Three-velocity hydrodynamics of superfluid solutions

A. F. Andreev and E. P. Bashkin

Institute for Physical Problems, USSR Academy of Sciences
(Submitted February 11, 1975)
Zh. Eksp. Teor. Fiz. 69, 319–326 (July 1975)

The equations of three-velocity hydrodynamics, which describe the properties of solutions of He^3 in liquid He^4 below the point of the transition of the Fermi component to the superfluid state, are determined by specifying the thermodynamic functions and symmetric 2×2 matrix playing the role of the density of the superfluid part. A calculation of the elements of this matrix is carried out on the basis of BCS theory. As a result it is shown that each of the two superfluid flows is accompanied by transport of both components of the solution. The velocities of three types of sound vibrations are calculated.

PACS numbers: 67.20., 67.60.F

After the experimental discovery of the finite solubility of He^3 in superfluid He^4 at zero temperature[1] and the theoretical investigation[2] of the character of the interaction of the dissolved He^3 atoms, it became clear that, at a sufficiently low temperature, a phase transition accompanied by Cooper pairing of He^3 Fermi particles should occur in the solution. Interest in this possibility has recently been particularly heightened in connection with the discovery of the transition of pure He^3 to a superfluid state[3-5].

Below the phase-transition point the solution should be a unique system, in which two forms of condensate, and, correspondingly, two forms of superfluid flow, exist simultaneously. The properties of such a liquid should be described by the equations of three-fluid hydrodynamics, with two superfluid and one normal flow-rate. The elucidation of the form of the equations of three-fluid hydrodynamics has been the subject of papers by Khalatnikov[6], Galasiewicz[7] and Mineev[8], in which, in particular, it was shown that these equations permit the existence of sound waves of three types.

Although the qualitative results of these papers do not raise doubts, from a quantitative point of view they lead to a strange and, as we shall see, incorrect picture, according to which each of the superfluid flows is accompanied by transport of particles of only one kind. This means, in particular, that the superfluid flow of Cooper pairs of He^3 atoms is not accompanied by mass transfer of He^4. The incorrectness of this statement is clear, if only from the fact that, as a result of the extremely strong interaction with the surrounding He^4, an He^3 atom is transformed, as is well known, into a quasi-particle with an effective mass 2.3 times greater than the mass of the He^3 atom itself. The motion of the quasi-particle therefore transports, in addition to the He^3 mass, a mass of He^4 that is by no means small. The Cooper pairs are formed as a result of the weak interaction between the fermions. They are a bound state of quasi-particles, the properties of the latter being practically unchanged in the onset of the superfluidity. It is clear, therefore, that the superfluid flow of He^3 should also be accompanied by He^4 mass transfer.

It is necessary to stress, incidentally, that in the presence of two superfluid flows there is ambiguity in the definition of the corresponding velocities. In principle one could define each of the velocities as the ratio of the mass flux of particles of a given kind to their density. Then the effect discussed above would be absent by definition. It is important, however, that with such a definition the superfluid flows are not potential. Moreover the potentiality condition plays an extremely important role in the formulation of the complete system of hydrodynamic equations. In retaining the potentiality condition, we must necessarily take into account the possibility of transport of both components of the solution by each of the superfluid flows.

1. From the analysis of the conservation laws performed by Khalatnikov[6], it follows that the complete system of equations of three-fluid hydrodynamics should have the following form:

$$\begin{gathered}\dot\rho_1+\operatorname{div}(\rho_1\mathbf{v}_n+\mathbf{p}_1)=0,\quad \dot\rho_2+\operatorname{div}(\rho_2\mathbf{v}_n+\mathbf{p}_2)=0,\\ \dot j_i+\frac{\partial\Pi_{ik}}{\partial x_k}=0,\quad \dot S+\operatorname{div}S\mathbf{v}_n=0,\\ \dot{\mathbf{v}}_1+\nabla(\mu_1-{}^1/{}_2v_n^2+\mathbf{v}_n\mathbf{v}_1)=0,\quad \dot{\mathbf{v}}_2+\nabla(\mu_2-{}^1/{}_2v_n^2+\mathbf{v}_n\mathbf{v}_2)=0,\\ \operatorname{rot}\mathbf{v}_1=\operatorname{rot}\mathbf{v}_2=0.\end{gathered}\tag{1}$$

