Remark concerning the article by G. I. Barenblatt and A. A. Gavrilov, "Contribution to the theory of self-similar degeneracy of homogeneous isotropic turbulence"

A. I. Korneev

Mechanics Institute of the Moscow State University
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In connection with the paper by Barenblatt and Gavrilov (BG)[1] and with my paper[2], the following remarks are in order.

The theory of degeneracy of free homogeneous turbulent motion of an incompressible viscous liquid can be based on different hypotheses concerning the behavior of the velocity correlation moments of second and third order, for example the Karman hypotheses:

$$b_d{}^d(r,t)=\langle u(0,t)u(r,t)\rangle=b(t)f(\chi), \quad (1)$$
$$b_d{}^{nn}(r,t)=\langle u(0,t)v^2(r,t)\rangle=b^{3/2}h(\chi), \quad (2)$$

where

$$b(t)=b_d{}^d(0,t)=\langle u^2(0,t)\rangle=\langle v^2(0,t)\rangle,\qquad \chi=r/l(t),$$

r is the distance, t is the time, u is the projection of the velocity pulsation on the direction r, and v is the projection on a direction orthogonal to r. The angle brackets denote the averaging operation. I have also considered more general hypotheses:

$$b_d{}^d(r,t)=b(t)-b_2(t)\beta(\chi), \quad (3)$$
$$b_d{}^{nn}(r,t)=b_1^{3/2}(t)h(\chi). \quad (4)$$

On the basis of (1) and (2), L. I. Sedov obtained, without any additional hypotheses, the possible laws governing the variation of the functions b(t), $l(t)$, $f(\chi)$ and $h(\chi)$. If we use formulas (3) and (4), then L. I. Sedov's analysis is also applicable, but it is necessary in this case to use additional hypotheses for the determination of the additional functions $b_1(t)$ and $b_2(t)$.

The comparison on the theoretical conclusions based on (1) and (2), given in[2], shows that the Karman hypotheses (1) and (2) agree well with the experimental data, so that there is no need to use hypotheses (3) and (4), which are connected with the corresponding additional assumption.

In BG it is also stated that hypothesis (2) agrees with experiment at large Reynolds numbers, and hypothesis (1) is always accepted without stipulation. However, in the light of the experiments of Ling, Huang, and Wan,[3,4] BG attempt to question the applicability of Karman's hypothesis (2) and use in lieu of (1) and (2) hypotheses (1) and (4), which are a particular case of hypotheses (3) and (4), but with $b(t)=b_2(t)\neq b_1(t)$.

In[2] I analyzed and used many published experiments, including those of[3,4]. A detailed comparison of the theory with the experimental data on B(t), $\lambda(t)\approx l(t)$, and $f(\chi)$ (and in one case also on $h(\chi)$), carried out in[2], confirms the applicability of hypotheses (1) and (2), and therefore the main conclusion of BG, that the Karman hypothesis (2) is not valid, is based on an approximation of the experimental data by means of the function $f(\chi)=(1+\chi^2)^{-1}$ and by the power-law formulas $b\approx(T-t_0)^{-n}$ and $l\approx(t-t_0)^{-1/2}$, and cannot be regarded as reliable enough. The reason is that the BG conclusions lie beyond the accuracy limits of the comparison of the theory with the experiments.

In[2] I compared the solution of L. I. Sedov, based on the Karman hypotheses, with only those experiments of[3,4] in which they measured f, and only at values of the time larger than or equal to those for which hypothesis (1) had been experimentally confirmed. The determination of b^* and λ^* in accord with the scheme employed in[2] (by comparing the theoretical $\lambda(b)$ curve with the corresponding experimental points) is simple and lucid enough, but results in no better a degree of accuracy—the rms deviation of the experimental points of b(t) and $\lambda(t)$ from the theory is about 10%. Taking into account the experimental errors (about 10%) due to the finite averaging time, to the influence of the noise, and to deviations from homogeneity and isotropy, this accuracy of the correspondence between theory and experiment was regarded as acceptable.

Nonetheless, the theory and the experiments of[3,4] were compared anew for the purpose of (1) increasing the accuracy with which the theory agrees with experiment; (2), increasing the time interval in which the theory agrees with experiment; (3) establishing agreement between the theory (with $\alpha_-=0.08$) and experiments in which f was not measured. The remaining parameters of the theory (b^*, λ^*, t^*) were chosen from the condition that $\Sigma[1-t(b_k)/t_k]^2$ be a minimum (b_k and t_k are the experimental points, and $t(b_k)$ is the plot with $\alpha_-=0.08$[2]).

