Hydrogen line contour deformation induced in a plasma by a strong electromagnetic field

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Deformation of the Holtsmark absorption spectrum of hydrogen in a plasma exposed to strong electromagnetic (laser) radiation $E_0 e^{i\omega t}$ is considered. The character of the spectrum is determined by the parameter $V_0 T_{ph}(\Delta\omega)$, where V_0 is the interaction between the atom and the field E_0, and T_{ph} is the time of phase coherence loss by the atom due to thermal motion of the ions ($T_{ph} \sim (T_F/\Delta\omega)^{1/2}$, where T_F is the Chandrasekhar-von Neumann ion-microfield lifetime). The Holtsmark theory is valid if $V_0 T \ll 1$. At $V_0 T \gg 1$, the absorption decreases by a factor $(V_0 T_{ph})^3$, depending on the fluctuation rate of the ion field. The effect can be used to determine ion temperatures or to study the dynamics of the ion microfield in a plasma.

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1. INTRODUCTION

The Holtsmark theory [1] for the spectrum of atomic hydrogen in a plasma presupposes from the very outset that the interaction of the atom with the emission (absorption) field E_0 is arbitrarily weak. Therefore, within the framework of this very theory, the dependence of the absorbed power Q on the light intensity E_0^2 is trivial, $Q \propto E_0^2$. This paper deals with the manner in which the absorption spectrum of hydrogen changes in the plasma in strong electromagnetic (say, laser) fields. This question is of interest both from the fundamental point of view in the sense of the limitations of Holtsmark's theory in strong fields E_0, and from the point of view of plasma diagnostics with a laser.

Let us dwell first on the premises underlying Holtsmark's theory:

(1) The spectrum is formed as a result of Stark frequency shifts of the hydrogen atom $\kappa = \alpha F$ in the microfield F produced by the plasma ions (α is the Stark constant of the considered transition).[1)]

(2) This microfield is static and has a Holtzmark distribution [1,2]: $W(\mathbf{F})d\mathbf{F} = F_0^{-1}\mathcal{H}(F/F_0)dF$, where $\mathcal{H}(\beta)$ is the Holtsmark function, $F_0 = \lambda N^{2/3}$ is the "normal" Holtsmark field ($\lambda = 2\pi(4/15)^{2/3} \approx 2.603\ldots$, N is the ion density).

(3) The intensity distribution $Q(\Delta\omega)$ ($\Delta\omega \equiv \omega - \omega_0$, ω is the observed frequency and ω_0 is the unperturbed frequency of the atom) at fixed F is proportional to $(\mathbf{dE}_0)^2\delta(\Delta\omega - \kappa)$, which obviously corresponds to the energy conservation law upon absorption of a quantum $\hbar\omega$ by an atom in the field $\mathbf{F}$ ($\mathbf{d}$ is the dipole moment of the transition).

(4) The total intensity is obtained by averaging the δ-function $\delta(\Delta\omega - \kappa)$ over the field distribution $W(\mathbf{F})$, which reduces to a direct substitution of the value $F = \Delta\omega/\alpha$ into the argument of the Holtsmark function.

The question of the limitations imposed on the Holtsmark theory by the electromagnetic field intensity E_0 never arose before at all, since E_0 usually pertained to the spontaneous emission. It is clear, however, that such limitations should exist in principle. This follows at least from the fact that for a static field $\mathbf{F}$, at a sufficiently large E_0, it is impossible to determine (without introducing additional parameters), even the stationary absorption velocity Q, inasmuch as it is impossible to satisfy the energy conservation law in absorption: the arguments of the δ-function in (3) now contains instead of $\Delta\omega - \kappa$ the quantity $((\Delta\omega - \kappa)^2 + 4(\mathbf{dE}_0)^2)^{1/2}$, which never vanishes (see Sec. 2). It is clear from the foregoing that a generalization of the Holtsmark theory to include the case of large E_0 should require that we give up the premise (2) that the ionic microfield is static. It is precisely allowance for the ion motion which makes it possible to determine a finite (nonzero) value of the absorption Q in a strong electromagnetic field. The energy conservation law is satisfied in this case as a result of a change in the ion velocity. We note that a nonzero contribution to the absorption can be also obtained by taking into account the additional relaxation processes due to radiative damping or to electron collisions. In what follows, however, these extraneous (relative to ions) processes will not be taken into account; the corresponding criteria under which the ion width predominates are sufficiently well known in broadening theory [3,4] (see also Sec. 2).