Here ρ_1 and ρ_2 are the densities of particles of each kind (the total density ρ of the solution is equal to the sum $\rho_1+\rho_2$), $\mathbf{v}_n$, $\mathbf{v}_1$ and $\mathbf{v}_2$ are respectively the velocities of the normal and the two superfluid flows, $\mathbf{j}=\rho\mathbf{v}_n+\mathbf{p}_1+\mathbf{p}_2$ is the momentum per unit volume, and S is the entropy per unit volume.

The meaning of the quantities μ_1, μ_2, $\mathbf{p}_1$ and $\mathbf{p}_2$ is determined by the thermodynamic identity for the energy per unit volume of the solution (ϵ) in the coordinate system in which $\mathbf{v}_n=0$:

$$d\varepsilon=TdS+\mu_1d\rho_1+\mu_2d\rho_2+\mathbf{p}_1d(\mathbf{v}_1-\mathbf{v}_n)+\mathbf{p}_2d(\mathbf{v}_2-\mathbf{v}_n).\tag{2}$$

The energy in the laboratory frame is then equal to

$$E={}^1/{}_2\rho v_n^2+(\mathbf{p}_1+\mathbf{p}_2)\mathbf{v}_n+\varepsilon.\tag{3}$$

The momentum-flux tensor has the form

$$\Pi_{ik}=\rho v_{ni}v_{nk}+(p_{1i}+p_{2i})v_{nk}+(p_{1k}+p_{2k})v_{ni}+p_{1k}(v_{1i}-v_{ni})+p_{2k}(v_{2i}-v_{ni})+P\delta_{ik},\tag{4}$$

where $P=-\epsilon+\mu_1\rho_1+\mu_2\rho_2+TS$ is the pressure.

All the formulas written out up to now differ from the corresponding formulas of two-fluid hydrodynamics only by the trivial doubling of the number of terms describing the relative motion of the superfluid and normal parts. An important distinctive feature appears, however, when quantities playing the role of the densities of the superfluid parts are introduced. In fact, if the rates of flow are not too great, the relative momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ can be expanded in powers of the small relative velocities $\mathbf{v}_1-\mathbf{v}_n$ and $\mathbf{v}_2-\mathbf{v}_n$, with constant coefficients:

$$\begin{gathered}\mathbf{p}_1=\rho_{11}^{(s)}(\mathbf{v}_1-\mathbf{v}_n)+\rho_{12}^{(s)}(\mathbf{v}_2-\mathbf{v}_n),\\ \mathbf{p}_2=\rho_{21}^{(s)}(\mathbf{v}_1-\mathbf{v}_n)+\rho_{22}^{(s)}(\mathbf{v}_2-\mathbf{v}_n).\end{gathered}\tag{5}$$

The aggregate of these coefficients $\rho_{\alpha\beta}^{(s)}$ ($\alpha,\beta=1,2$) is the analog of the density of the superfluid part in two-fluid hydrodynamics. As can be seen from the thermodynamic identity (2), the quantities $\rho_{\alpha\beta}^{(s)}$ are equal to the second derivatives of the energy ϵ with respect to the relative velocities. Therefore, the matrix $\rho_{\alpha\beta}$ is symmetric, i.e.,

$$\rho_{12}^{(s)}=\rho_{21}^{(s)}. \quad (6)$$

We may say that, in three-fluid-dynamics, in place of the density of the superfluid part there appear three independent quantities: $\rho_{11}^{(s)}$, $\rho_{22}^{(s)}$ and $\rho_{12}^{(s)}$. The last of these describes the above-mentioned effect of the drag of both components of the solution by each of the superfluid flows.

