Scattering of atoms by light

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The possibility of scattering of atoms by a strong resonant field of standing wave is studied. Two scattering regimes are considered, when the phase varies in a random manner and stochastic acceleration (heating) of the atoms occurs, and when the phase varies in a regular manner. The atomic diffusion coefficient due to quantum and classical gradient-force fluctuations is calculated by means of the kinetic equation and by taking recoil into account. The possibilities of heating metastable atoms in a beam and of isotope separation are discussed.

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1. INTRODUCTION

The possibility of accelerating atoms by pressure of light has been under discussion of late.[1-4] When light acts on a massive dielectric or on a dense gas, the acceleration due to the pressure is usually small and can be neglected. But when it comes to a low-density gas, then the effect of light pressure can become quite appreciable.

Since the force acting on an atom in an electromagnetic field is resonant in character, it becomes possible to use light pressure for such applications as separation of isotopes and of excited and unexcited atoms. Thus, Picque and Vialle[5] and Schieder et al.[6] have deflected a beam of sodium atoms by pressure from a mercury lamp and from a laser. If metastable helium atoms (or helium-like ions) are separated from an atomic beam with the aid of light pressure, then it becomes possible to effect lasing on a principal resonant transition located in the far ultraviolet or the soft x-ray region.[4]

Let us explain first the physical nature of the considered phenomena. The forces acting on an atom in a resonant field can be mainly of three types.

1) In a field of a plane traveling wave, the atom is acted upon by the force due to the scattering of photons with a spontaneous emission rate γ[1,2]

$$F=\hbar k\gamma W, \tag{1}$$

where W is the probability of populating the upper working level. At saturation we have $W = \frac{1}{2}$. The change of the critical energy of the atom under the influence of the force (1) is conveniently expressed in terms of the number of scattered photons $N = \gamma t/2$:

$$\varepsilon_k=Mv^2/2=(\hbar k)^2N^2/2M. \tag{2}$$

Since the acceleration is coherent in this case, the change of the energy ϵ_k is proportional to N^2. To impart to an atom an energy comparable with thermal energy at room temperature we need $N \sim 10^3-10^4$.

2) We consider now the field of a standing wave

$$E(x)e^{-i\omega t}+\text{c.c.}, \quad E(x)=E_0\cos(kx+\varphi). \tag{3}$$

In this case the atom is acted also by a gradient force whose form near resonance is given approximately by (*D is the dipole moment of the transition*)[1)]

$$F\approx\hbar\partial V(x)/\partial x, \quad V=dE(x)/\hbar. \tag{4}$$

If the variation of the phase of the wave (3) is given by $\varphi \sim t^2$, then the atoms captured by the wave can be coherently accelerated to high energies, on the order of several keV.[3,4] On the other hand, if the phase varies randomly with time, then stochastic acceleration (heating) of the atoms occurs. The energy ϵ_{inc} of the atoms in the case of this incoherent acceleration is proportional to the effective number N′ of the scattered quanta

$$\varepsilon_{inc}=(\hbar k)^2N'/2M, \quad N'=tV_0^2/\Gamma, \quad V_0=dE_0/\hbar. \tag{5}$$

We have introduced here the width Γ of the spectrum of the phase fluctuations, which is defined by the relation

$$\langle(\varphi(t)-\varphi(0))^2\rangle=2\Gamma t. \tag{6}$$

It is important that the heating rate is proportional to the radiation power. Comparing cases (2) and (5), we have

$$\frac{\varepsilon_{inc}}{\varepsilon_k}=\frac{N'}{N^2}=\left(\frac{2V_0}{\gamma}\right)^2\Big/\Gamma t. \tag{7}$$

In cases of practical interest we have $\epsilon_{inc}/\epsilon_k \sim 10^2$ (see Sec. 3).

3) When the atom moves in the field of a standing wave, a certain phase difference is produced between the dipole moment of the atom and the field, owing to the spontaneous emission. The atom is therefore acted upon by a certain average force that depends in resonant fashion on the velocity of the atom relative to the wave.

