\quad \alpha_{lk}-\alpha_{kl}=e_{klj}H_j, \quad (1.2)$$

e_{klj} is a completely antisymmetrical unit tensor of third rank.

In the adiabatic approximation, the dependence of the elastic displacement and of the modulus of the wave function of the electron on the time is connected only with the motion of the polaron as a whole. Therefore, if we introduce the radius vector **R**(t) of the center of gravity of the polaron, then

$$u=u(\mathbf{x}),\quad \Psi=\exp\left[i\frac{e}{\hbar c}A_l(\mathbf{R})r_l\right]\tilde{\psi}(\mathbf{x}), \quad (1.3)$$

where

$$\mathbf{x}=\mathbf{r}-\mathbf{R}(t). \quad (1.4)$$

Substitution of (1.3) in (1.1) yields

$$\mathscr{L}=\int d^3x\left\{\dot{R}_l\tilde{j}_l+\frac{\rho}{2}\left(\dot{R}_l\frac{\partial u}{\partial x_l}\right)^2-\frac{1}{2m^*}\left|\left[-i\hbar\frac{\partial}{\partial x_l}-\frac{e}{c}A_l(\mathbf{x})\right]\tilde{\psi}\right|^2-\frac{4\pi\beta e}{\varepsilon}u|\tilde{\psi}|^2-\frac{\lambda}{2}(\nabla u)^2-\frac{e}{c}\alpha_{kl}\dot{R}_lR_k|\tilde{\psi}|^2-U(\mathbf{x}+\mathbf{R})|\tilde{\psi}|^2\right\}, \quad (1.5)$$

$$\tilde{j}_l=\frac{i\hbar}{2}\left(-\tilde{\psi}^*\frac{\partial\tilde{\psi}}{\partial x_l}+\tilde{\psi}\frac{\partial\tilde{\psi}^*}{\partial x_l}\right)-\frac{e}{c}\alpha_{kl}x_k|\tilde{\psi}|^2. \quad (1.6)$$

The wave function and the elastic displacement determined with the aid of the Lagrangian (1.5) depend on **R** and $\dot{\mathbf{R}}$ as parameters. But before deriving the equations, it is convenient to make one more canonical transformation

$$\tilde{\psi}=\exp\left(i\frac{m^*}{\hbar}\dot{R}_lx_l\right)\psi. \quad (1.7)$$

As a result

$$\mathscr{L}=\int d^3x\left\{\frac{\rho}{2}\left(\dot{R}_l\frac{\partial u}{\partial x_l}\right)^2-\frac{1}{2m^*}\left|\left[-i\hbar\frac{\partial}{\partial x_l}-\frac{e}{c}A_l(\mathbf{x})\right]\psi\right|^2-\frac{e}{c}(\alpha_{kl}-\alpha_{lk})x_k|\psi|^2\dot{R}_l-\frac{4\pi\beta e}{\varepsilon}u|\psi|^2-\frac{\lambda}{2}(\nabla u)^2-U(\mathbf{x}+\mathbf{R})|\psi|^2\right\}+\frac{m^*}{2}\dot{\mathbf{R}}^2-\frac{e}{c}\alpha_{kl}\dot{R}_lR_k, \quad (1.8)$$

where the condition for the normalization of the wave function is used. The external field U(**r**) is assumed to be too weak to influence the polaron structure. In this case the equations for ψ and u take the form

$$\frac{1}{2m^*}\left[-i\hbar\frac{\partial}{\partial x_l}-\frac{e}{c}A_l(\mathbf{x})\right]^2\psi+\frac{4\pi\beta e}{\varepsilon}u\psi=E\psi, \quad (1.9)$$

$$\lambda\Delta u-\rho\dot{R}_l\dot{R}_k\frac{\partial^2 u}{\partial x_l\partial x_k}=\frac{4\pi\beta e}{\varepsilon}|\psi|^2, \tag{1.10}$$

and the equation describing the motion of the polaron as a unit is

$$\frac{d}{dt}M_{lk}\dot{R}_k=\frac{e}{c}e_{ljk}\dot{R}_jH_k-\frac{\partial\bar{U}}{\partial R_l}. \tag{1.11}$$