By means of the identity (2) and formulas (5), it is easy to find the dependence of the energy ϵ on the relative velocities:

$$\varepsilon=\varepsilon_0(S,\rho_1,\rho_2)+{}^1/_2[\rho_{11}^{(s)}(\mathbf{v}_1-\mathbf{v}_n)^2+2\rho_{12}^{(s)}(\mathbf{v}_1-\mathbf{v}_n)(\mathbf{v}_2-\mathbf{v}_n)+\rho_{22}^{(s)}(\mathbf{v}_2-\mathbf{v}_n)^2],$$

where ϵ_0 is the energy of the solution at rest. After substituting ϵ into (3) we arrive at the following expression for the energy E of the solution in the laboratory coordinate frame:

$$E={}^1/_2\{\rho^{(n)}v_n^2+\rho_{11}^{(s)}v_1^2+2\rho_{12}^{(s)}\mathbf{v}_1\mathbf{v}_2+\rho_{22}^{(s)}v_2^2\}+\varepsilon_0(\rho_1,\rho_2,S). \quad (7)$$

Here we have introduced the quantity $\rho^{(n)} \equiv \rho - \rho_{11}^{(s)} - \rho_{22}^{(s)} - 2\rho_{12}^{(s)}$, which obviously plays the role of the density of the normal part.

In an analogous way, we can write an expression for the momentum-flux tensor:

$$\Pi_{ik}=\rho^{(n)}v_{ni}v_{nk}+\rho_{11}^{(s)}v_{1i}v_{1k}+\rho_{22}^{(s)}v_{2i}v_{2k}+\rho_{12}^{(s)}(v_{1i}v_{2k}+v_{1k}v_{2i})+P\delta_{ik}, \quad (8)$$

which, as it should be, is symmetric.

In order that the kinetic part of the energy of the solution be positive-definite, in addition to the positivity of $\rho^{(n)}$, $\rho_{11}^{(s)}$ and $\rho_{22}^{(s)}$ the further condition

$$\rho_{12}^{(s)2}<\rho_{11}^{(s)}\rho_{22}^{(s)}. \quad (9)$$

is necessary, as can be seen from (7). The quantity $\rho_{12}^{(s)}$ itself can in principle be either positive or negative.

We shall also write expressions for the mass-flux densities $\mathbf{j}_1$ and $\mathbf{j}_2$ of the first and second components of the solution. As can be seen from the first two equations of (1), these fluxes are equal to

$$\mathbf{j}_\alpha=\rho_\alpha\mathbf{v}_n+\mathbf{p}_\alpha.$$

Substituting (5) into this, we obtain

$$\mathbf{j}_1=(\rho_1-\rho_{11}^{(s)}-\rho_{12}^{(s)})\mathbf{v}_n+\rho_{11}^{(s)}\mathbf{v}_1+\rho_{12}^{(s)}\mathbf{v}_2,$$

$$\mathbf{j}_2=(\rho_2-\rho_{22}^{(s)}-\rho_{21}^{(s)})\mathbf{v}_n+\rho_{21}^{(s)}\mathbf{v}_1+\rho_{22}^{(s)}\mathbf{v}_2. \quad (10)$$

The total mass flux $\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2$ is expressed in terms of the velocities in the following way:

$$\mathbf{j}=\rho^{(n)}\mathbf{v}_n+(\rho_{11}^{(s)}+\rho_{21}^{(s)})\mathbf{v}_1+(\rho_{22}^{(s)}+\rho_{12}^{(s)})\mathbf{v}_2. \quad (11)$$

Equations (1) with the conditions (5) and (8) are the complete system of equations of three-fluid hydrodynamics, if the thermodynamic functions of the solution and all three superfluid densities are known.

2. The thermodynamic functions of weak solutions of He^3 in liquid He^4 at temperatures below the point of the transition of the Fermi component to the superfluid state are exhaustively described by the formulas of the BCS theory of superconductivity. Moreover, on the basis of this theory it is easy to give a microscopic derivation of the formulas (10) for the mass fluxes, and thereby to calculate the quantities $\rho_{\alpha\beta}^{(s)}$.