In this paper we investigate in detail the acceleration of atoms by forces of the second and third type. We calculate the average force and the diffusion coefficient of the atoms in velocity space. We discuss the possibility of using this acceleration mechanism to heat atoms and separate isotopes. We note that the phenomenon under consideration is analogous to some degree to the Kapitza-Dirac effect (electron scattering by a standing-wave field). The difference lies in the large mass of the atoms (which permits the use of a quasiclassical approximation) and in the resonant character of the interaction with the field.

2. SOLUTION OF KINETIC EQUATIONS

The rate of heating of the atoms is determined by fluctuations of the dipole moment. To take consistent account of the classical and quantum fluctuations, we start with the quantum kinetic equation for the density matrix of the atoms. The exact kinetic equation with allowance for the recoil effect, which was solved by Kol'chenko, Rautian, and Sokolovskiĭ[8] for the case of a weak electromagnetic field, is quite complicated. This equation can be simplified by treating the recoil

by perturbation theory and expanding the Hamiltonian of the system in terms of the parameter $\hbar k/Mv \ll 1$ (v is the velocity of the atoms relative to the standing wave).[7] In this approximation the initial system of equations for the atom distribution function f(xvt) and for the dipole-moment distributions p(xvt) (measured in units of d), and the population difference between the lower and upper levels q(xvt) take the following forms:

$$\frac{df}{dt}=\frac{\hbar}{M}\left(\frac{\partial V^*}{\partial x}\frac{\partial p}{\partial v}+\text{c.c.}\right), \tag{8}$$

$$\frac{dp}{dt}+(i\Delta+\gamma/2)p=iVq+\frac{\hbar}{2M}\frac{\partial V}{\partial x}\frac{\partial f}{\partial v}, \tag{9}$$

$$\frac{dq}{dt}+\gamma q=-\gamma f+2i(pV^*-\text{c.c.}). \tag{10}$$

Here $d/dt = \partial/\partial t + v\partial/\partial x$ is the total derivative with respect to time.

The problem consists of calculating the average acceleration and the diffusion coefficient of the atoms in velocity space. We are interested in the case of a strong field and of noise that is not too narrow-band:

$$V_0\gg\Gamma\gg\Gamma_0, \quad \Gamma_0=k(\hbar V_0/M)^{1/2}. \tag{11}$$

Γ_0 is the frequency of the small oscillations of the atoms captured in the periodic potential V(x). At $\Gamma < \Gamma_0$, the captured atoms follow adiabatically the variation of the field, and there is no heating. The criterion for the onset of stochastic acceleration is the inequality $\Gamma > \Gamma_0$. The condition $\Gamma \gg \Gamma_0$ makes it possible to use perturbation theory and obtain for the atom distribution function a Fokker-Planck equation. We note that in the cases of greatest practical interest we have $\Gamma \gtrsim \gamma$. Therefore we neglect the spontaneous relaxation in comparison with Γ wherever this is permissible.

At arbitrary values of the detuning, our problem is quite complicated. We consider therefore two limiting cases: $\Delta < \Gamma$ and $\Delta > \Gamma$. We start with the case of large Δ. At $\Delta \gg \Gamma$ the solution of (9) can be obtained in the form of an expansion in $1/\Delta$:

$$p\approx\frac{1}{\Delta}\left(Vq-\frac{i\hbar}{2M}\frac{\partial V}{\partial x}\frac{\partial f}{\partial v}\right)+i\left(\frac{d}{dt}+\frac{\gamma}{2}\right)\frac{Vq}{\Delta^2}. \tag{12}$$

Substituting (12) in (8) and (10), we get

$$\frac{dq}{dt}(1+\varepsilon)+\left(\gamma+\frac{1}{2}\gamma\varepsilon+\frac{1}{2}\frac{d\varepsilon}{dt}\right)q=-\gamma f+\frac{\hbar\Delta}{4M}\frac{\partial\varepsilon}{\partial x}\frac{\partial f}{\partial v}, \tag{13}$$

$$\frac{df}{dt}=\frac{\hbar\Delta}{4M}\frac{\partial\varepsilon}{\partial x}\frac{\partial q}{\partial v}, \quad \varepsilon=\frac{4V^2}{\Delta^2}. \tag{14}$$