In the derivation of (1.9) and (1.11) we used the relation

$$\int \mathbf{x}|\psi|^2d^3x=0, \tag{1.12}$$

which is the consequence of (1.10) and expresses the fact that R is rigidly connected with the center of gravity of the polaron. In the expression for the polaron effective mass

$$M_{lk}=\rho\int\frac{\partial u}{\partial x_l}\frac{\partial u}{\partial x_k}d^3x \tag{1.13}$$

the free-electron mass can be neglected. In order of magnitude, M_{lk} is equal to that fraction of the crystal mass in the polaron volume which is proportional to the square of the strain (cf.[4]). In an external potential field, the polaron is acted upon by a force proportional to the gradient of the average potential

$$\bar{U}(\mathbf{R})=\int U(\mathbf{R}+\mathbf{x})|\psi|^2d^3x. \tag{1.14}$$

Using Eq. (1.10), we can represent the elastic displacement in the form of an integral of the square of the modulus of the wave function

$$u(\mathbf{x})=-\frac{\beta e}{\varepsilon\lambda}\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{-1/2}\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1/2}\int\Bigl[\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1}(x-x')^2$$
$$+(y-y')^2+\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{-1}(z-z')^2\Bigr]^{-1/2}|\psi(\mathbf{x}')|^2d^3x'. \tag{1.15}$$

Here $w = (\lambda/\rho)^{1/2}$ is the speed of sound, the polaron velocity vector $\dot{\mathbf{R}} = \dot{\mathbf{R}}_\parallel + \dot{\mathbf{R}}_\perp$ has been resolved into components along and across the magnetic field, the z axis is directed along the magnetic field, and the x axis is directed along $\dot{\mathbf{R}}_\perp$.

The problem reduces now to solving the Schrödinger equation (1.9) with the potential (1.15) and to calculating the dependence of the polaron energy on its momentum and the angular-momentum projection along the z axis.

2. INTERNAL STRUCTURE OF POLARON AND ITS OPTICAL PROPERTIES

In a strong magnetic field the wave function ψ can be expanded in the ratio of the electron binding energy $\mathscr{E}_e$ in the polaron well to the cyclotron energy $\hbar\Omega_e$ ($\Omega_e \equiv eH/m^*c$):

$$\psi(\mathbf{x})=\Phi_{nm}(\mathbf{x}_\perp)f(z)+\psi^{(1)}(\mathbf{x})+\dots, \tag{2.1}$$

where $\Phi_{nm}(\mathbf{x}_\perp)$ satisfies the equation

$$\frac{1}{2m^*}\left[-i\hbar\frac{\partial}{\partial x_l}-\frac{e}{c}A(\mathbf{x}_\perp)\right]^2\Phi_{nm}=\left(n+\frac{1}{2}\right)\hbar\Omega_e\Phi_{nm}, \tag{2.2}$$

n is the Landau quantum number, and m is the quantum number in which the states of the free electron in the magnetic field are degenerate. Functions with different values of m are orthogonal and are normalized to unity.

From the condition that Eq. (1.9) have a solution we obtain for the first approximation correction $\psi^{(1)}$ the equation satisfied by f(z)

$$-\frac{\hbar^2}{2m}\frac{d^2f}{dz^2}+\frac{4\pi\beta e}{\varepsilon}u_nf=\mathscr{E}_ef, \tag{2.3}$$

where

$$\mathscr{E}_e=E-(n+1/2)\hbar\Omega_e,$$
$$u_n(z)=\int u(\mathbf{x})|\Phi_{nm}(\mathbf{x}_\perp)|^2d^2\mathbf{x}_\perp \tag{2.4}$$

(the subscript m will be omitted where there is no danger of misunderstanding), and also the condition that determines the choice[1)] of $\Phi_{nm}(\mathbf{x}_\perp)$:

$$\int u(\mathbf{x})\Phi^*_{nm'}(\mathbf{x}_\perp)\Phi_{nm}(\mathbf{x}_\perp)d^2\mathbf{x}_\perp=0. \tag{2.5}$$

The elastic displacement in (2.4) and (2.5) must be calculated with account taken of only the zeroth-approximation term in the expansion (2.1).

