Interference of two-particle states in elementary-particle physics and astronomy

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Two types of experiments on the interference of two-particle states of identical particles, namely, space-time and momentum-energy experiments, are compared. Either can be used to determine the size of a particle source. An experiment is now proposed which can be used, at least in principle, to investigate the transition from one of these mutually exclusive formulations to the other.

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A new method of measuring stellar diameters was introduced in astronomy twenty years ago, namely, the so-called intensity interferometry[1-3] based on interference between a pair of photons emitted by independent sources on a star and detected by two receivers on the earth (when the two photons are detected, the random phase difference between them is unimportant and interference becomes observable). A similar idea has been put forward in elementary-particle physics, namely, it has been suggested that it may be possible to use the interference between pairs of identical particles to determine the size, shape, and lifetime of an excited region in which elementary particles are produced (see[4] and[5]).

In addition to similarities, there are also differences. In astronomy, one measures the time difference between the arrival of the identical particles (photons) and the difference in the position of the detectors. This then may be looked upon as a space-time formulation of the experiment. In elementary-particle physics, on the other hand, one must measure the energy and momentum differences between the recorded identical particles (for example, positive pions). If they are of the order of m_π, the space-time parameters of the meson-creation process determined from them are of the order of $\hbar/m_\pi$, i.e., they lie in the required range of distances and times. In elementary-particle physics, therefore, one is concerned with the momentum-energy formulation of the interference experiment. A simple theory of this experiment and its possible implications are discussed in detail in a number of papers[6-9] (see also[10]).

Our present paper is concerned with the comparison of these two variants of the experiment. In Sec. 1, we briefly analyze the momentum-energy variant and are mainly interested in the temporal parameters that may be determined in this case.

In Sec. 2, we give a brief theory of the space-time variant and compare it with the momentum-energy variant. In particular, we examine a useful symmetry, namely, the fact that the expressions for the phases of the interfering states in one variant of the experiment are obtained in the other by interchanging particle sources with particle detectors. Finally, in Sec. 3, we propose a gedanken experiment, two limiting cases of which are the space-time and momentum-energy variants discussed in Secs. 1 and 2.

Some additional details are given in[11]. Throughout this paper, we use the system of units in which $\hbar = 1$ and $c = 1$.

1. MOMENTUM-ENERGY CORRELATIONS

When a pair of identical particles (to be specific, we suppose that they are pions) is emitted by two point sources 1 and 2 with lifetime τ, which are switched on at times t_1 and t_2 and are separated in space by the distance $\mathbf{r}_1 - \mathbf{r}_2$, the probability of observing particles with 4-momenta $p_3 = \{\omega_3, \mathbf{p}_3\}$ and $p_4 = \{\omega_4, \mathbf{p}_4\}$ is given by (see[6])

$$W\sim\left|\frac{\exp(ip_3r_{13}+i\omega_3t_1)}{\omega_3-\omega_1+i\Gamma/2}\frac{\exp(ip_4r_{24}+i\omega_4t_2)}{\omega_4-\omega_2+i\Gamma/2}+\frac{\exp(ip_4r_{14}+i\omega_4t_1)}{\omega_4-\omega_1+i\Gamma/2}\frac{\exp(ip_3r_{23}+i\omega_3t_2)}{\omega_3-\omega_2+i\Gamma/2}\right|^2. \quad (1)$$

In these expressions, $\Gamma = 1/\tau$ is the energy width of each of the sources, and ω_1 and ω_2 are the mean energies of the emitted particles. If the quantities ω_1 and ω_2 can be varied and are distributed uniformly in a sufficiently broad energy band, equation (1) must be integrated with respect to ω_1 and ω_2. The result is [see equation (63) in[8]]

$$W\sim1+\frac{\cos[\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)-q_0(t_1-t_2)]}{1+(q_0\tau)^2}, \quad (1')$$

where $q_0 = \omega_3 - \omega_4$ and $\mathbf{q} = \mathbf{p}_3 - \mathbf{p}_4$. By measuring W as a function of the difference between $\mathbf{q}$ and q_0, we can determine $(\mathbf{r}_1 - \mathbf{r}_2)$ and $(t_1 - t_2)$.