An analysis of the temporal fluctuations of the ion microfield, based on allowance for the action of all the ions on the atom, encounters considerable difficulties, since even the bivariate distribution $W(\mathbf{F}, \dot{\mathbf{F}})$ for the field $\mathbf{F}$ and its second derivative $\dot{\mathbf{F}}$ cannot be calculated exactly.[5] Therefore all the results that are pertinent here must of necessity be limited to the first few moments of the distributions $\langle \dot{\mathbf{F}}_\sigma \rangle_{\mathbf{F}}$ and $\langle \dot{\mathbf{F}}_\sigma^2 \rangle_{\mathbf{F}}$, which were first obtained by Chandrasekhar and von Neumann.[2] As applied to calculations of the hydrogen spectrum, allowance for the thermal motion of the ions has reduced to determination of the corrections to the static (Holtsmark) contour, and on their basis to a determination of the criteria for quasistatic behavior. Within the framework of the adiabatic model of broadening, these corrections were calculated by Kogan [6] and later by Wimmel [7] [2)]. The non-adiabaticity effect and rotation of the atom were taken into account in [9]. We emphasize that the difficulties of taking into account the ion dynamics are due to the multiparticle character of the problem, which greatly complicates all the calculations in comparison with the binary limit. In the latter case, however, the problem of hydrogen broadening can be solved exactly [10].

2. QUALITATIVE TREATMENT

We consider first, following [7], the formation of a Holtsmark spectrum in a weak field E_0. If the field F(t) varies slowly with time, we can confine ourselves to the first terms of the expansion of κ in powers of t,

$$Q(\Delta\omega)\propto E_0^2\frac{1}{\pi}\mathrm{Re}\int_0^\infty dF\,W(F)\int_0^\infty d\tau\, e^{i(\Delta\omega-\alpha F)\tau}\langle e^{i\alpha\dot F(0)\tau^2/2}\rangle_F, \qquad (2.1)$$

where $\langle\ldots\rangle_F$ denotes averaging at a fixed value of the modulus.

The internal integral with respect to τ yields the line profile at a fixed ion field F, and should be proportional, according to condition (3) of Sec. 1, to $\delta(\Delta\omega-\alpha F)$. The factor under the $\langle\ldots\rangle_F$ sign describes the change of the phase due to the thermal motion of the ions and obviously leads, after averaging at a given F, to the loss of coherence by a train of waves radiated at a given value of the field F. The coherence-loss time $T_{ph}(F)$ can be estimated by considering that the phase shift during the time T_{ph} should be of the order of unity, so that

$$T_{ph}(F)\sim\left\langle\frac{1}{(\alpha|\dot F(0)|)^{1/2}}\right\rangle_F\sim\left(\frac{T_F}{\alpha F}\right)^{1/2}, \qquad (2.2)$$

where T_F is the time of variation of the field F, see [2].

From this we easily obtain the criterion for the applicability of the Holtsmark static theory, corresponding to the condition of formation of the δ-function in (2.1) (cf. [7]):

$$\Delta\omega T_{ph}(\Delta\omega/\alpha)\gg 1. \qquad (2.3)$$

Within the framework of the adiabatic model, for which formula (2.1) was in fact written, the criterion (2.3) is too stringent, [11,12] but when the non-adiabaticity effects are taken into account [9] the condition (2.3) is qualitatively correct.

We now consider the process of absorption of a strong electromagnetic field. The probability w of a transition (by the instant T) from a lower level i of the atom to an upper level f at a constant field **F** oscillates with frequency $\Omega=[(\Delta\omega-\kappa)^2+4V^2]^{1/2}$, where $V=\mathbf{d_{if}}\cdot\mathbf{E}_0$ ($\mathbf{d_{if}}$ is the dipole moment of the transition), see [13], p. 173.

The stationary probability Q = w/T of the transitions per unit time can be estimated only as $V\to 0$, when $Q\propto\delta(\Delta\omega-\kappa)$.

In the general case, however, it is impossible to determine Q from the expression for w, without introducing some relaxation mechanism that cuts off the sinusoidal oscillations of the atom in the field $\mathbf{E}_0e^{i\omega t}$. Such a mechanism is usually connected with radiative damping or with electron collisions, which make it possible to introduce Q by averaging w with a weight factor $\gamma e^{-\gamma t}$. If we then integrate the results with the distribution function $W(\kappa)$, then we obtain at $\Delta\omega\gg\max(V,\gamma)$ the formula of the inhomogeneous-broadening theory ([14], Sec. 17, formula (17.80)).

