Critical current density for type-II superconductors in the mixed state

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A phenomenological approach to the description of the resistive state (Krasnov and Shukhman, 1971) is applied to the case of interaction between vortices with pinning centers. The equations of motion of a vortex in a pinning force field are derived. An exact solution of the dynamic equations for concrete interaction potentials between the vortex and the pinning center is presented. It is shown that a distinction should be made between two characteristic values of the transport current, one related to the detachment of the vortex from the pinning center and the other to capture of a vortex scattered by the center. Both values are shown to depend on the Hall angle, the detachment current being also dependent on the rate at which the external current is switched on. The influence of these effects on the current-voltage characteristics in the mixed state is discussed.

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1. INTRODUCTION

A phenomenological approach to the motion of Abrikosov vortices was developed in a number of recent papers.[1-4] Within the framework of this approach, resistive phenomena in both alternating and direct current were described. In particular, the relaxation time of the natural oscillations of the vortex filament was obtained and the resonant character of the ac losses was described.[4]

The main relation of this approach is the effective Ohm's law[4]

$$\rho_f \mathbf{j} = \mathbf{e} + \frac{\delta}{nec}[\mathbf{j}\times\mathbf{b}], \tag{1}$$

where $\mathbf{b}$ and $\mathbf{e}$ are the magnetic induction and the electric field intensity at the observation point, $\mathbf{e} = -\mathbf{v}_L \times \mathbf{b}/c$, $\mathbf{v}_L$ is the transport velocity of the vortex, n is the electron density, and $\rho_f = \mu b/nec$ is the local resistivity. The ratio δ/μ yields the tangent of the Hall angle in the field H_{c2}; this angle will be designated θ_H. The local current density $\mathbf{j}$ in (1) is the total local current density after subtracting the "London" part connected with the field of the vortex by the Londons' equation.

The crucial concept in the introduction of the effective Ohm's law is that at each given instant of time the vortex is in local equilibrium with the lattice. This means that the vortex velocity at any given instant of time is determined completely by the total force acting on it at that instant, and does not depend on the prior history of the vortex motion. We shall call this assumption henceforth the locality hypothesis.

The range of validity of (1) becomes much wider if we assume in addition the so-called "equivalence" hypothesis (see[1] and [4]), the gist of which is the following: The action of the local force $\mathbf{F}$ on the vortex is replaced by the interaction between the vortex and a certain equivalent current chosen such that

$$\mathbf{F} = [\mathbf{j}_e \times \boldsymbol{\varphi}_0]/c, \tag{2}$$

where φ_0 is the magnetic flux carried by the vortex. Then, substituting for $\mathbf{j}$ in (1) the "equivalent" current $\mathbf{j}_e$ calculated from (2) summed with the transport current $\mathbf{j}_T$ and solving (1) for $\mathbf{v}_L$, we obtain the transport velocity of the vortex under the action of the external current $\mathbf{j}_T$ in the field of the extraneous force $\mathbf{F}$. The "equivalence" hypothesis consists in essence of the assumption that such a calculation procedure describes correctly the influence of the extraneous forces on the motion of the vortex.

In this paper we apply consistently the described approach to the case of the interaction of the vortex with a pinning center (the pinning phenomenon). We do not touch upon questions connected with the elastic properties of the Arbikosov vortices, and consider only their transport motion, i.e., the motion as a whole in the model of non-interacting vortices.

In Sec. 2 are discussed certain general properties of vortex motion in the field of pinning forces.

In Sec. 3 we consider the limiting case of materials with $\theta_H = \pi/2$. This case admits of an exact solution of the dynamic equations for any extraneous force, and makes it possible by the same token to explain a whole number of singularities of vortex motion in the field of pinning forces. In particular, it becomes necessary to spell out in detail the criterion for the critical current in the mixed state.

In Sec. 4 we analyze the case of material with finite Hall-angle tangents. Using a parabolic potential as an example, we investigate the exact solution of the dynamic equation and the ensuing singularities of the vortex motion.