A similar possibility arises when the particles are emitted not by two sources but by a set of sources distributed in a sufficiently narrow space-time interval.

Suppose that the sources are switched on at the same time $t = 0$ and are distributed near the origin in accordance with the law

$$U(\mathbf{r})\sim\exp\left[-\frac{1}{2}\left(\frac{x^2}{A^2}+\frac{y^2}{B^2}+\frac{z^2}{C^2}\right)\right].$$

We shall use equations (57) and (58) from[8] and will integrate (1′) throughout all space with the weight $U(\mathbf{r}_1)U(\mathbf{r}_2)$. This yields

$$W\sim1+I^2(\mathbf{q})/[1+(q_0\tau)^2], \quad (2)$$

where

$$I^2(\mathbf{q})=\exp[-(q_x^2A^2+q_y^2B^2+q_z^2C^2)]. \quad (3)$$

We shall assume that the narrow particle pairs in which we are interested are observed in the direction of the z axis. We shall also suppose that, when $\mathbf{p}_1 \approx \mathbf{p}_2$, we have $q_0 = \mathbf{q}\cdot\mathbf{v}$, where $\mathbf{v}$ is the particle velocity ($\mathbf{v}_1 \approx \mathbf{v}_2 \approx \mathbf{v}$); since $\mathbf{v}$ has the single component v_z, we have $q_z = q_0/v$. The quantity $q_z^2C^2$ in (3) should now be written in the form $q_0^2(t^L)^2$, where $t^L = C/v$. Consequently, the formula given by (2) assumes the form

$$W\sim1+\frac{\exp[-q_x^2A^2-q_y^2B^2-q_0^2(t^L)^2]}{1+(q_0\tau)^2}. \quad (4)$$

This expression contains a new parameter, t^L, which is the time for the created particles to traverse the excited volume (see also[10]). To estimate the role of this parameter, let us suppose that $t^L \gg \tau$. Wave packets produced by sources in the front and rear (relative to the direction of observation $\mathbf{n}$) parts of the excited volume will not then succeed in interfering: by the time the packet from the rear reaches the point at which the packet from the front is produced, the latter disappears. The interference term does, in fact, vanish in the region $q_0 \sim 1/\tau$ when $t^L \gg \tau$.

Finally, when the sources are not turned on at the same time, but the times at which they begin to operate are distributed uniformly within the interval $(-T, T)$, the expression given by (4) assumes the form

$$W \sim 1+\frac{\exp[-q_x^2A^2-q_y^2B^2-q_0^2(t^L)^2]}{1+(q_0\tau)^2}\left(\frac{\sin q_0T}{q_0T}\right)^2. \qquad (5)$$

It follows from (2)–(5) that it is possible, at least in principle, to determine the space-time parameters of a multiple-creation process by carrying out a momentum-energy experiment. Practical details of this experiment are discussed in[6-8,12].

2. SPACE-TIME CORRELATIONS

1. In the preceding section, we considered three types of temporal parameter, namely, τ, T, and t^L. When any of these become very large, it is no longer possible to measure the size of the radiating system because the interference effect remains in an unobservably small range of values of q_0. On the other hand, in astronomy, observations of two-photon interference correlations can be used to determine the angular parameters of stars[2,3] despite the fact that their lifetime (which has the same significance as T) can be regarded as infinitely long. In actual fact, this is not in conflict with the conclusion that there is no interference, which follows from formulas such as (5).

We shall now show that, in the space-time variant of the interference experiment used in astronomy, the time parameters t^L and T are absent from the final expressions, in contrast to the momentum-energy variant discussed above.[1)] Instead of these parameters, there is another temporal parameter which is analogous to τ, namely, the correlation interval or the time of correlated emission of recorded particles.