In this formula, the usual result of the static broadening theory is obtained at $V\ll\gamma$, whereas at $V\gg\gamma$ there takes place a decrease of the quasistatic absorption by a factor γ/V.

In the case of interest to us, of broadening by ions, the situation is in many respects analogous to that considered above. For a static field F, there is no absorption, inasmuch as at $V\neq 0$ it is impossible to satisfy the "energy conservation law" $\Omega=0$. However, if the thermal motion of the ions is taken into account, the absorption becomes possible because of the finite coherence-loss time T_{ph}, which plays the role of the relaxation parameter γ^{-1}. Just as in the case of inhomogeneous broadening, the ordinary Holtsmark theory is valid at $VT_{ph}(\Delta\omega)\ll 1$, whereas at $VT_{ph}(\Delta\omega)\gg 1$ the absorption is decreased by a factor $(VT_{ph})^k$. The exponent k is determined by the concrete mechanism that causes the detuning from coherence and, as shown by calculation, is equal to three (see formula (4.4) below). We note that the time T_{ph}, as will be shown below, is in this case a parameter of the inelastic relaxation, since this time is connected with the inelastic process wherein the ion velocity is changed when the atom absorbs light. In contrast to the ordinary treatment of saturation in inhomogeneous broadening [14], we consider here a new inelastic-relaxation channel connected with the motion of the broadening particles themselves.

3. CALCULATION OF THE ABSORPTION PROBABILITY

We consider the absorption of electromagnetic radiation $\mathbf{E}_0e^{i\omega t}$ by a hydrogen atom in a plasma. We are interested in the case of quasistatic broadening, which is realized at sufficiently low velocities v and large densities N of the plasma ions (criteria will be given below; see also [3,4]). Under these conditions, the Stark components of a hydrogen atom situated in the electric field **F** of the plasma ions can be regarded independently of one another. Then, taking into account the interaction with the radiation, it suffices to confine oneself to a two-level scheme corresponding to a transition between Stark components of the upper and lower levels.

We write down the Schrödinger equation for the amplitudes of the lower (a_i) and upper (a_f) components of the levels:

$$\begin{aligned}\dot a_i&=U_i(t)a_i+V(t)e^{i\Delta\omega t}a_f,\\ \dot a_f&=U_f(t)a_f+V(t)e^{-i\Delta\omega t}a_i.\end{aligned} \qquad (3.1)$$

Here $U_{i,f}=\alpha_{i,f}|\mathbf{F}(t)|$ are the Stark shifts of the corresponding components (α_i and α_f are the Stark constants); $V(t)=\mathbf{d_{if}}(t)\mathbf{E}_0$. Equations (3.1) were written in a coordinate system with the oZ axis along F(t), where the Hamiltonian of the interaction of the atom with the ion field is diagonalized. In the course of time, this system rotates with the field **F**(t), so that the angle between $\mathbf{d_{if}}$ and $\mathbf{E}_0$, and by the same token also V(t), depends on the time.

The system (3.1) has been written, as already noted, on the assumption that the Stark components are sufficiently well isolated. This means, first, that the nonadiabatic transitions between the components can be neglected and, second that the interaction with the field V is sufficiently weak so that the Stark structure of the terms is not too distorted. The first of these conditions is ensured by the known criterion (2.3) that the ion be quasistatic.

The second condition requires, obviously, satisfaction of the inequality

$$\Delta\omega\gg|V|. \qquad (3.2)$$

When conditions (2.3) and (3.2) are satisfied, the problem of finding of the $i\to f$ transition probability, which determines the absorption of the light, is equivalent to the problem of the elastic transition between two terms in

slow collisions of particles ([13], Sec. 90). It is known in this case that the main contribution to the transition probability is made by the term-intersection point t_k corresponding to the energy conservation law:

$$U_i(t_k)-U_f(t_k)\equiv\varkappa(t_k)=\Delta\omega, \tag{3.3}$$

where the instantaneous frequency shift of the transition is $\kappa = \alpha F(t)$ ($\alpha \equiv \alpha_i - \alpha_f$).

The condition (3.3) can be regarded as the condition for the intersection of the terms of the composite "atom + electromagnetic field" system (for details see [15,16]). On the other hand, the interaction with the electromagnetic field causes these terms to repel each other by an amount V which is small in comparison with $\Delta\omega$.