Inasmuch as the velocities $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_n$ appear linearly in the formulas (10), it is sufficient to calculate the fluxes $\mathbf{j}_1$ and $\mathbf{j}_2$ for cases when only one of the three velocities is nonzero.

First of all, however, it is necessary to give a microscopic definition of the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ of the superfluid flows. The quantum-mechanical state of the solution is characterized by the presence of two condensates and, correspondingly, two phases. Let φ_1 be the phase of the order parameter (the energy gap) associated with the condensate of Cooper pairs. We shall define the velocity $\mathbf{v}_1$ as a quantity proportional to the gradient $\nabla\varphi_1$. The constant of proportionality is uniquely determined by the requirement that, in a Galilean transformation to a coordinate frame moving with velocity $\mathbf{V}$ relative to the initial frame, the velocity $\mathbf{v}_1$ be transformed to $\mathbf{v}_1' = \mathbf{v}_1-\mathbf{V}$. The phase φ_1 transforms, as is well-known, in the following way:

$$\varphi_1'(\mathbf{r},t)=\varphi_1(\mathbf{r},t)-2m\mathbf{V}\hbar^{-1}(\mathbf{r}+\mathbf{V}t/2),$$

where m is the mass of an isolated He^3 atom. Therefore, the velocity $\mathbf{v}_1$ should be defined by the equality

$$\mathbf{v}_1=\hbar\nabla\varphi_1/2m. \quad (12)$$

The fact that the mass m, and not the effective mass of the Fermi excitation, appears in (12) ensures that the superfluid flow is potential. The velocity $\mathbf{v}_2$ of the second superfluid flow is defined in an analogous way in terms of the phase φ_2 of the boson condensate: $\mathbf{v}_2 = (\hbar/M)\nabla\varphi_2$, where M is the mass of the He^4 atom.

Let $\mathbf{v}_2 = \mathbf{v}_n = 0$, but $\mathbf{v}_1 \neq 0$. When $\mathbf{v}_2 = 0$ the Hamiltonian of He^3 Fermi particles in He^4 coincides with the Hamiltonian of free particles with a mass m^* equal to the effective mass of the quasi-particle associated with the He^3 atom. If the He^3 atom possessed a charge e, the superconducting current $\mathbf{j}_e$ would be determined by the well-known formula of the theory of superconductivity:

$$\mathbf{j}_e=\frac{\hbar e N_s}{2m^*}\nabla\varphi_1,$$

where N_s is the number of "superfluid" He^3 atoms, expressed in terms of the total number N of He^3 atoms per unit volume by the relation

$$\frac{N_s(T)}{N}=\left(1-T\frac{\partial\ln\Delta}{\partial T}\right)^{-1},$$

where $\Delta(T)$ is the temperature-dependent energy gap.

The mass flux $\mathbf{j}_1$ is obtained from $\mathbf{j}_e$ by replacing e by m. Therefore, using (12) we obtain

$$\mathbf{j}_1=m^2N_s\mathbf{v}_1/m^*. \quad (13)$$

The total mass $\mathbf{j}$ differs from $\mathbf{j}_1$ by the factor m^*/m, inasmuch as $\mathbf{j}$ coincides with the momentum per unit volume. From this we easily find the He^4 mass flux:

$$\mathbf{j}_2=\mathbf{j}-\mathbf{j}_1=m(m^*-m)N_s\mathbf{v}_1/m^*. \quad (14)$$

If $\mathbf{v}_2 \neq 0$, the energy $\epsilon(\mathbf{p})$ of the He^3 quasi-particle in the approximation linear in $\mathbf{v}_2$ can be written (cf.[2]) in the form

$$\varepsilon(\mathbf{p})=\frac{p^2}{2m^*}+\mathbf{p}\left(1-\frac{m}{m^*}\right)\mathbf{v}_2\approx\frac{1}{2m^*}[\mathbf{p}+(m^*-m)\mathbf{v}_2]^2.$$