Let us briefly consider the fundamentals of the theory of the space-time experiment. Suppose we have two excited atoms at points 1 and 2. The photons emitted by them are recorded by detectors 3 and 4 at times t_3 and t_4 (see the figure). Emission by the atoms is governed by currents whose time dependence is

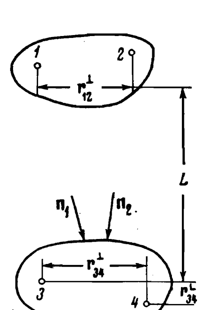

FIG. 1. Arrangement of sources and detectors of particles.

$$g_1(t)e^{-i\omega_1 t} \text{ and } g_2(t)e^{-i\omega_2 t},$$

where $g_1(t)$ and $g_2(t)$ are slowly-varying (as compared with the exponentials) functions of time. To be specific, let us suppose that they are functions of the form

$$\sum_l \theta(t-t_l)A_l \exp\left(i\delta_l-\frac{t-t_l}{2\tau}\right),$$

where (see, for example,[13]) the amplitudes A_l and phases δ_l are random quantities, and the times t_l in the argument of the θ function are distributed in time in accordance with the Poisson rule. Finally, $\tau \gg \omega_1^{-1}$, ω_2^{-1}. It will be convenient to introduce the two correlation functions:[2)]

$$\lambda_j(\vartheta)=\langle g_j(t)g_j^*(t+\vartheta)\rangle \qquad (j=1, 2),$$

such that $\lambda_j(-\vartheta) = \lambda_j^*(\vartheta)$.

The amplitude for a double count at times t_3 and t_4 can be written, apart from an unimportant common factor, in the form

$$\begin{aligned} A=&g_1(t_3-r_{13})\exp(-i\omega_1t_3+i\omega_1r_{13})g_2(t_4-r_{24})\\ &\times \exp(-i\omega_2t_4+i\omega_2r_{24})+g_1(t_4-r_{14})\exp(-i\omega_1t_4\\ &+i\omega_1r_{14})g_2(t_3-r_{23})\exp(-i\omega_2t_3+i\omega_2r_{23}), \end{aligned} \qquad (6)$$

where the symbols r_{13}, r_{24}, and so on represent distances between points 1 and 3, 2 and 4, and so on. When we calculate the probability of a double count, we take into account the fact that the distance between the sources and the detectors is much greater than the distance between the sources and between the detectors. Under these conditions,

$$r_{13}-r_{14}=(\mathbf{r}_4-\mathbf{r}_3)\mathbf{n}_1=\mathbf{r}_{34}\mathbf{n}_1,$$

where $\mathbf{n}_1$ is the direction of the detectors seen from source 1; similarly, $r_{23} - r_{24} = \mathbf{r}_{34}\cdot\mathbf{n}_2$.

Next, substituting $t_3 - t_4 = \vartheta$, $\omega_1\mathbf{n}_1 = \mathbf{k}_1$, and $\omega_2\mathbf{n}_2 = \mathbf{k}_2$, we obtain

$$\begin{aligned} |A|^2=&|g_1(t_3-r_{13})g_2(t_4-r_{24})|^2+|g_1(t_4-r_{14})g_2(t_3-r_{23})|^2\\ &+g_1(t_3-r_{13})g_1^*(t_4-r_{14})g_2(t_4-r_{24})g_2^*(t_3-r_{23})\cdot\\ &\times\exp(-i\omega_1\vartheta+i\mathbf{k}_1\mathbf{r}_{34})\exp(i\omega_2\vartheta-i\mathbf{k}_2\mathbf{r}_{34})+\text{к.с.} \end{aligned}$$

It will be convenient to substitute $T_1 = \mathbf{r}_{34}\cdot\mathbf{n}_1$, $T_2 = \mathbf{r}_{34}\cdot\mathbf{n}_2$, and to average $|A|^2$ over the times of detection t_3 and t_4 for a fixed value of $\vartheta = t_3 - t_4$.