The indicated analogy with the theory of inelastic slow collisions makes it possible, naturally, to write down immediately an expression for the transition probability by using the Landau-Zener result [12,15,16]. This result, however, contains the quantities $|V(t_k)|^2$ and $|\dot\kappa(t_k)|$ taken at the intersection point t_k. Therefore, to obtain the total transition probability it is necessary to sum over all the points t_k with allowance for the stochastic time variation of $\mathbf{F}(t)$. Whereas in the case of pair collisions this summation reduces simply to a determination of the corresponding transition cross sections, [13,16] in the general case this summation cannot be carried out because of the very complicated dependence of t_k on the coordinates of all the ions, see (3.3).

To avoid the difficulties that arise in the Landau-Zener method, it is necessary to start by obtaining for (3.1) a general solution that is not connected beforehand with the singular points of the potential U(t). Unfortunately, there is no exact solution of the system (3.1). There exist, however, approximate general solutions of (3.1), which in certain limiting cases coincide with the exact solutions. For our purposes, the most convenient is the formula proposed by Vaĭnshteĭn, Presnyakov, and Sobel'man.[3,17] According to this formula, the probability w of a transition from the lower to the upper level by the instant of time T is equal to

$$w(T)=|a_f(T)|^2=\left|\int_{-\tau}^{T}dt\,V(t)\cos\int_0^t\left\{[\Delta\omega-\varkappa(\tau)]^2+4V^2(\tau)\right\}^{1/2}d\tau\right|^2. \tag{3.4}$$

In the case of a sufficiently slow (adiabatic) variation of $\mathbf{F}(t)$, formula (3.4) yields the Landau-Zener result, [13] and the fact that the intersection points t_k are real follows from (3.4) automatically. We indicate that at large V (the case of an exponential fall-off) the probability w(∞) differs from the exact Zener result by a preexponential factor. This leads, however, only to an additional numerical coefficient on the order of unity in the final result. Since we are interested not in the absolute value of w(T) but only in the line profile, this difference is immaterial and the corresponding numerical factors will henceforth be omitted.

It is important to note that in a constant field ($\mathbf{F}$ = const) expression (3.4) yields the exact result [13] for the transition probability in a two-level system under the influence of monochromatic radiation. From this point of view, formula (3.4) can be regarded as generalization of the well known model of a variable-frequency oscillator to include the case of strong electromagnetic fields.

This last remark allows us to go over from the time-dependent picture to an ensemble picture in the spirit of ordinary broadening theory. Namely, we assume that the change of the field $\mathbf{F}(t)$ (i.e., of V(t) and of $\kappa(t)$) in (3.4) has a random character due to the random locations and velocities of the ions; then, transforming (3.4) in accordance with the general correlation-theory formulas ([3], Sec. 36), we obtain

$$w=QT, \tag{3.5}$$

$$Q=\int_{-\infty}^{\infty}dt\left\langle V(0)V(t)\cos\left\{\int_0^t\{[\Delta\omega-\varkappa(\tau)]^2+4V^2(\tau)\}^{1/2}d\tau\right\}\right\rangle. \tag{3.6}$$

Here the symbol $\langle\ldots\rangle$ denotes averaging over the ensemble (under the usual assumption that the random process is stationary).

Formula (3.6) determines the stationary velocity of the transitions (per unit time) between levels under the influence of an electromagnetic field, i.e., it determines the power (after multiplying by the energy). Naturally, formulas (3.5) and (3.6) are meaningful if the integral with respect to t in (3.6) converges. At F = const, expressions (3.5) and (3.6) are meaningful only as $V \to 0$, when $Q \propto \delta(\Delta\omega - \kappa)$, and we arrive at the result of [18], which is valid for weak fields. It will be shown below that when account is taken of the time variation of F due to the thermal motion of the ions, Q turns out to be finite also at sufficiently large V. We note, finally, that for a two-level system the results (3.5) and (3.6) are not connected, generally speaking, with the assumption that the perturbation is slow.

Let us investigate expression (3.6) for the case of slow variation of F(t). In this case we can expand in powers of t in (3.6), and this yields

$$Q\approx\int_{-\infty}^{\infty}dt\left\langle V^2(0)\cos\left\{\int_0^t\sqrt{[\Delta\omega-\varkappa(0)-\dot\varkappa(0)\tau]^2+4V^2(0)}\,d\tau\right\}\right\rangle. \tag{3.7}$$

We have discarded here terms of higher order in t and the derivatives $\dot V(0)$ on the basis of conditions (2.3) and (3.2). The arguments of the functions $\kappa(0)$, $\dot\kappa(0)$, and V(0) will henceforth be omitted for brevity.