The dimensionless field intensity $\epsilon(xt)$ consists of constant and rapidly-alternating parts

$$\varepsilon=\varepsilon_0+\varepsilon_1, \quad \varepsilon_0=2V_0^2/\Delta^2, \quad \varepsilon_1=\varepsilon_0\cos(2kx+2\varphi).$$

The functions f and q are similar in form. Solving Eq. (13), it suffices to use for the function f in the right-hand side the slow part $f_0(v)$ of this function

$$q(xvt)=(1+\varepsilon(xt))^{-1/2}\int_{-\infty}^{t}dt'(1+\varepsilon(x't'))^{-1/2}\exp\left\{-\gamma\int_{t'}^{t}d\tau\frac{1+\varepsilon(x'\tau)/2}{1+\varepsilon(x'\tau)}\right\}$$
$$\times\left[-\gamma f_0+\frac{\hbar\Delta}{4M}\frac{\partial\varepsilon(x't')}{\partial x}\frac{\partial f_0}{\partial v}\right]. \tag{15}$$

The function q(xvt) contains here a slow part and a fast part. The latter, however, drops out after averaging over the fast oscillations with $\partial\epsilon/\partial x$ in (14). We arrive as a result at a Fokker-Planck equation for the distribution function $f_0(vt)$:

$$\frac{\partial f_0}{\partial t}=\frac{\partial}{\partial v}\left(-a(v)f_0+D(v)\frac{\partial f_0}{\partial v}\right), \tag{16}$$

$$a(v)=\frac{\gamma\,dE_0k^2v\,\operatorname{sgn}\Delta}{M(4\Gamma^2+(kv)^2)}C(\Delta), \tag{17}$$

$$D(v)=\left(\frac{\hbar\Delta}{2M}\right)^2\frac{\Gamma(1+\varepsilon_0)}{(4\Gamma^2+(kv)^2)}\left(1-\sqrt{1-\left(\frac{\varepsilon_0}{1+\varepsilon_0}\right)^2}\right). \tag{18}$$

The function $C(\Delta)$ was calculated earlier ([4], Fig. 3). Thus, formulas (16)—(18) are solutions of the posed problem for the case $\Delta > \Gamma$.

We consider now the case of small detunings. Putting $\Delta = 0$, we obtain

$$D(v)=\frac{(k\,dE_0)^2\Gamma}{2M^2(\Gamma^2+(kv)^2)}, \quad \Delta=0. \tag{19}$$

The Diffusion Coefficient

Summing the results of the calculations for the resonant and nonresonant cases, we can represent the diffusion coefficient graphically in the form shown in Fig. 1. Curve 1 corresponds to the low-velocity limit, when $kv \ll \Gamma$. It follows from (18) and (19) that D(0) as a function of the detuning Δ at the maximum exceeds D(0) in the plateau region $\Gamma < \Delta < 2V_0$ by a factor of four. The physical cause of this behavior of the diffusion coefficient is the following:

At low atom velocities the diffusion coefficient is proportional to the random-field correlation time, $D \sim \Gamma^{-1}$. In the case of exact resonance the atom acts with the field amplitude E(t) and the diffusion coefficient is proportional to the quantity

$$\int_0^\infty\frac{dt\langle E(t)E(0)\rangle}{\langle E^2(0)\rangle}=\frac{1}{\Gamma}. \tag{20}$$

In the nonresonant case the atom interacts with intensity $E^2(t)$ and the diffusion coefficient is given by

$$\int_0^\infty\frac{dt\langle E_1^2(t)E_1^2(0)\rangle}{\langle E_1^4(0)\rangle}=\frac{1}{4\Gamma}, \tag{21}$$

i.e., in the nonresonant case the field correlation time is decreased by a factor of four.