The result is

$$\begin{aligned} \langle|A|^2\rangle=&2\lambda_1(0)\lambda_2(0)+\lambda_1^*(\vartheta-T_1)\lambda_2(\vartheta-T_2)\\ &\times\exp[-i(\omega_1-\omega_2)\vartheta+i(\mathbf{k}_1-\mathbf{k}_2)\mathbf{r}_{34}]+\text{c.c.} \end{aligned}$$

If we suppose that $\lambda_1(\vartheta)$ and $\lambda_2(\vartheta)$ are equal, the last expression assumes the simpler form

$$\langle|A|^2\rangle=2\lambda^2(0)+\lambda^*(\vartheta-T_1)\lambda(\vartheta-T_2)\exp[-i(\omega_1-\omega_2)\vartheta+i(\mathbf{k}_1-\mathbf{k}_2)\mathbf{r}_{34}]+\text{c.c.} \qquad (7)$$

We have $(\vartheta - T_2) - (\vartheta - T_1) = T_1 - T_2 = \mathbf{r}_{34}\cdot(\mathbf{n}_1 - \mathbf{n}_2)$ and this can be rewritten in the form $T_1 - T_2 = \mathbf{r}_{34}^{\perp}\cdot\mathbf{r}_{12}^{\perp}/L$, where $\mathbf{r}_{34}^{\perp}$ and $\mathbf{r}_{12}^{\perp}$ are the projections of $\mathbf{r}_{34}$ and $\mathbf{r}_{12}$ onto the plane perpendicular to the direction of observation, and L is the distance between the regions containing the sources and detectors (see the figure). The argument of the exponential contains a similar term:

$$(\mathbf{k}_1-\mathbf{k}_2)\mathbf{r}_{34}=\mathbf{r}_{34}^{\perp}\mathbf{r}_{12}^{\perp}\omega/L+(\omega_1-\omega_2)r_{34}^{L}, \qquad (8)$$

where $\omega = (\omega_1 + \omega_2)/2$ and r_{34}^L is the longitudinal component of $\mathbf{r}_{34}$.

Thus, the distance r_{12} between the source enters in (7) only in the form of the projection onto the plane perpendicular to the direction of the line of sight $\mathbf{n} \sim \mathbf{n}_1 + \mathbf{n}_2$. Consequently, the longitudinal size of the source cannot be determined through experiments based on (7). However, the formula given by (7) contains the longitudinal characteristics of the detector (namely, r_{34}^L, T_1, and T_2).

It is interesting to compare the structure of the arguments of the rapidly varying factors in (1′) and (7) corresponding to the momentum-energy and space-time experiments.

In the momentum-energy variant, the argument of the exponential contains the phase

$$\alpha=(\mathbf{k}_3-\mathbf{k}_4)(\mathbf{r}_1-\mathbf{r}_2)-(\omega_3-\omega_4)(t_1-t_2). \tag{9}$$

In this quantity, the momenta and energies are determined by the properties of the detectors, and the coordinates and times refer to the sources. In the space-time experiment, the argument contains the phase

$$\beta=(\mathbf{k}_1-\mathbf{k}_2)(\mathbf{r}_3-\mathbf{r}_4)-(\omega_1-\omega_2)(t_3-t_4). \tag{9'}$$

Here, the momenta and energies are determined by the properties of the sources, and the coordinates and times refer to the detectors, i.e., the sources and detectors have changed places. In the approximation in which the first term in (9′) becomes identical with (8), the phase α in (9) can be written in the form

$$\alpha=\mathbf{r}_{34}^{\perp}\mathbf{r}_{12}^{\perp}\omega/L-(\omega_3-\omega_4)(t_1-t_2-r_{12}^L). \tag{8'}$$

Consequently, in the momentum-energy variant, the formulas contain the longitudinal dimensions of the source whilst the longitudinal size of the region occupied by the detectors is unimportant.

2. Let us now return to (7). The correlation functions $\lambda(\vartheta - T_1)$ and $\lambda(\vartheta - T_2)$ in this expression vanish for sufficiently large values of the arguments. Under real conditions, these values are usually very large in comparison with $(T_1 - T_2)$. From now on, we therefore assume that $T_1 = T_2 = \tilde{T}$ and, instead of (7), we use

$$\langle|A|^2\rangle\sim1+\left|\frac{\lambda(\vartheta-\tilde{T})}{\lambda(0)}\right|^2\cos\left[(\omega_1-\omega_2)(\vartheta-\tilde{T})-\frac{\omega}{L}\mathbf{r}_{34}^{\perp}\mathbf{r}_{12}^{\perp}\right]. \tag{10}$$