2. VORTEX EQUATIONS OF MOTION IN THE FIELD OF EXTRANEOUS FORCES

Let the pinning center produce around itself an axially-symmetric force field, and let the vortex be directed along the symmetry axis of this field, the Z axis. We investigate the motion of such a vortex under the influence of an external current $\mathbf{j}_T$. Following the scheme outlined in the Introduction, we substitute in (1) in place of $\mathbf{j}$ the sum of the external current $\mathbf{j}_T$ and the current $\mathbf{j}_e$ calculated from (2). Solving the obtained equation with respect to $v_{Lx} = \dot{x}$ and $v_{Ly} = \dot{y}$, we have

$$ne\dot{x} = \delta j_{Tx} + \mu j_{Ty} + \frac{c}{\varphi_0}(\mu F_x - \delta F_y), \tag{3}$$

$$ne\dot{y} = \delta j_{Ty} - \mu j_{Tx} + \frac{c}{\varphi_0}(\delta F_x + \mu F_y),$$

where x and y are the running coordinates of the vortex axis.

The system (3) has a simple physical meaning. It is easily seen that (3) expresses the transport velocity of the vortex at any instant of time as the sum of the velocities that are imparted to it separately by the external current (at $\mathbf{F} = 0$)

$$\mathbf{v}_j=\frac{1}{ne}(\mu[\mathbf{j}\times\mathbf{e}_z]+\delta\mathbf{j})$$

and by the extraneous force (at $j_T = 0$)

$$\mathbf{v}_F=\frac{c}{ne\varphi_0}(\mu\mathbf{F}-\delta[\mathbf{F}\times\mathbf{e}_z]).$$

In other words, the vortex velocity at each given instant of time is determined completely by the Lorentz force $\mathbf{F}_L = \mathbf{j}_T \times \varphi_0/c$ exerted on it by the external current and by the extraneous force $\mathbf{F}(\mathbf{r}, t)$ exerted by the pinning center, and does not depend on the prior history of the vortex motion. It is precisely this circumstance that makes it possible to regard the system (3) as the dynamic equations for the transport motion of the vortex.

The main consequence of this approach is that under the action of only the extraneous force $\mathbf{F}$ (i.e., at $j_T = 0$) the vortex is displaced at an angle θ_H to the direction of the action of this force (gyroscopic effect). This effect exerts an appreciable influence on the vortex motion.

Since

$$v_j=\frac{1}{ne}\sqrt{\mu^2+\delta^2}\,j,\quad v_F=\frac{c}{ne\varphi_0}\sqrt{\mu^2+\delta^2}\,|F|,$$

it follows that in polar coordinates, when v_j is directed along the X axis, the system takes the form

$$\dot\rho=v_j\cos\varphi-v_F\cos\theta_H,\qquad \rho\dot\varphi=-v_j\sin\varphi\mp v_F\sin\theta_H. \tag{4}$$

The choice of the sign is determined here by whether $\mathbf{F}$ is an attraction or repulsion force, respectively. In the absence of an external current ($v_j = 0$) the vortex trajectory in the field of the force $\mathbf{F}$ takes the form

$$\rho=\rho_0\exp(\varphi\operatorname{ctg}\theta_H), \tag{5}$$

i.e., the vortex moves along a spiral, the form of which does not depend on the value of the force $\mathbf{F}$. The value of $\mathbf{F}$ determines only the time of displacement of the vortex along the spiral, and the direction of this force (as well as the direction of the field in the vortex) determines only the direction of the vortex motion along the spiral. Therefore when the external current is turned on it is natural to expect the vortex trajectory to be asymmetrical in the field of the pinning forces relative to the direction of this current, and also the direction of the external field.

It is easily seen from (3) that the points of stable equilibrium of the vortex under the influence the pinning force and of the external current lie on the ray $\varphi = -\theta_H$ and their distance from the pinning center is given by

$$\mathbf{F}(\rho)=\frac{1}{c}[\mathbf{j}_T\times\boldsymbol{\varphi}_0].$$

3. THE CASE $\theta_H = \pi/2$

At $\theta_H = \pi/2$ we have $\mu = 0$ and $\delta = 1$.[4] For a potential attraction force $\mathbf{F} = -\text{grad}\, u(\rho)$ we can easily integrate (4) and obtain the trajectory

$$(j_T\varphi_0/c)\rho\sin\varphi=-u(\rho)+u(\rho_0). \tag{6}$$

In a Cartesian coordinate system in which Z represents the values of $u(\rho)$ the solution (6) is the projection on the XY plane of the line of intersection of the surface of revolution $z = u(\rho)$ with the plane $z = -j_T\varphi_0 c^{-1} y$ passing parallel to the X axis through the point ρ_0 of the initial vortex position.