Curve 2 pertains to the case $kv \gg \Gamma$. The effective correlation time in this case is $\Gamma/(kv)^2$. On going from the resonant region to the nonresonant one, Γ increases by four times and k increases by two times, so that the diffusion coefficient does not change essentially at $\Delta < 2V_0$.

Average Effective Force

The dependence of the acceleration a (the average effective force) on the detuning Δ is determined by the function $C(\Delta)$. At $\Delta > 0$ we have $a > 0$ and we are dealing with an accelerating force; at $\Delta < 0$ the aver-

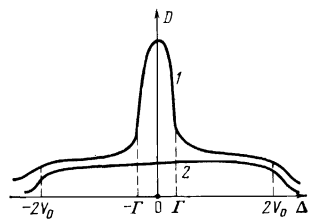

FIG. 1

age force is a decelerating one. At small Δ we have $a \sim \Delta$, and at low radiation intensity $a \sim \epsilon^3$. In contrast to the case of a monochromatic standing wave, considered earlier in[4] The denominator in the formula (17) for the average force contains $4\Gamma^2$ in place of γ^2.

At $\Delta \approx \Delta_m \approx 0.2V_0$ the function $C(\Delta)$ reaches a maximum $C_m \approx 0.1$. Thus, the maximum acceleration is given approximately by the formula

$$a_m \approx \frac{2.5 \cdot 10^{-2}\, \gamma\, dE_0 k^2 v}{M(\Gamma^2 + (kv)^2/4)}. \qquad (22)$$

3. HEATING AND ACCELERATION OF ATOMS

We now estimate with the aid of the kinetic equation (16) the effect of acceleration and heating of the atoms. We consider first the limit of low velocities, $kv < \Gamma$. In this case the equations of motion for the first two moments (the average velocity $\bar{v}$ and the mean-squared velocity $\overline{v^2}$) take the form

$$\dot{\bar{v}} = \frac{\bar{v}}{\tau}, \quad \tau^{-1} = \frac{\partial a}{\partial v}\Big|_{v=0},$$

$$\dot{\overline{v^2}} = 2D(0) + \overline{2v^2}/\tau. \qquad (23)$$

If the atoms had a zero velocity at $t = 0$, then we obtain for the mean-squared velocity

$$\overline{v^2} = D(0)\tau(e^{2t/\tau} - 1). \qquad (24)$$

For short times $t < \tau$ we have

$$\overline{v^2} = 2D(0)t. \qquad (25)$$

Heating of Atoms

In the initial stage of heating it is desirable to have small Γ. However, the atom heating linear in time [Eq. (25)] continues only so long as $kv < \Gamma$. At $kv > \Gamma$ the heating rate slows down and $\dot{\overline{v^2}} \sim (t)^{1/2}$. Therefore to accelerate the atoms in the linear regime (25) to a velocity v_0 we must have $\Gamma \sim kv_0$. From this condition we obtain for the radiation intensity the relation

$$E_0^2 = (kv_0 t)^{-1}(Mv_0^2/d)^2, \qquad (26)$$

where t is the heating time. We have used the maximum diffusion coefficient, taken at $\Delta = 0$.

Let us estimate with the aid of this relation the radiation power needed to heat the $He(2^3S)$ atoms by a resonant field (wavelength 1.06 $m\mu$) to a temperature 300°K. At $t = 10^{-4}$ sec the required power is 10^5 W/cm^2. Then $\Gamma = 6 \times 10^9$ Hz, and $\Gamma_0 = 2 \times 10^8$ Hz, so that the condition $\Gamma > \Gamma_0$ is satisfied with a large margin.

Isotope Separation

To effect separation, a given isotope traveling in the atom beam with velocity v_0 must be imparted a small transverse velocity αv_0, where $\alpha \ll 1$ is the scattering angle (see Fig. 2). In this case $\Gamma = \alpha k v_0$ and $t = l/v_0$, where l is the width of the light beam. Then the field intensity needed to scatter the atoms through a given angle α is determined by the relation

$$E_0^2 = \alpha^3 (kl)^{-1}(Mv_0^2/d)^2. \qquad (27)$$

Assuming the atom energy in the beam to be 300°K, $l = 1$ cm, and $\alpha = 0.1$ for radiation at resonance with the D line of the sodium atom, we obtain the required

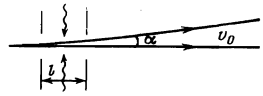

FIG. 2

power 2 kW/cm^2. Of course, at smaller values of α we need less radiation power. We note that in the experiments[5,6] the typical order of magnitude of the scattering angle was $\alpha \sim 10^{-4}$.