Since (3.7) depends only on the field F and its first derivative $\dot F$, the bivariate distribution[3)] $W(F, \dot F)$ alone suffices to carry out the averaging in (3.7). It must also be recognized that the angle θ_{FE_0} between the vectors $\mathbf{F}$ and $\mathbf{E}_0$ can have an arbitrary value in a plasma. We carry out the averaging in two stages: we first average at a fixed value of the modulus of the field F (designated $\langle\ldots\rangle_F$), and then average over all F with the Holtsmark distribution function W(F). Thus,

$$\langle\ldots\rangle=\frac{1}{F_0}\int_0^\infty dF\,\mathcal{H}\left(\frac{F}{F_0}\right)\frac{1}{2}\int_0^\pi d\theta_{FE_0}\cos\theta_{FE_0}\langle\ldots\rangle_F. \tag{3.8}$$

We first transform the integral (3.7). Introducing the dimensionless variables $z = |\dot\kappa|\tau/2V$ and $p = |\dot\kappa|t/2V$, we obtain with the aid of simple transformations

$$Q=\langle{}^1/_2|V|J(4V^2/|\dot\varkappa|;\ |\Delta\omega-\varkappa|/2V)\rangle, \tag{3.9}$$

where

$$J(y;x)=y\cos\left[y\int_0^x dz\sqrt{1+z^2}\right]\int_{-\infty}^{\infty}dp\cos\left[y\int_0^p dz\sqrt{1+z^2}\right]. \tag{3.10}$$

Writing down further the averaging in (3.9), we obtain according to (3.8)

$$Q=\frac{1}{F_0}\int_0^\infty dF\,\mathcal{H}\left(\frac{F}{F_0}\right)\frac{1}{2}\int_{-1}^{+1}d\cos\theta_{FE_0}\left\langle\frac{|V|}{2}J\left(\frac{4V^2}{|\dot\varkappa|};\ \left|\frac{\Delta\omega-\varkappa}{2V}\right|\right)\right\rangle_{|F} \tag{3.11}$$

or, putting $(\Delta\omega - \kappa)/2V = x$,

$$Q=\frac{1}{\alpha F_0}\int_{-\Delta\omega/2V}^{\infty}dx\,\mathcal{H}\left(\frac{\Delta\omega-2Vx}{\alpha F_0}\right)\frac{1}{2}\int_{-1}^{+1}d\cos\theta_{FE_0}V^2\left\langle J\left(\frac{4V^2}{\alpha|F|};\ |x|\right)\right\rangle. \tag{3.12}$$

Expressions (3.11) and (3.12) are essentially convolutions of the Holtsmark function with the function $J(4V^2/|\dot{\kappa}|; x)$, defined at a given value of the field F (or at a given x). This function, if the conditions (2.3) and (3.2) are satisfied, oscillates rapidly everywhere with the exception of the region near the point $x \approx 0$ ($\kappa \equiv \alpha F \approx \Delta\omega$ is the point where the oscillations are damped out). The width x_{eff} of this region depends, as seen from (3.11) and (3.12), on the parameter $4V^2/|\dot{\kappa}|$. Under the condition

$$\Delta\omega \gg |Vx_{eff}| \tag{3.13}$$

we can introduce in (3.12) a slowly-varying Holtsmark distribution at the point x = 0, and extend the integration with respect to x to $-\infty$. Simultaneously, we can replace the mean value at a given F (or x) to the mean value at $F = \Delta\omega/\alpha$ (x = 0). The integration with respect to x in (3.12) can then be carried out explicitly. We thus have

$$Q = \frac{1}{\alpha F_0}\mathcal{H}\left(\frac{\Delta\omega}{\alpha F_0}\right)\frac{1}{2}\int_{-1}^{+1} d\cos\theta_{FE_0} V^2 \left\langle \int_{-\infty}^{\infty} dx J\left(\frac{4V^2}{|\dot{x}|}; x\right)\right\rangle_{F=\Delta\omega/\alpha}. \tag{3.14}$$

We consider the integral with respect to x in (3.14):

$$J(y) = \int_{-\infty}^{\infty} dx J(y; x) = y\left[\int_{-\infty}^{\infty} dx \cos\left(y\int_0^x \sqrt{1+z^2}\, dz\right)\right]^2. \tag{3.15}$$

At $y \ll 1$, the effective values are $x_{eff} \sim 1/\sqrt{y} \gg 1$, so that

$$J(y) \approx y\left[\int_{-\infty}^{\infty} dx \cos\left(\frac{yx^2}{2}\right)\right]^2 = \pi. \tag{3.16}$$

At $y \gg 1$, the main contribution to (3.15) is made by the branch point of the root: $x_{eff} = i$. Estimating the integral (3.15), we get

$$J(y) \propto e^{-\pi y/2}, \tag{3.17}$$

where we have omitted an inessential pre-exponential factor.