In the problem of vortex scattering by a pinning center, the trajectories take the form

$$\frac{j_T\varphi_0}{c}(y-y_\infty)=-u(\rho), \tag{7}$$

where y_∞ is the impact parameter. It is easily seen on the basis of the indicated geometric interpretation that at any y_∞ all the trajectories are infinite, i.e., for any arbitrarily small external transport current the vortex scattered by a potential well of arbitrary depth cannot be captured by the well. If the external current is turned on after the capture of the vortex by the pinning center, then the trajectories take the form

$$\frac{j_T\varphi_0}{c}(y-y_0)-u(\rho_0)=-u(\rho). \tag{8}$$

Here, too, it is easily seen on the basis of the indicated geometric interpretation that we must have here a finite density of the external current in order to transfer the vortex to an infinite trajectory.

There are thus two characteristic values of the external current. The first is equal to that minimal value at which the transfer of the vortex to an infinite trajectory, i.e., detachment of the vortex from the pinning center, becomes possible. We call this value the detachment current and designate it j_{c1}. (By way of illustration, Fig. 1 shows the trajectories of a vortex moving in a normalized Gaussian well as functions of the "criticality" parameter α, equal to the ratio of the Lorentz force exerted on the vortex by the given external current to the maximum pinning force.) The second value of the current is equal to that minimal value above which it is impossible to capture a vortex scattered by a given pinning center. We call this value the capture current and designate it j_{c2}. As we have shown, at $\theta_H = \pi/2$ the detachment current is finite and the capture current is equal to zero. The difference between j_{c1} and j_{c2} should be preserved also for materials with $\theta_H \neq \pi/2$. In the next section we shall corroborate this statement by a calculation for a concrete potential.

So far, we have in fact implied everywhere that the external current is turned on instantaneously. This situation changes appreciably if the external current is turned on at a finite rate. The slower the external current reaches its asymptotic values, the larger the time intervals τ over which it is correct to regarded the current as constant. Using the geometrical interpretation indicated above for the trajectories and considering the motion of the vortex in successive time intervals τ, it is easy to draw the following general conclusion concerning the influence of the rate of turning on the external current on the vortex motion: as this rate decreases, the limiting value of the "criticality" parameter α_{c1} at which the vortex becomes detached from the pinning center tends to unity, and the vortex trajectory is local-

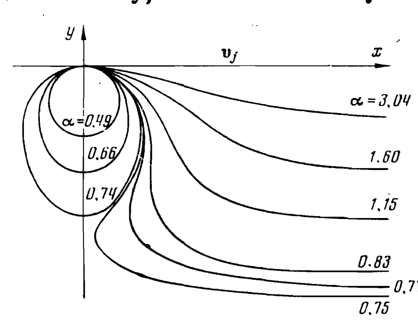

FIG. 1. Trajectory of vortex in a Gaussian potential field at $\theta_H = \pi/2$ for various values of the "criticality" parameter α.

ized in an ever decreasing region about the ray $\varphi = -\theta_H$. In the limit of a very slow rate of current application, the vortex moves practically along the ray $\varphi = -\theta_H$, and becomes deteached from the pinning force when the Lorentz force becomes equal to the maximum pinning force. By way of illustration, Figs. 2 and 3 show the vortex trajectories and the values of α_{c1} at various rates of current switching for a parabolic potential field ($u = u_0 - k\rho^2/2$ at $\rho \leq 1$ and is equal to zero at $\rho > 1$) and for the external current $j_T = J_0(1 - e^{-\gamma t})$ ($\beta = \gamma/\omega$, where $\omega = ck/ne\varphi_0$).

4. MATERIALS WITH FINITE HALL-ANGLE TANGENT

We consider now the motion of a vortex in a parabolic well in a material having a finite Hall-angle tangent. Let the vortex be at rest on the Z axis at the instant when the external current is turned on. Then the system (3) for the force

$$F(\rho)=\begin{cases} -k\rho, & \rho\leq 1 \\ 0, & \rho>1 \end{cases} \tag{9}$$

yields, when the external current is turned on in accord with the law $j_T = j_0(1 - e^{-\gamma t})$, a vortex trajectory in the form

$$x(t)=\frac{v_{j0}}{\omega}\cos\theta_H+\frac{v_{j0}\beta e^{-\omega t\cos\theta_H}}{\omega(1-2\beta\cos\theta_H+\beta^2)}[\cos(\omega t\sin\theta_H+2\theta_H)-\beta\cos(\omega t\sin\theta_H+\theta_H)]+\frac{\beta-\cos\theta_H}{1-2\beta\cos\theta_H+\beta^2}\frac{v_{j0}}{\omega}e^{-\gamma t},$$

$$y(t)=-\frac{v_{j0}}{\omega}\sin\theta_H+\frac{v_{j0}\beta e^{-\omega t\cos\theta_H}}{\omega(1-2\beta\cos\theta_H+\beta^2)}[\beta\sin(\omega t\sin\theta_H+\theta_H)-\sin(\omega t\sin\theta_H+2\theta_H)]+\frac{v_{j0}e^{-\gamma t}\sin\theta_H}{\omega(1-2\beta\cos\theta_H+\beta^2)},$$

where $\omega = \mu ck/ne\varphi_0 \cos\theta_H$ and $\beta = \gamma/\omega$.