From this it is clear that the presence of $\mathbf{v}_2$ is equivalent to the introduction of a vector potential

$$\mathbf{A}=-(m^*-m)\mathbf{v}_2/e. \quad (15)$$

into the Hamiltonian. Therefore, for $\mathbf{v}_1 = \mathbf{v}_n = 0$ and

$\mathbf{v}_2 \neq 0$, the mass current $\mathbf{j}_1$ is easily obtained by means of the London formula $\mathbf{j}_e = -(e^2N_s/m^*)\mathbf{A}$. We have

$$\mathbf{j}_1 = m\mathbf{j}_e/e = m(m^*-m)N_s\mathbf{v}_2/m^*. \quad (16)$$

The operator of the total mass flux is equal to

$$\mathbf{j} = \rho_2\mathbf{v}_2 + \sum_{\mathbf{p}} \mathbf{p} n_{\mathbf{p}}, \quad n_{\mathbf{p}} = \sum_{\sigma} a_{\mathbf{p}\sigma}^{+} a_{\mathbf{p}\sigma}, \quad (17)$$

where $a_{\mathbf{p}\sigma}^{+}$ and $a_{\mathbf{p}\sigma}$ are the creation and annihilation operators for the He^3 quasi-particles (not for the Bogolyubov excitations, but for the bare quasi-particles, whose number is specified!); σ is the spin index.

Rewriting (17) in the form

$$\mathbf{j} = \rho_2\mathbf{v}_2 + \sum_{\mathbf{p}} (\mathbf{p} - e\mathbf{A}) n_{\mathbf{p}} + e\mathbf{A}N$$

and taking into account that the flux operator is equal to

$$\mathbf{j}_e = \frac{e}{m^*} \sum_{\mathbf{p}} (\mathbf{p} - e\mathbf{A}) n_{\mathbf{p}},$$

we find

$$\mathbf{j} = \rho_2\mathbf{v}_2 - N_n(m^*-m)\mathbf{v}_2,$$

where $N_n = N - N_s$.

The He^4 mass flux, in the case under consideration, is thus determined by the formula

$$\mathbf{j}_2 = \mathbf{j} - \mathbf{j}_1 = \rho_2\mathbf{v}_2 - (m^*-m)N_n\mathbf{v}_2 - m(m^*-m)N_s\mathbf{v}_2/m^*. \quad (17')$$

Finally, suppose that only the velocity $\mathbf{v}_n$ of the normal flow is nonzero. The distribution function of the Bogolyubov excitations differs from the equilibrium value $\nu^{(0)}(E)$, which is a function of the energy E of these excitations, by the quantity

$$\delta\nu_{\mathbf{p}\sigma} = -\mathbf{p}\mathbf{v}_n \partial\nu^{(0)}/\partial E. \quad (18)$$

The total mass flux is calculated in the usual way as the momentum of the excitations and is equal to

$$\mathbf{j} = m^* N_n \mathbf{v}_n.$$

By means of a Bogolyubov transformation the operator of the He^3 mass flux

$$\mathbf{j}_1 = \frac{m}{m^*} \sum_{\mathbf{p}} \mathbf{p}\,\delta n_{\mathbf{p}}$$

can be expressed in terms of $\delta\nu_{\mathbf{p}\sigma}$ in the following way:

$$\mathbf{j}_1 = \frac{m}{m^*} \sum_{\mathbf{p}\sigma} \mathbf{p}(u_{\mathbf{p}}^2 \delta\nu_{\mathbf{p}\sigma} - v_{\mathbf{p}}^2 \delta\nu_{-\mathbf{p},\sigma}),$$

where $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are the coefficients of the transformation. Inasmuch as $\delta\nu_{\mathbf{p}\sigma}$, as can be seen from (18), is an odd function of $\mathbf{p}$, and $u_{\mathbf{p}}^2 + v_{\mathbf{p}}^2 = 1$, the flux $\mathbf{j}_1$ differs from $\mathbf{j}$ by the factor m/m^*:

$$\mathbf{j}_1 = m\mathbf{j}/m^* = mN_n\mathbf{v}_n. \quad (19)$$

We again calculate the flux $\mathbf{j}_2$ as the difference $\mathbf{j} - \mathbf{j}_1$:

$$\mathbf{j}_2 = (m^*-m)N_n\mathbf{v}_n. \quad (20)$$

In the general case when all velocities are nonzero, $\mathbf{j}_1$ and $\mathbf{j}_2$ are determined respectively by the sum of the expressions (13), (16) and (19) and by the sum of (14), (17′) and (20). As a result, formulas of the form (10) are obtained, with

$$\rho_{11}^{(s)} = m^2N_s/m^*, \quad \rho_{12}^{(s)} = \rho_{21}^{(s)} = m(m^*-m)N_s/m^*,$$
$$\rho_{22}^{(s)} = \rho_2 - (m^*-m)N_n - m(m^*-m)N_s/m^*. \quad (21)$$

Here the density of the normal part is equal to $\rho^{(n)} = m^*N_n$. At zero temperature it disappears. In this case, the following simple expression for the total mass flux is obtained:

$$\mathbf{j} = \rho_2\mathbf{v}_2 + mN\mathbf{v}_1.$$

We note that in weak solutions the inequality (9) is fulfilled by a large margin, inasmuch as $\rho_{11}^{(s)}$ and $\rho_{12}^{(s)}$ are proportional to the concentration while $\rho_{22}^{(s)}$ does not contain this small factor.

3. We shall calculate the velocities of the sound vibrations of the superfluid solution. This is easily done by means of the linearized equations (1), if we take into account that, as will be seen from the result, the velocities of the three types of sound vibration satisfy the condition $u_1 \gg u_2 \gg u_3$, where u_2 has a power dependence on the concentration of the solution and u_3 an exponential dependence (like the temperature of the transition of the Fermi component to the superfluid state). Without discussing the simple calculations, we give the result.

The velocity of the first type of vibration is given in the usual way in terms of the compressibility of the solution:

$$u_1^2 = \partial P/\partial\rho,$$

depends weakly on the temperature, and is close to the velocity of first sound in pure He^4.

The vibrations of the second type are oscillations of the concentration of the solution. The corresponding velocity is determined by the formula

$$u_2^2 = \frac{\rho^{(n)}\rho_{11}^{(s)} + (\rho_1 - \rho_{11}^{(s)} - \rho_{12}^{(s)})^2}{\rho^2\rho^{(n)}} \left(\frac{\partial^2\varepsilon}{\partial c^2}\right)_{P,T}, \quad (22)$$

where $c = \rho_1/(\rho_1 + \rho_2)$ is the concentration.

The vibrations of the third type are principally temperature oscillations, and in this sense are analogous to second sound in an ordinary superfluid liquid. The square of the velocity of the vibrations is equal to

$$u_3^2 = \frac{TS^2}{C} \frac{\rho_{11}^{(s)}}{\rho^{(n)}\rho_{11}^{(s)} + (\rho_1 - \rho_{11}^{(s)} - \rho_{12}^{(s)})^2}, \quad (23)$$

which is strongly reminiscent of the expression for the velocity of second sound. Here $C = T\partial S/\partial T$ is the specific heat per unit volume of the solution.

To calculate the second derivative of ϵ with respect to the concentration, appearing in (22), it is sufficient to note that in the expansion of the energy ϵ in c there is, in addition to a constant and a term proportional to c, a term

$$\frac{3}{10}(3\pi^2)^{2/3}\frac{\hbar^2}{m^*}\left(\frac{\rho c}{m}\right)^{5/3},$$

which gives the main contribution to the above derivative and represents the energy of a strongly degenerate ideal Fermi gas with mass m^* and particle number per unit volume equal to $N = \rho c/m$. Substituting also into (22) the expressions given above for $\rho^{(n)}$ and $\rho_{\alpha\beta}^{(s)}$, we find that $u_2 = v_F\sqrt{3}$, where v_F is the velocity of the He^3 quasi-particles at the Fermi surface. Thus, the vibrations of the second type are none other than the well-known[9-11] collective oscillations of the superfluid Fermi gas of the impurity He^3 particles.