Further progress can be made only if the transverse velocity of the polaron is not too high:

$$\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)\gg\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)\frac{l^2}{a^2}, \tag{2.6}$$

where $l \equiv (2c\hbar/eH)^{1/2}$ is the magnetic length, and a is the longitudinal dimension of the polaron. It is important to note that the right-hand side of (2.6) is much smaller than unity, since

$$l^2/a^2\sim|\mathscr{E}_e|/\hbar\Omega_e\ll 1. \tag{2.7}$$

When (1.15) is substituted in (2.4), the significant regions in the integrals with respect to $\mathbf{x}_\perp$ and $\mathbf{x}'_\perp$ are $|\mathbf{x}'_\perp| \sim |\mathbf{x}_\perp| \sim l$, whereas $|z| \sim |z'| \sim a$. Therefore if the inequality (2.6) is satisfied, then it is easy to verify that the resultant integral is logarithmically large. To separate the logarithmically large contribution, it is convenient to use the integral of the Bessel function of imaginary argument and its expansion at small values of the argument ([5], pp. 746 and 975):

$$\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{-1/2}\left[\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1}(x-x')^2+(y-y')^2+\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{-1}(z-z')^2\right]^{-1/2}$$
$$=\frac{1}{\pi}\int e^{i(z-z')s}K_0\left\{|s|\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{1/2}\left[\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1}(x-x')^2+(y-y')^2\right]^{1/2}\right\}ds$$
$$\approx-\frac{1}{\pi}\int e^{i(z-z')s}\ln\left\{\frac{|s|\exp C}{2}\left(1-\frac{\dot{R}_\parallel^2}{w^2}\right)^{1/2}\right.$$
$$\left.\times\left[\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1}(x-x')^2+(y-y')^2\right]^{1/2}\right\}ds, \tag{2.8}$$

where C is the Euler constant. Substituting (1.15) in (2.4) and using the representation (2.8), we obtain

$$u_n(z)=-\frac{\beta e}{\varepsilon\lambda}\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1/2}[2\Lambda|f(z)|^2+u_n^{(1)}(z)], \tag{2.9}$$

where

$$\Lambda=\ln[a/l(1-\dot{R}_\parallel^2/w^2)^{1/2}], \tag{2.10}$$
$$u_n^{(1)}(z)=\frac{1}{\pi}\int|f(z')|^2dz'\int e^{i(z-z')s}ds\int d^2x_\perp d^2x_\perp'|\Phi_n(\mathbf{x}_\perp)|^2$$
$$\times|\Phi_n(\mathbf{x}_\perp')|^2\ln\left\{\frac{2l}{a|s|\exp C}\left[\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{-1}(x-x')^2+(y-y')^2\right]^{-1/2}\right\}. \tag{2.11}$$

The argument of the logarithm in (2.11) is of the order of unity, and therefore the second term in the square brackets of (2.9) is of the order of unity, whereas the first one contains a large logarithm. We expand f(z) in reciprocal powers of Λ:

$$f(z)=F(z)+f^{(1)}(z)+\dots. \tag{2.12}$$

Neglecting $u^{(1)}z$, we find that Eq. (2.3) admits of a unique solution that tends to zero as $|z| \to \infty$:

$$F(z)=\left(\frac{a_0}{2a^2\Lambda}\right)^{1/2}\left(1-\frac{\dot{\mathbf{R}}_\perp^2}{w^2}\right)^{1/4}\frac{1}{\operatorname{ch}(z/a)}, \tag{2.13}$$

where

$$a_0=\frac{1}{K^2}\frac{\hbar^2\varepsilon}{m^*e^2}=\frac{1}{\alpha}\frac{\hbar}{m^*w} \tag{2.14}$$

is the radius of the polaron in the absence of a magnetic field, $K^2 \equiv 4\pi\beta^2/\epsilon\lambda$, $\alpha = K^2e^2/\epsilon\hbar w$ is the dimensionless coupling constant, and the longitudinal dimension of the polaron in a magnetic field is determined by

$$a=(2m^*|\mathscr{E}_e|)^{1/2}/\hbar. \tag{2.15}$$

In contrast to the usual Schrödinger equation for a particle in a specified potential, the solution (2.13) exists for all values $\mathscr{E}_e < 0$. The quantity $\mathscr{E}_e$ must be determined from the normalization condition

$$\frac{a_0}{a\Lambda}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{1/2}=1. \tag{2.16}$$