Acceleration of Atoms

With increasing Γ, the acceleration decreases like Γ^{-2}, and the diffusion coefficient is $D \sim \Gamma^{-1}$. Therefore, when the width of the spectrum is appreciably larger than the natural line width, the heating turns out to be more effective than the acceleration. By way of illustration we present the values of the acceleration time τ for the particular cases considered above. Thus, at those values of the parameters for which we estimated the heating of the $He(2^3S)$ atoms we obtain, using (22), $\tau = 3.5 \times 10^{-4}$ sec. This is about three times longer than the heating time. In the case of small-angle deflection of the Na atoms, exceeds the heating time by approximately 15 times.

The average effective force can be used more efficiently in the field of a monochromatic uniformly accelerated waves. We put $\varphi = \omega t^2$. Depending on the wave acceleration, ω/k, the atom will be acted upon by a different average force. The maximum possible accelerating force is[4]

$$F_m = 0.1\gamma(dE_0 M)^{1/2}. \qquad (28)$$

This force can accelerate the $He(2^3S)$ atoms to 10^5 cm/sec in a field of 10^5 W/cm^2 within 10^{-5} sec. The acceleration time is in this case one-tenth the heating time. To obtain an accelerated traveling wave we must change the frequency of one of the opposing waves by 6×10^6 Hz within an acceleration time 10^{-5} sec.

To deflect sodium atoms through an angle $\alpha = 0.1$ with the aid of an accelerated wave, the required radiation power is 125 W/cm^2. Then the field frequency should change by 6×10^8 Hz within the acceleration time 10^{-5} sec.

4. CONCLUSION

Resonant radiation of power 10^2–10^5 W/cm^2 can thus be used to heat or accelerate atoms in the energy interval 10^{-4} to 10^{-2} eV. We are dealing here with relatively small flight acceleration times $\sim 10^{-5}$ sec.

Deflection of atoms under the influence of the light pressure produced by spontaneous emission was observed in recent experiments.[5,6] In the present study we considered atom acceleration caused by induced transitions. To principal regimes are possible: heating of the atoms in a standing-wave field with random phase, and acceleration with a field of an accelerated traveling wave. In strong field, the acceleration of the atoms by induced transitions is more effective than that by spontaneous transitions. For comparison we indicate that in the cases considered here the atom energy acquired by heating is approximately 100 times larger than that provided by the accelerating force (1) in the same time interval.

Note added in proof (24 April 1975). The scattering of atoms by the field of a standing monochromatic wave was recently considered by A. P. Kazantsev and G. I. Surdutovich (ZhETF Pis. Red. 21, 346 (1975), JETP Lett. 21, 158 (1975)).

[1)]We note that far from resonance $\hbar\Delta \gtrsim dE_0$ ($\Delta = \omega - \omega_0$ is the detuning of the field frequency relative to the transition frequency ν_0) the gradient force is [2]

$$F = \frac{d}{dx}\frac{[dE(x)]^2}{\hbar\Delta}$$

At $\hbar\Delta \sim dE_0$ we have approximately formula (4). At $\hbar\Delta < dE_0$, as the resonance is approached, the average gradient force tends to zero, owing to the saturation effect, and the fluctuations of this force increase. [3,4,7] At $\hbar\Delta = 0$ the fluctuating gradient force can be represented in the form $\pm d[dE(x)]/dx$. [7] Since both the average and the fluctuating forces lead to scattering of the atoms, we can assume approximately that formula (4) holds also for the frequency interval $\hbar\Delta \lesssim dE$.

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