Substituting (3.16) in (3.14), we verify that in weak fields the absorption is proportional, as it should be to $Q \propto E_0^2$, and its contour is described by a Holtsmark distribution.

In strong fields, substituting (3.17) in (3.13) and averaging over the angle θ_{Fe_0} with allowance for $V^2 = V_0^2\cos^2\theta_{Fe_0}$ ($V_0 \equiv |d_{if}|E_0$), we obtain

$$Q \propto \frac{1}{\alpha F_0}\mathcal{H}\left(\frac{\Delta\omega}{\alpha F_0}\right)\langle|\dot{x}|^{1/2}\rangle_{F=\Delta\omega/\alpha}/V_c. \tag{3.18}$$

As seen from (3.18), in strong fields the character of the absorption changes qualitatively: first, the absorption decreases with increasing field ($Q \propto 1/E_0$), and second, its spectrum is determined not by the static Holtsmark distribution, but by the rate of the field fluctuation $|\dot{F}|^{3/2}$.

We note that after substitution of x_{eff} the criterion for the applicability of (3.12) reduces in cases (3.16) and (3.17) to the conditions (2.3) and (3.2), respectively.

It is clear from the derivation that the transition to the case of strong fields occurs at $V_0^2 \sim |\dot{\kappa}|$.

4. AVERAGING AND TOTAL LINE PROFILE

We consider the absorption contour in a strong electromagnetic field, which is expressed, according to (3.18), in terms of the fluctuation rate $\langle|\dot{F}|^{3/2}\rangle_{F=\Delta\omega/\alpha}$ of the total field. Although there is a standard mark of procedure [2] for the calculation of this last quantity, it can be carried through to conclusion only in the limiting cases of large and small values of F (this being due to the fractional exponent of $|\dot{F}|^{3/2}$). On the other hand, it is clear from physical considerations that if the operations of averaging the modulus $|\dot{F}|$ and of raising it to a power are interchanged, then the two results should differ only by a numerical factor on the order of unity. Therefore, to find the approximate shape of the contour (3.18), it is expedient to use the Chandrasekhar–von Neumann result [2] for the quantity $\langle|\dot{F}|^2\rangle_F$, introducing the approximation

$$\langle|\dot{F}|^{1/2}\rangle_F \approx \{\langle|\dot{F}|^2\rangle_F\}^{1/4} \propto (Nv)^{1/2}[\beta^{1/2}I(\beta)/\mathcal{H}(\beta)]^{1/4} \propto (Nv)^{1/2}\begin{cases}\text{const}, & \beta \ll 1\\ \beta^{1/4}, & \beta \gg 1,\end{cases} \tag{4.1}$$

where $\beta \equiv F/F_0$ and $I(\beta)$ is the Chandrasekhar–von Neumann integral. [2]

Substituting (4.1) in (3.18), we get

$$Q \propto Q_{st}(\beta)\left(\frac{\alpha Nv}{V_0^2}\right)^{1/2}\left[\frac{\beta^{1/2}I(\beta)}{\mathcal{H}(\beta)}\right]^{1/4}, \quad \beta = \frac{\Delta\omega}{\alpha F_0}, \tag{4.2}$$

where $Q_{st}(\beta)$ is the static contour.

It is convenient to rewrite the results by introducing the time T_{ph} of loss of the phase coherence by a wave train emitted at a given value of the field $F = \Delta\omega/\alpha$:

$$T_{ph} = [\alpha^2\langle|\dot{F}|^2\rangle_{F=\Delta\omega/\alpha}]^{-1/4} = \frac{(5\pi)^{1/4}}{2\pi}\frac{1}{\sqrt{\alpha Nv}}\left[\frac{\mathcal{H}(\beta)}{\beta^{1/2}I(\beta)}\right]^{1/4}. \tag{4.3}$$

Using (4.3) and (4.2), we get

$$Q \propto Q_{st}(\beta)\,[V_0 T_{ph}(\beta)]^{-2}. \tag{4.4}$$

On the other hand, for the condition of the transition from the strong field to the weak field we obtain

$$|V_0 T_{ph}(\beta)| \sim 1. \tag{4.5}$$

Thus, in accordance with the qualitative analysis of Sec. 2, in strong fields $|V_0 T_{ph}| \gg 1$ a decrease takes place in the absorption, by a factor $(V_0 T_{ph})^3$ in comparison with the case of weak fields.