The polaron states in this approximation turn out to be degenerate, just as the states of a free electron in a magnetic field. The degeneracy is lifted in the first-order approximation in Λ^{-1}. The normalization condition, accurate to first-order approximation terms, is

$$\frac{a_0}{a\Lambda}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{1/2}+\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1/2}\int F(z)g(z)dz=1, \tag{2.17}$$

where $g(z) = (1-\dot{R}_\perp^2/w^2)^{1/2}(f^{(1)}+f^{(1)*})$. This function satisfies the following linear inhomogeneous equation that is obtained from (2.3):

$$\frac{d^2g}{dz^2}+\frac{12}{a_0}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1/2}\Lambda F^2g-\frac{1}{a^2}g=-\frac{4}{a_0}u_n^{(1)}F. \tag{2.18}$$

The solution of this equation is

$$g(z)=\frac{4}{a_0}\left[G_1(z)\int_0^z G_2(z')u^{(1)}(z')F(z')dz'-G_2(z)\int_{-\infty}^z G_1(z')u^{(1)}(z')F(z')dz'\right], \tag{2.19}$$

where

$$G_1=dF/dz,\quad G_2=\tilde{G}G_1 \tag{2.20}$$

are the solutions of the homogeneous equation and

$$d\tilde{G}/dz=1/G_1^2. \tag{2.21}$$

With the aid of (2.19)–(2.21) we obtain after simple transformations

$$\int Fgdx=-\frac{2}{a_0}\int\left[F^2(0)G_2(z)+G_1(z)\int_0^z\frac{F^2(z')-F^2(0)}{G_1^2(z')}dz'\right]u^{(1)}(z)F(z)dz.$$

We now use the explicit expression (2.13) with allowance for (2.16)

$$\frac{F^2(z)-F^2(0)}{G_1^2(z)}=-a^2\,\mathrm{ch}^2(z/a),$$

$$\tilde{G}(z)=-2a^4\frac{\mathrm{ch}^3(z/a)}{\mathrm{sh}(z/a)}+6a^3\int_0^z\mathrm{ch}^2(z'/a)dz';$$

as a result we get

$$\int Fgdz=\frac{2a^2}{a_0}\int F^2u_n^{(1)}\left(\frac{z}{a}\,\mathrm{th}\frac{z}{a}-1\right)dz. \tag{2.22}$$

It is necessary to substitute in the resultant integral the expression (2.11), in which $f(z')$ is replaced by $F(z')$. The integrals with respect to z and z′ reduce to tabulated integrals ([5], p. 519):

$$\int e^{-isz'}\frac{dz'}{\mathrm{ch}^2(z'/a)}=\frac{\pi a^2s}{\mathrm{sh}(\pi as/2)},$$

$$\int e^{isz}\left(\frac{z}{a}\,\mathrm{th}\frac{z}{a}-1\right)\frac{dz}{\mathrm{ch}^2(z/a)}=-a\frac{(\pi as/2)^2\,\mathrm{ch}(\pi as/2)}{\mathrm{sh}^2(\pi as/2)}, \tag{2.23}$$

after which the integration with respect to s is also carried out with the aid of the tabulated integral ([5], p. 366):

$$\int\left(\frac{\pi as}{2}\right)^3\frac{\mathrm{ch}(\pi as/2)}{\mathrm{sh}^3(\pi as/2)}ds=\frac{6}{\pi a}\int_0^\infty\frac{\tau^2d\tau}{\mathrm{sh}^2\tau}=\frac{\pi}{a}, \tag{2.24}$$

$$\int\left(\frac{\pi as}{2}\right)^3\ln\left(\frac{\pi a|s|}{2}\right)\frac{\mathrm{ch}(\pi as/2)}{\mathrm{sh}^3(\pi as/2)}ds=\frac{2}{\pi a}\int_0^\infty\left(\frac{3\tau^2\ln\tau}{\mathrm{sh}^2\tau}+\frac{\tau^2}{\mathrm{sh}^2\tau}\right)d\tau$$

$$=\frac{2}{\pi a}\left\{\frac{d}{d\nu}[2^{1-\nu}\Gamma(1+\nu)\zeta(\nu)]_{\nu=2}\right\}=\frac{\pi}{a}\left(\frac{11}{6}-\ln 2-C-\sum_{k=2}^\infty\frac{\ln k}{k^2}\right).$$