In the case of very rapid switching of the current ($\beta \gg 1$) within a time $t \gg 1/\gamma$, when the terms with $\exp(-\gamma t)$ can be neglected, we obtain for the vortex trajectory

$$\left[x(t)-\frac{v_{j0}\cos\theta_H}{\omega}\right]^2+\left[y(t)+\frac{v_{j0}\sin\theta_H}{\omega}\right]^2=\left(\frac{v_{j0}}{\omega}\right)^2e^{-2\omega t\cos\theta_H},$$

i.e., the vortex rotates along a spiral around the point $(\omega^{-1}v_{j0}\cos\theta_H, -\omega^{-1}v_{j0}\sin\theta_H)$, and the radius of the spiral diminishes like

$$R=(v_{j0}/\omega)e^{-\omega t\cos\theta_H}.$$

Figure 4 shows the vortex trajectories in the field of the force (9) for $\theta_H = 69°$ at different values of the "criticality" parameter α.

The values of the detachment currents at which the vortex goes over to an infinite trajectory are obtained from the condition that they reach the limit of the action of the force (9). Figure 5 shows a plot of α_{c1} against the values of $\cot\theta_H$. We see that with decreasing θ_H the detachment current changes from a value 0.5 $(c/\varphi_0)|F_{max}|$ (at $\theta_H = \pi/2$) to $(c/\varphi_0)|F_{max}|$ (at $\theta_H = 0$). In the case of very slow switching of the external current ($\beta \ll 1$) the trajectory of the vortex in the field of the forces (9) is given by

$$x(t)=\frac{v_{j0}}{\omega}\cos\theta_H(1-e^{-\gamma t}),\quad y(t)=-\frac{v_{j0}}{\omega}\sin\theta_H(1-e^{-\gamma t}),$$

i.e., the vortex practically approaches the equilibrium point $(\omega^{-1}v_{j0}\cos\theta_H, -\omega^{-1}v_{j0}\sin\theta_H)$ along the ray $\varphi = -\theta_H$, as already discussed in Sec. 3. The detachment current at any θ_H corresponds here to a

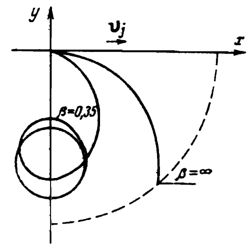

FIG. 2

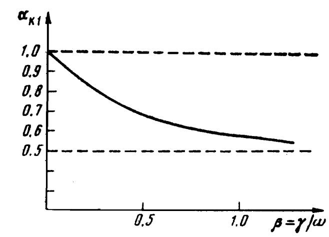

FIG. 3

FIG. 2. Trajectory of vortex in a parabolic potential field at $\theta_H = \pi/2$.

FIG. 3. Dependence of the detachment current on the rate of turning on the external current at $\theta_H = \pi/2$ for a parabolic potential.

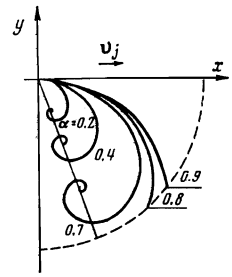

FIG 4

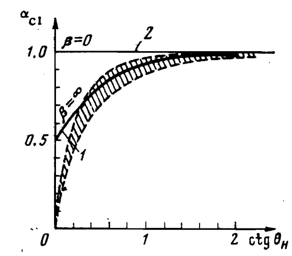

FIG. 5

FIG. 4. Vortex trajectories in a parabolic potential field at $\theta_H = 69°$ for different values of the "criticality" parameter (for a current turned on sufficiently rapidly).

FIG. 5. Dependence of the detachment current on the Hall angle for a parabolic potential of interaction between the vortex and the pinning center. The detachment current at intermediate values of the parameter β corresponds to points lying in the region bounded by curves 1 and 2. The dashed curves delimit the regions of the capture-current values corresponding to the possible impact parameters.