At $\beta \gg 1$ ($\Delta\omega \gg \alpha F_0$), Eq. (4.2) leads to the boundary result

$$Q_{bin} \propto Nv^{3/4}\alpha^{1/4}/V_0(\Delta\omega)^{1/4}. \tag{4.6}$$

This result can, of course, be obtained also directly by the Landau-Zener method. Indeed, applying the results of [16] obtained in this manner to hydrogen broadening and averaging them in addition over the angle θ_{FE_0}, we arrive at (4.6). This demonstrates, in particular, the equivalence of the time-dependent and ensemble approaches for the binary region.

At $\beta \ll 1$ ($\Delta\omega \ll \alpha F_0$) we obtain from (4.2)

$$Q \propto \Delta\omega^2 v^{1/2}/\alpha^{1/2}N^{1/2}V_0 = \Delta\omega^2/V_0\sqrt{h}, \tag{4.7}$$

where we have introduced the characteristic parameter $h \equiv N(\alpha/v)^3$, which determines (in order of magnitude) the number of particles in a sphere of radius α/v (Weisskopf sphere).

The result (4.7), just like formula (4.2) as a whole, is essentially nonbinary ($Q \propto 1/\sqrt{N}$).

For the determination of the contour (4.4) (and also of the criterion (4.5)) in the intermediate region $\beta \sim 1$, Fig. 1 shows the dependence of the phase-coherence-loss time T_{ph} on the dimensionless field $\beta = F/F_0$.

A general delineation of the region of applicability of the theory at $h \gg 1$ is shown in Fig. 2. The quasistatic theory is applicable in the region $\Delta\omega \gg \sqrt{\alpha Nv}$ and $\Delta\omega \gg V_0$. Above the curve $V_0 = T_{ph}^{-1}(\Delta\omega)$ there appear non-

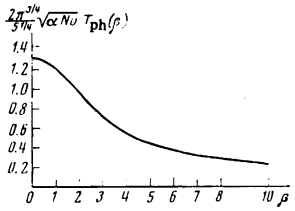

FIG. 1

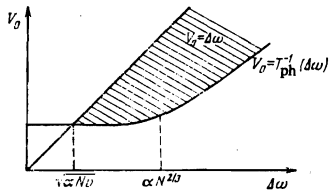

FIG. 2

FIG. 1. Change of the phase-coherence-loss time as a function of the ion field $\beta = F/F_0$. The ordinates are the values of $2\pi^{3/4} \cdot 5^{1/4}\sqrt{\alpha N v}\, T_{ph}(\beta)$.

FIG. 2. Regions of applicability of the approximations on the ($\Delta\omega$, V_0) plane. The shaded region corresponds to the nonlinear effects.

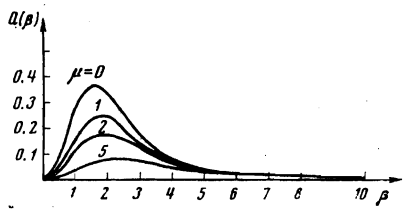

FIG. 3. Deformation of Holtsmark spectrum of $Q(\beta)$ in a strong electromagnetic field at various values of the parameter $\mu \equiv V_0^2/\alpha Nv$. The curve $\mu = 0$ corresponds to the Holtsmark spectrum.

linear effects corresponding to the formula (4.4). Below this curve, the ordinary Holtsmark theory is valid. At $h \ll 1$, the nonlinear effects are realized only in the binary region.

It is of interest to obtain an analytic expression for the total line profile (3.14) for arbitrary V_0. This can be done by using, as in (4.1) an approximate averaging procedure. Indeed, approximating J(y) by expression (3.17) and averaging directly in the argument of the exponential, we obtain from (3.14)

$$Q(\Delta\omega) = \frac{V_0^2}{\alpha F_0}\mathcal{H}(\beta) J\left(\frac{V_0^2}{\alpha N v}\sqrt{\frac{5\mathcal{H}(\beta)}{4\pi\beta^{3/2} I(\beta)}}\right), \qquad (4.8)$$

where $\beta \equiv \Delta\omega/\alpha F_0$ and

$$J(x) = \sqrt{\pi}\,\Phi(\sqrt{x})/4x^{1/2} - e^{-x}/2x \qquad (4.9)$$

($\Phi(z)$ is the error integral).