As a result we have

$$\int Fgdz=\frac{a_0}{a\Lambda^2}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)\left(\frac{11}{6}-\ln 2\pi-\sum_{k=2}^\infty\frac{\ln k}{k^2}-A_{nn}^{mm}\right), \tag{2.25}$$

where

$$A_{nn'}^{mm'}=\int|\Phi_{nm}(\mathbf{x}_\perp)|^2|\Phi_{n'm'}(\mathbf{x}_\perp')|^2\ln\left\{l\left[\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1}(x-x')^2+(y-y')^2\right]^{-1/2}\right\}d^2x_\perp d^2x_\perp'. \tag{2.26}$$

Substitution of (2.25) in (2.17) leads to the following expression for the electron energy:

$$\mathscr{E}_e=-\frac{\hbar^2\Lambda^2}{2m^*a_0^2}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1}\left[1-\frac{2}{\Lambda}\left(\frac{11}{6}-\ln 2\pi-\sum_{k=2}^\infty\frac{\ln k}{k^2}-A_{nn}^{mm}\right)\right]. \tag{2.27}$$

Owing to the dependence of A_{nn}^{mm} on the state of the electron, the degeneracy is lifted. The electron wave functions can be calculated only in the limiting case

$$\dot{R}_\perp^2/w^2\ll 1. \tag{2.28}$$

Then m is an integer characterizing the projection of the angular momentum of the electron on the z axis[2)]. But even in this case A_{nn}^{mm} cannot be calculated in final form for arbitrary n and m. By way of example we present its value at n = 0, when

$$\Phi_{0m}(\mathbf{x}_\perp)=(\pi|m|!l^2)^{-1/2}(x_\perp/l)^{|m|}\exp(im\varphi-x_\perp^2/2l^2)$$

(here φ is the azimuthal angle of the vector $\mathbf{x}_\perp$):

$$A_{00}^{mm}=\sum_{k=0}^{|m|}\frac{(|m|+k)!}{k!|m|!}2^{-|m|-k-1}[\psi(|m|+k+1)-\ln 2]-\psi(|m|+1), \tag{2.29}$$

where $\psi(k)$ is the logarithmic derivative of the Γ function. With increasing |m|, the absolute value of A_{nn}^{mm} increases but its sign remains negative.

The optical properties of the polaron are determined by the solutions of (1.9) at fixed $u(\mathbf{x})$. In the case when the electromagnetic wave is polarized perpendicular to the constant magnetic field, ordinary cyclotron resonance takes place. The dependence of $u(\mathbf{x})$ on the initial state of the electron leads to a fine structure of the cyclotron resonance. The frequency of the transition n, m → n + 1, m′ (m′ = m + 1 under the condition (2.28)) is

$$\omega_{n\,n+1}^{mm'}=\Omega_e+\frac{\hbar^2\Lambda}{m^*a_0^2}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1}(A_{nn}^{mm}-A_{n+1\,n}^{m'm}). \tag{2.30}$$

In addition, transitions without a change of the Landau quantum number are possible. The frequency of such a transition is

$$\omega_{nn}^{mm'}=\frac{\hbar^2\Lambda}{m^*a_0^2}\left(1-\frac{\dot{R}_\perp^2}{w^2}\right)^{-1}(A_{nn}^{mm}-A_{nn}^{m'm}). \tag{2.31}$$