After substitution of formulas (21), the expression (23) for the velocity of the third sound acquires the following form:

$$u_3^2 = \frac{TS^2}{C}\frac{N_s}{m^*NN_n}. \quad (24)$$

At the point $T = T_c$ of the transition of the Fermi component to the superfluid state, u_3 goes to zero like $(T_c - T)^{1/2}$. For $T \ll T_c$ it is proportional to the first power of the temperature:

$$u_3 = (9/2\pi)^{1/2} T/p_F. \qquad (25)$$

Here $p_F = m^* v_F$ is the Fermi momentum.

Formulas (24) and (25) cease to be valid at very low temperatures, when the principal contribution to the temperature dependence of the thermodynamic quantities and superfluid densities is made not by the Fermi excitations (as was assumed in the derivation of (24) and (25)) but by the phonons. In the given case there are two types of phonons, corresponding to the first two types of sound vibrations of the solution. However, the principal contribution to the temperature dependence of all the quantities is made by the second-sound phonons, inasmuch as their velocity is much smaller. The third-sound waves are then sound vibrations in the phonon gas of the second sound, and their velocity of propagation is equal to $u_3 = u_2/\sqrt{3} = v_F/3$. It must be emphasized that the frequency of the third-sound vibrations in the temperature range considered should be extremely low, since the wavelength should be considerably greater than the mean free path of the thermal excitations, which is extremely large at such low temperatures.

In conclusion we express our gratitude to I. E. Dzyaloshinskiĭ, M. I. Kaganov, I. M. Lifshitz, L. P. Pitaevskiĭ and I. M. Khalatnikov for useful discussion.

[1]D. O. Edwards, D. F. Brewer, P. Seligman, M. Skertic and M. Yaqub, Phys. Rev. Lett. **15**, 773 (1965).
[2]J. Bardeen, G. Baym and D. Pines, Phys. Rev. **156**, 207 (1967).
[3]D. D. Osheroff, W. J. Gully, R. C. Richardson and D. M. Lee, Phys. Rev. Lett. **29**, 920 (1972).
[4]T. A. Alvesalo, Yu. D. Anufrier, H. K. Collan, O. V. Lounasmaa and P. Wennerström, Phys. Rev. Lett. **30**, 962 (1973).
[5]H. Kojima, D. N. Paulson and J. C. Wheatley, Phys. Rev. Lett. **32**, 141 (1974).
[6]I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **32**, 653 (1957) [Sov. Phys.-JETP **5**, 542 (1957)]; ZhETF Pis. Red. **17**, 534 (1973) [JETP Lett. **17**, 386 (1973)].
[7]Z. M. Galasiewicz, Phys. Lett. **43A**, 149 (1973); Phys. Kondens. Materie **18**, 141, 155 (1974).
[8]V. P. Mineev, Zh. Eksp. Teor. Fiz. **67**, 683 (1974) [Sov. Phys.-JETP **40**, 338 (1975)].
[9]N. N. Bogolyubov, V. V. Tolmachev and D. V. Shirkov, Novyĭ metod v teorii sverkhprovodimosti (A New Method in the Theory of Superconductivity), Izd. AN SSSR, M., 1958 (English translation published by Consultants Bureau, N.Y., 1959).
[10]V. M. Galitskiĭ, Zh. Eksp. Teor. Fiz. **34**, 1011 (1958) [Sov. Phys.-JETP **7**, 698 (1958)].
[11]P. W. Anderson, Phys. Rev. **112**, 1900 (1959).

Translated by P. J. Shepherd
34