This formula describes the emission of photons by two elementary sources. Suppose now that there is a large number of these sources, and that they are distributed uniformly over the surface of a circular disk of radius R, perpendicular to the direction of observation (or over the surface of a sphere radiating in accordance with Lambert's law). Equation (10) must then be integrated over the surface of the disk, and this yields

$$\langle|A|^2\rangle\sim1+\left|\frac{\lambda(\vartheta-\tilde{T})}{\lambda(0)}\right|^2\left(\frac{2J_1(\rho)}{\rho}\right)^2\cos[(\omega_1-\omega_2)(\vartheta-\tilde{T})], \tag{11}$$

where the Bessel-function argument

$$\rho=\omega|\mathbf{r}_{34}^{\perp}|\varphi$$

depends on the angular radius φ of the star. If the detectors define the frequency band between $\omega - \Delta\omega/2$ and $\omega + \Delta\omega/2$, then (11) must be integrated with respect to ω_1 and ω_2. The result is

$$\langle|A|^2\rangle\sim1+\left|\frac{\lambda(\vartheta-\tilde{T})}{\lambda(0)}\right|^2\left[\frac{2J_1(\rho)}{\rho}\frac{\sin{}^1\!/_2\Delta\omega(\vartheta-\tilde{T})}{{}^1\!/_2\Delta\omega(\vartheta-\tilde{T})}\right]^2. \tag{12}$$

In the special case where $g_1(t)$ and $g_2(t)$ are given by the expression at the beginning of this section, we have

$$|\lambda(\vartheta-\tilde{T})/\lambda(0)|^2=\exp(-|\vartheta-\tilde{T}|/\tau).$$

The expressions given by (10) and (12) show that it is possible, at least in principle, to determine the angular size of stars and the correlation time for elementary sources in the space-time variant of the interference experiment. By varying ϑ, it is possible to ensure that the argument $\vartheta - \tilde{T}$ becomes zero, so that (10) and (12) must be of the form

$$\langle|A|^2\rangle_{\vartheta=\tilde{T}}\sim1+[2J_1(\rho)/\rho]^2. \tag{13}$$

Subsequent variation of the parameter ρ can be used to determine the angular radius φ. On the other hand, by measuring the delay ϑ for fixed values of ρ, we can establish the structure of the correlation function and determine the correlation time. It is important to note, by the way, that the last quantity is of no interest in astronomy.

3. SYNTHESIS OF THE TWO TYPES OF INTERFERENCE CORRELATIONS

We have considered two different versions of the correlation experiment, namely, the momentum-energy and space-time variants. We shall now show that, in principle, it is possible to formulate a unified approach in which the momentum-energy and space-time variants are limiting cases.

Let us suppose that excited atoms located at points 1 and 2, and have identical natural widths Γ (see figure). At points 3 and 4, there are identical resonance scatterers with natural widths γ. A pair of counters is located near these points and each records the time of arrival of photons from its own scatterer. The character of interference phenomena produced in this system depends on the ratio γ/Γ. When $\gamma/\Gamma \gg 1$, we have the space-time variant and when $\gamma/\Gamma \ll 1$, we have the momentum energy variant.

We now calculate the probability that counters 3 and 4 will fire at times t_3 and t_4, assuming for simplicity that atoms 1 and 2 are excited simultaneously at time $t = 0$. The amplitude for the emission of a photon of frequency ω by atom 1 is proportional to $\Gamma/(\omega - \omega_1 + i\Gamma/2)$. The amplitude for the scattering of this photon by target 3 is proportional to $\gamma/(\omega - \omega_3 + i\gamma/2)$.[14] The amplitude for the firing of counter 3 at time t_3 is proportional to the product of these two fractions, and contains in addition the factor $\exp[-i\omega(t_3 - r_{13})]$ which takes into account the delay in the propagation of the field from point 1 to point 3. The amplitude for the firing of counter 4 due to a photon frequency Ω emitted by atom 2 can be obtained in a similar way. The final result is

$$A'(\omega,\Omega)=\frac{\Gamma\gamma\exp[-i\omega(t_3-r_{13})]}{(\omega-\omega_1+i\Gamma/2)(\omega-\omega_3+i\gamma/2)}\cdot\frac{\Gamma\gamma\exp[-i\Omega(t_4-r_{24})]}{(\Omega-\omega_2+i\Gamma/2)(\Omega-\omega_4+i\gamma/2)}.$$