If the electric field in the electromagnetic wave is parallel to the constant magnetic field, then the possible solutions are determined by the solutions of the one-dimensional Schrödinger equation (2.3) with fixed potential $u_n(z)$. The ground state in this potential is the state of the electron producing the potential well, i.e., the potential $u_n(z)$ itself. If we discard $u_n^{(1)}(z)$, then the equation has an exact solution, and the first excited state $f \sim \tanh(z/a)$ lies already at the boundary of the continuous spectrum. The potential $u_n^{(1)}$ contains a long-range part that leads to the appearance of an infinite number of new bound states.[3] However, the binding energy of such states is of the order of $\hbar^2/m^*a_0^2$ and lies beyond the limits of the accuracy of the present calculation. Thus, the existence of the polaron leads to the frequency threshold

$$\omega=\hbar^2\Lambda^2/2m^*a_0^2. \tag{2.32}$$

3. MOTION OF POLARON AS A UNIT. THE DISPERSION LAW

The polaron equation of motion (1.11) can be written in the form

$$\frac{d}{dt}M_{\|}\dot{R}_{\|}=-\frac{\partial\bar{U}}{\partial R_{\|}}, \tag{3.1}$$

$$\frac{d}{dt}M_{\perp}\dot{\mathbf{R}}_{\perp}=\frac{e}{c}[\dot{\mathbf{R}}_{\perp}\times\mathbf{H}]-\frac{\partial\bar{U}}{\partial\mathbf{R}}, \tag{3.2}$$

where the longitudinal and transverse masses of the polaron are defined by the relations

$$M_{\|}=\rho\int\left(\frac{\partial u}{\partial z}\right)^2 d^3x,\quad M_{\perp}=\rho\int\left(\frac{\partial u}{\partial x}\right)^2 d^3x. \tag{3.3}$$

To calculate them it is convenient to use the Fourier representation for the elastic displacement. We then obtain with the aid of (2.23)

$$M_{jl}=\frac{1}{8}\frac{\hbar^2a}{m^*w^2\Lambda}\int\left[\left(1-\frac{\dot{R}_{\|}^2}{w^2}\right)k_{\|}^2+\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)k_x^2+k_y^2\right]^{-2}$$
$$\times\left|\int|\Phi_n(\mathbf{x}_{\perp})|^2e^{i\mathbf{k}_{\perp}\mathbf{x}_{\perp}}d^2\mathbf{x}_{\perp}\right|^2\frac{k_{\|}^2k_jk_ld^3k}{\mathrm{sh}^2(\pi ak_{\|}/2)}. \tag{3.4}$$

The main contribution to the longitudinal mass is made by the region of integration $k_{\perp}\sim(1-\dot{\mathbf{R}}_{\|}^2/w^2)^{-1/2}a^{-1}$. The exponential in the integral with respect to $\mathbf{x}_{\perp}$ can be set equal to unity, and with the aid of (2.24) we obtain

$$M_{\|}=\frac{1}{3}\frac{\hbar^2\Lambda}{m^*a_0^2w^2}\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)^{-1/2}\left(1-\frac{\dot{\mathbf{R}}_{\|}^2}{w^2}\right)^{-1} \tag{3.5}$$

A logarithmically large contribution to the transverse mass is made by the region $(1-\dot{\mathbf{R}}_{\|}^2/w^2)^{-1/2}a^{-1}\ll k_{\perp}\ll l^{-1}$. In the integral with respect to $\mathbf{x}_{\perp}$, the exponential can again be set equal to unity. As a result we have

$$M_{\perp}=\frac{1}{3}\frac{\hbar^2\Lambda^2}{m^*a_0^2w^2}\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)^{-1/2}. \tag{3.6}$$

In the case when $\bar{U}=0$, Eqs. (3.1) and (3.2) are solved practically in the same way as the equations of motion of a free electron in a magnetic field. It follows from (3.2) that

$$M_{\perp}\dot{\mathbf{R}}_{\perp}=\frac{e}{c}[\mathbf{R}_{\perp}\times\mathbf{H}], \tag{3.7}$$

i.e., the polaron rotates about an axis parallel to the magnetic field, with angular frequency

$$\Omega=eH/M_{\perp}c. \tag{3.8}$$

The projection of the angular momentum on the magnetic field direction

$$\hbar L_z=[\mathbf{R}\times M_{\perp}\dot{\mathbf{R}}_{\perp}]_z \tag{3.9}$$

is an integral of the motion. Expression (3.9) is obtained from the Lagrangian (1.1) if one neglects the contribution of the electron in comparison with the contribution of the phonon field. The quantity L_z is connected with the square of the transverse velocity by the relation