"criticality" parameter $\alpha_{c1} = 1$. At an arbitrary value of β, the plot of the detachment current against the Hall angle lies in the region bounded in Fig. 5 by the curve 1 ($\beta = \infty$) and the straight line 2 ($\beta = 0$).

We now calculate the capture current. To this end we must integrate the system (3) with the force (9) under the initial conditions

$$x|_{t=0}=\cos\psi_0,\quad y|_{t=0}=\sin\psi_0,$$

where ψ_0 is the polar angle of the radius vector of the point of approach of the vortex to the boundary of the action of the force (9). It is easy to obtain the result of the integration, namely

$$\begin{aligned} x(t)&=\alpha\cos\theta_H+e^{-\omega t\cos\theta_H}[\cos(\psi_0-\omega t\sin\theta_H)-\alpha\cos(\omega t\sin\theta_H+\theta_H)],\\ y(t)&=-\alpha\sin\theta_H+e^{-\omega t\cos\theta_H}[\sin(\psi_0-\omega t\sin\theta_H)+\alpha\sin(\omega t\sin\theta_H+\theta_H)],\end{aligned} \tag{10}$$

which is a spiral with center at the point $(\alpha\cos\theta_H, -\alpha\sin\theta_H)$, and with a radius that decreases like

$$R=e^{-\omega t\cos\theta_H}\sqrt{1+\alpha^2-2\alpha\cos(\psi_0+\theta_H)}.$$

If α is smaller than a certain critical value α_{c2}, then the obtained spiral lies entirely in a circle of unit radius, i.e., the vortex is captured. On the other hand if $\alpha > \alpha_{c2}$, then the spiral crosses the boundary of the action of the force (9), i.e., the vortex remains on an infinite trajectory and there is no capture. Since it follows from (10) that for a verotex far from the pinning center

$$\rho^2(t)=\alpha^2+[1+\alpha^2-2\alpha\cos(\psi_0+\theta_H)]e^{-2\omega t\cos\theta_H}+2\alpha e^{-\omega t\cos\theta_H}[\cos(\psi_0+\theta_H-\omega t\sin\theta_H)-\alpha\cos(\omega t\sin\theta_H)],$$

it follows that α_{c2} is the minimal value of α at which $\rho(t) = 1$ for a certain $t > 0$. Figure 5 shows the dependence of the capture current on the Hall angle for scattering by a parabolic potential. The dashed lines outline the region in which are located the capture-current values for different impact parameters. We see that the capture current decreases with increasing Hall angle and drops to zero at $\theta_H = \pi/2$.

5. DISCUSSION OF RESULTS

The main result of the proposed approach is the notion of the gyroscopic character of the forces acting on the Abrikosov vortices, including the pinning forces. Let us discuss some consequences that result from this concept and may manifest themselves in experiment.

(a) The vortex trajectory and the field of the pinning force have asymmetry relative to the direction of the external field and the field that produces the vortices. This singularity could manifest itself in an asymmetry of the current-voltage characteristics in the mixed state with changing direction of the external current or field.

(b) A distinction must be made between two characteristic values of the external current: the detachment current, at which the vortex breaks away from the pinning centers and goes over to an infinite trajectory, and the capture current, below which the pinning center can capture the vortex scattered by it. These values differ from each other. With decreasing Hall angle, the detachment current increases and in the limit $\theta_H = 0$ it coincides with the value at which the Lorentz force becomes equal to the maximum pinning force. The capture current is equal to zero at $\theta_H = \pi/2$ and tends with decreasing Hall angle to the same value as the detachment current. The difference between these currents should lead to a hysteresis in the current-voltage characteristics of the mixed state, depending on the sign of the current increment.

(c) The detachment and capture currents, which depend on the Hall angle, should be determined not only by the pinning centers proper, but also by all other defects; while the latter do not act effectively as pinning centers, they influence appreciably the Hall angle, e.g., point defects. We note that in[5-7] the increase of the critical current with increasing irradiation dose during a stage in which only point defects were introduced was attributed to fluctuations of the density of these defects. In our opinion the decrease of the Hall angle with increasing number of point defects could make an appreciable contribution in this case.

(d) The vortex trajectory depends strongly on the rate of turning on the external transport current. In particular, the detachment current tends, when this rate decreases, to that value at which the Lorentz force becomes equal to the maximum pinning force.

We note once more that our results were obtained by using a model in which the vortices were independent. Allowance for the interaction between the vortices can alter these results.

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