It is easy to verify that formulas (4.8) and (4.9) give the investigated limiting case (4.6) and (4.7). As to the accuracy of the employed averaging procedure, it appears that numerical factors close to unity are lost here, for in the case of small V_0, according to (3.6), the result is generally independent of the parameter $V_0^2/\alpha Nv$, while at large V_0 the averaging results, according to (4.1), in an interchange of exponents that differ by a factor 3/4. We note that (4.8) takes into account all the numerical coefficients that enter both in the exponent of (3.17) and in $\langle|\dot{F}|^2\rangle_F$. Although the retention of the numerical factor in (4.8) is superfluous within the framework of the employed accuracy, nonetheless we see that it likewise does not differ strongly from unity.

Thus, expression (4.8) describes the distortion of the Holtsmark absorption spectrum by an external electromagnetic field. We see that this distortion is homogeneous in the spectrum, i.e., it depends not only on V_0 but also on $\Delta\omega$. In addition, it is in the general case nonlinear in the density. The character of the deformation of the Holtsmark spectrum as a function of the parameter $\mu \equiv V_0^2/\alpha nV$ is shown in Fig. 3.

5. DISCUSSION

Let us estimate the order of magnitude of the laser field E_0 in which a noticeable distortion occurs in the Holtsmark spectrum (see (4.8)). Assuming, in atomic units, $\alpha \sim 10$, $N \sim 10^{-10} \sim 10^{15}$ cm^{-3} and $v \sim 10^{-3}$, we get for $\Delta\omega \sim \alpha F_0 (\beta \sim 1)$ the value $V_0 \sim E_0 \sim 10^{-6} \sim 10^4$ V/cm. Obtaining such values of E_0 is no problem at present. It appears that the main difficulty lies in finding a high-power laser whose emission wavelength λ is close to the wavelength of the hydrogen line (they should agree within 10 Å). Nonetheless, there is a sufficiently large choice of possibilities of observing the effect, both on hydrogen-like lines of atoms, and on lines that experience the quadratic Stark effect (see, e.g., [4]). The latter is reached by direct generalization of the results to include the case $\kappa = CF^2$.

The line contour in strong fields depends, according to (4.8), on the ion density as well as ion temperature. Therefore, we can determine in principle these two parameters by measuring the absorption of the strong electromagnetic radiation. To this end, however it is necessary that the plasma contain a sufficient concentration N_H of neutral hydrogen. Let us estimate the required value of N_H for an absorption line $L \sim 10$ cm and a wavelength of light $\lambda \sim 10^{-4}$ cm. For the Holtsmark width $\alpha F_0 \sim \alpha N^{2/3} \sim 10^{11}$ sec^{-1} ($N \sim 10^{15}$ cm^{-3}, $\alpha \sim 10$) and for the radiation width $\gamma \sim 10^8$ sec^{-1} we obtain from the formula $L^{-1} \sim N_H \lambda^2 \gamma/\alpha F_0$ the value $N_H \sim 10^{10}$ cm^{-3}.

Formula (4.8) determines the rate of the transitions Q between the lower and upper levels under the influence of the electromagnetic radiation. If we are interested in the stationary absorbed power, then Q must be substituted in the kinetic equation, which includes also other relaxation mechanisms (for example, radiative decay), see [16], and also [19]. One can observe the absorption described by expression (4.8), and the absorption in "pure" form. This is made possible by laser pulses which, on the one hand, ensure a sufficient field intensity E_0, and on the other hand have a duration τ short in comparison with the times required to establish the stationary picture via radiative relaxation. It is clear that the time τ must simultaneously be large in comparison with the time of the fluctuation of the ion field $(N^{1/3}v)^{-1}$.

We indicate in conclusion that the considered effect affords an interesting possibility of directly investigating the dynamics itself of an ion microfield in a plasma.

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[1)] We put $e = \hbar = 1$.

[2)] The numerical errors of [7] were corrected in [8].

[3)] The final answer will then contain the coments of $W(F, \dot{F})$ for which, in contrast to the distribution itself, analytic expressions are available.

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