$$\hbar|L_z|=\frac{cM_{\perp}^2}{eH}\dot{\mathbf{R}}_{\perp}^2. \tag{3.10}$$

From (3.1) with allowance for (3.5) we obtain the following expression for the kinetic energy of the longitudinal motion of the polaron:

$$\mathscr{E}_{\|}=M_{\|0}w^2\left[\frac{\dot{R}_{\|}^2/w^2}{1-\dot{R}_{\|}^2/w^2}+\frac{1}{2}\ln\left(1-\frac{\dot{R}_{\|}^2}{w^2}\right)\right], \tag{3.11}$$

where

$$M_{\|0}=(\hbar^2\Lambda^2/3m^*a_0^2w^2)(1-\dot{\mathbf{R}}_{\perp}^2/w^2)^{-1/2}.$$

In the calculation of the projection of the polaron momentum on the magnetic-field direction we can neglect the electron momentum. Then

$$P=-\rho\int\frac{\partial u}{\partial t}\frac{\partial u}{\partial z}d^3r=M_{\|}\dot{R}_{\|}. \tag{3.12}$$

The dispersion law for the longitudinal motion is obtained by eliminating $\dot{\mathbf{R}}_{\|}$ from (3.11) and (3.12). It is meaningless to write out the complete formula, since it is too cumbersome. It is easy to verify that it satisfies the relation $\partial\mathscr{E}_{\|}/\partial P=\dot{\mathbf{R}}_{\|}$. In the limiting cases

$$\mathscr{E}_{\|}=P^2/2M_{\|0}\quad\text{if}\quad\dot{R}_{\|}\ll w\quad\text{or}\quad P\ll M_{\|0}w,$$
$$\mathscr{E}_{\|}=Pw+{}^1/{}_2M_{\|0}w^2\ln\frac{P}{M_{\|0}w^2}\quad\text{if}\quad w-\dot{R}_{\|}\ll w\quad\text{or}\quad P\gg M_{\|0}w. \tag{3.13}$$

To calculate the total energy of the polaron it is necessary to use for the Hamiltonian an expression obtained from (1.1) by taking into account the transformations (1.4) and (1.7):

$$\mathscr{H}=\int\left\{\frac{\rho}{2}\left(\dot{R}_l\frac{\partial u}{\partial x_l}\right)^2+\frac{\lambda}{2}(\nabla u)^2+\frac{1}{2m^*}\left|\left[-i\hbar\frac{\partial}{\partial x_l}-\frac{e}{c}A_l(\mathbf{x})\right]\psi\right|^2\right.$$
$$\left.+\frac{4\pi\beta e}{\varepsilon}u|\psi|^2+\dot{R}_lj_l\right\}d^3x+\frac{m^*}{2}\dot{\mathbf{R}}^2, \tag{3.14}$$

$$j_l=\frac{i\hbar}{2}\left(\psi\frac{\partial\psi^*}{\partial x_l}-\psi^*\frac{\partial\psi}{\partial x_l}\right)-\frac{e}{c}A_l(\mathbf{x})|\psi|^2. \tag{3.15}$$

The integral of the current density for the bound state is equal to zero, and the last term in (3.14) can be neglected. As a result, the polaron energy reckoned from the bottom of the Landau band for the free electron is given by

$$\mathscr{E}=\mathscr{E}_e+\mathscr{E}_u, \tag{3.16}$$

where

$$\mathscr{E}_u=\frac{1}{2}\int\left[\rho\left(\dot{R}_l\frac{\partial u}{\partial x_l}\right)^2+\lambda(\nabla u)^2\right]d^3x$$
$$=\rho\int\left(\dot{R}_l\frac{\partial u}{\partial x_l}\right)^2d^3x-\frac{2\pi\beta e}{\varepsilon}\int u|\psi|^2d^3x. \tag{3.17}$$

The last integral is easy to calculate:

$$\int u|\psi|^2d^3x=-\frac{\beta e}{\varepsilon\lambda}\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)^{-1/2}\frac{\Lambda}{2a^2}\int\frac{dz}{\mathrm{ch}^4(z/a)}$$
$$=-\frac{2}{3}\frac{\beta e}{\varepsilon\lambda}\frac{\Lambda^2}{a_0}\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)^{-1}, \tag{3.18}$$

and the result can be represented in the form

$$\mathscr{E}=-\frac{1}{6}\frac{\hbar^2}{m^*a_0^2}\left(\ln\frac{a}{l}\right)^2+M_{\perp}\dot{\mathbf{R}}_{\perp}^2\left[1-\frac{1}{2}\left(1-\frac{\dot{\mathbf{R}}_{\perp}^2}{w^2}\right)^{1/2}\right]+\mathscr{E}_{\|}. \tag{3.19}$$