As seen from the table, which lists the refined values of the parameters of the theory and the mean squared deviations ϵ_b and ϵ_λ of the experimental b(t) and $\lambda(t)$ from the theory, the experimental data of Ling, Huang, and Wan[3,4] are in good agreement with the solution based on Karman's hypotheses. The deviations of the experimental points b(t), at suitable values of t^*, from the theoretical curve (at one and the same $\alpha_-=0.08$) do not exceed the corresponding deviations from the power-law relations (in which the values of n differ: n = 2, 1.73, and 1.35). The only discarded measured values of b and λ are those in the immediate vicinity of the grid (see the table), this being based on the universally accepted concept that the theory of isotropic turbulence can be compared with the experimental data on the degeneracy of the turbulence behind the grid only at $Ut/M = x/M > 20$—30. The following notation is used in the table: x = Ut is the distance to the grid, M is the pitch of the grid, d is the number of rods in the grid, U is the velocity of the average flow relative to the grid, V_g is the characteristic velocity of the mobile grids[4], and $R_M=UM/\nu$.

A few additional remarks are in order:

Grid	Experimental conditions				Values of the parameters			Mean-squared deviations %		Discarded measurement data at $Ut/M \leqslant a$
	R_M	M/d	U, cm/sec	M, cm	t^*, sec	U^2/b^*	λ^{*2}/ν, sec	ε_b	ε_λ	a
A	940	2.8	2.9	3.56	−22.2	15.9	14.3	4,5	–	17
B	470	2.8	2.9	1.78	−17.9	20	10.9	8	9	25
C	840	5	2.9	3.18	−11.8	84.7	6.97	1	6	14
A+B	–	–	2.9	–	−19.7	29.4	17.1	3,6	–	12
$V_p/U=3$	2000	5	3.14	6.4	−5.9	33.3	8.12	1	2.5	4
$V_p/U=17$	2000	5	3.14	6.4	1.25	0.445	1.57	2	3.8	All measurements taken into account

1. As a result of the power-law reduction of the experimental data,[3,4] employed also by BG, the following formula is obtained, with low accuracy, for the third-order moments:

$$b_1/b=(t-t_0)^{(n-1)/3}.$$

An approximation of the same experimental data in my article and in the present note leads to constancy of the ratio b_1/b. No direct measurements of b_1 were made in the experiments of[3,4], since they were beyond the limits of the measurement accuracy. In the reduction of the experiments of[3,4], the exponent $(n-1)/3$ is small. The ratio b_1/b (in the time-variation interval in which formula (1) is corroborated by the experimental data) lies within the limits of accuracy of the comparison of the theory with the experimental data on $b(t)$, $l(t)$, and $f(\chi)$, whereas the main conclusion of BG is based on the difference between the laws governing b_1 and b on account of the small exponent $(n-1)/3$.

2. In the case of power-law relations, the case $n = 1$ corresponds to $b = b_1$. In L. I. Sedov's theory he gets $b = b_1$ and no power laws in $t + t^*$ are obtained (t^* is determined from experiment, with $t^* \neq -t_0$). It is also important to emphasize that in a theory based on the equality $b = b_1$ (with the appropriate t^*), theoretically correct laws are obtained, with a minimum of assumption, also for the correlation functions. On the other hand, if the power-law relations $b \approx (t-t_0)^{-n}$ and $l \approx (t-t_0)^{1/2}$ are used, then within the framework of the initial BG assumptions (3) and (4), there are no correct theoretical formulas for the correlation functions at all.

We note that the theoretical $f(\chi)$ curve used in[2] and the empirical curve[3,4] practically coincide. The difference does not exceed 0.015, which is much less than the measurement error connected with the finite averaging time and amounting to $0.03\sqrt{1+f^2}$.

3. Power-law formulas for b and l lead to the relation[1,2]

$$b_d^{nn}(\chi_0, t)=\langle u(0, t)v^2(r_0, t)\rangle=h(\chi_0)b_1^{3/2}\approx(t-t_0)^{-n-1/2} \tag{5}$$

at any constant value $\chi_0 = r_0(t)/l(t) \neq 0$. In experiments in which t increases, no decrease of the similarity region $f(1)$ in χ was observed, so that χ_0 in (5) can be regarded as constant. Using the rigorous inequality

$$[b_d^{nn}(r, t)]^2 \leqslant \langle u^2(0, t)\rangle\langle v^4(0, t)\rangle,$$

we find from (5) that $\langle v^4\rangle/\langle v^2\rangle^2 \to \infty$ as $t \to \infty$ no slower than $(t-t_0)^{n-1}$. This conclusion contradicts the theoretical premises that the pulsations of the velocity v have a Gaussian distribution, and contradicts all the experimental data according to which the ratio $\langle v^4\rangle/\langle v^2\rangle^2$ is equal to three with approximate accuracy 10%[3,5].

[1]The main results of this paper were reported in April 1972, at the All-Union Seminar on Turbulence in Moscow.

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40