The first term is the binding energy of the polaron at rest, calculated in the lowest order in Λ^{-1}, and the second term is the energy of its transverse motion. Both terms are of the order of Λ^2, whereas the last terms are of order Λ. Nonetheless, expression (3.23) is not an exaggeration of the accuracy, since the corrections of order Λ to the first two terms contain $\dot{R}_{\parallel}^2$ only under the logarithm sign, and make no contribution to the energy of the longitudinal motion.

4. LIMITS OF APPLICABILITY OF THE THEORY

The limits of applicability of the results are bounded by two principal conditions. The first is that the adiabatic approximation be applicable, and the second is that the magnetic field be strong.

In the adiabatic approximation, the elastic displacement can be regarded as a c-number, i.e., we can neglect the amplitude of the zero-point oscillations in comparison with the characteristic elastic displacement. The same condition can be formulated as stating that the phonon energy $\hbar wq$ with characteristic wavelength q^{-1} be small in comparison with the polaron energy[6], or that the speed of sound be small in comparison with the uncertainty of the electron velocity $\hbar/m^*a$, which corresponds to the original meaning of the term "adiabatic." In this case a somewhat more stringent requirement must be satisfied: the phonon energy $\hbar wq$ must be small compared with the energy characterizing the fine structure of the polaron state (for simplicity, the estimates were made only for the case when the ratio R^2/w^2 is not too close to unity):

$$\hbar wq \ll \hbar\Lambda/m^* a_0^2. \tag{4.1}$$

The spectrum of the phonons making up the well is characterized by the wave vectors $q_{\parallel} \sim a^{-1}$, $a^{-1} < q_{\perp} < l^{-1}$, which is obvious, for example, from a calculation of the longitudinal and transverse masses. The most essential in the condition (4.1) are phonons with $q \sim l^{-1}$. Then this condition, with allowance for (2.7), reduces to

$$\alpha \gg \frac{a_0}{l}\frac{1}{\Lambda} \gg 1. \tag{4.2}$$

From (3.5) and (3.6) it follows that

$$\frac{m^*}{M_{\parallel}} \sim \frac{1}{\alpha^2\Lambda}, \quad \frac{m^*}{M_{\perp}} \sim \frac{1}{\alpha^2\Lambda^2}, \tag{4.3}$$

and thus neglect of the electron mass in comparison with the polaron mass is necessary.

When determining the internal structure of the polaron, no account was taken of the time dependence of $\dot{R}_{\perp}$. This is permissible if the angular frequency Ω of the polaron is small in comparison with the characteristic polaron frequencies

$$\Omega \ll \hbar\Lambda/m^* a_0^2. \tag{4.4}$$

This inequality is a consequence of (4.2), and is therefore automatically satisfied.

It must be noted that a polaron state appears in each Landau band. However, for large Landau quantum numbers n the results may turn out to be inapplicable, since the radius of the state of the electron turns out to be already not l but nl, and the right side of the inequality (4.2) ceases to be valid.

Satisfaction of the condition (4.2) can be expected in the strongest piezoelectrics. Thus, in Te, where $\alpha \sim 5$($K^2 \sim 0.3$; $\epsilon \sim 40$; $w \sim 3 \times 10^5$ cm/sec), and in fields on the order of 50 kOe, there should apparently exist piezopolaron states close to those described in this paper.

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1) It is easy to show with the aid of the approximation (2.8) that the z-dependence drops out of this condition.

2) The off-diagonal element of (2.5) are in this case not equal to zero but proportional to $\dot{R}_{\perp}^2/w^2$. Their effect can be neglected, since the degeneracy is lifted by diagonal elements that do not contain the small quantity (2.28).

3) This remark does not pertain to the case when u_n is defined in self-consistent fashion.

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