We also have the further expression

$$A''(\omega,\Omega)=\frac{\Gamma\gamma\exp[-i\omega(t_3-r_{23})]}{(\omega-\omega_2+i\Gamma/2)(\omega-\omega_3+i\gamma/2)}\cdot\frac{\Gamma\gamma\exp[-i\Omega(t_4-r_{14})]}{(\Omega-\omega_1+i\Gamma/2)(\Omega-\omega_4+i\gamma/2)},$$

corresponding to the emission of frequency ω by atom 2 and the emission of frequency Ω by atom 1. Since the times at which the counters fire are accurately known, the frequencies ω and Ω are completely undetermined. The resulting sum must therefore be integrated with respect to ω and Ω. The final expression for the amplitude is

$$A(t_3,t_4)\sim\int d\omega d\Omega[A'(\omega,\Omega)+A''(\omega,\Omega)]. \tag{14}$$

Evaluation of the integral yields

$$A(t_3,t_4)=F_{13}F_{24}+F_{14}F_{23}, \tag{15}$$

where

$$F_{jk}=\gamma\Gamma[\omega_j-\omega_k-i(\Gamma-\gamma)/2]^{-1}\theta(t_k-r_{jk})\cdot \times\{\exp[-i(\omega_j-i\Gamma/2)(t_k-r_{jk})]-\exp[-i(\omega_k-i\gamma/2)(t_k-r_{jk})]\}. \quad (16)$$

The required probability that the counters will fire is given by

$$W(t_3, t_4)=|F_{13}F_{24}+F_{14}F_{23}|^2. \quad (17)$$

The next problem is to elucidate the behavior of this probability when $\gamma/\Gamma \to 0$ and when $\gamma/\Gamma \to \infty$. In the former case, the frequencies of the scattered photons are, in fact, equal to the natural frequencies of the targets, ω_3 and ω_4. One would therefore expect the realization of the momentum-energy variant in which the frequency of the photons is determined and there is no dependence on t_3 and t_4. Conversely, when $\gamma/\Gamma \to \infty$, the scattering process has no effect on the frequency of the scattered photons and the counters determine only the times of detection. Under these conditions, one would, of course, expect the realization of the space-time variant.

In fact, when the width Γ is fixed, and $\gamma/\Gamma \to \infty$, the second exponentials in the numerator of (16) are found to vanish, and $\gamma/[\omega_j - \omega_k - i(\Gamma - \gamma)/2] \to -2i$. Therefore, the amplitude assumes the form

$$A(t_3, t_4)\sim\exp[-(i\omega_1+\Gamma/2)(t_3-r_{13})]\exp[-(i\omega_2-\Gamma/2)(t_4-r_{24})]\theta(t_3-r_{13})\theta(t_4-r_{24})+\exp[-(i\omega_1+\Gamma/2)(t_4-r_{14})]. \times\exp[-(i\omega_2+\Gamma/2)(t_3-r_{23})]\theta(t_4-r_{14})\theta(t_3-r_{23}), \quad (18)$$

which is identical with (6), since the currents g_1 and g_2 are equal to $\exp(-\Gamma t/2)\theta(t)$, apart from a constant factor.

Let us now suppose that Γ is fixed and $\gamma/\Gamma \to 0$, i.e., $\gamma \to 0$. For practically all values of t_3 and t_4, one can then neglect the first exponentials in the numerators of (16). The amplitude is therefore given by

$$A(t_3, t_4)\sim\frac{\exp[-i\omega_3(t_3-r_{13})]\exp[-i\omega_4(t_4-r_{24})]}{(\omega_1-\omega_3-i\Gamma/2)(\omega_2-\omega_4-i\Gamma/2)}\theta(t_3-r_{13})\theta(t_4-r_{24}) + \frac{\exp[-i\omega_4(t_4-r_{14})]\exp[-i\omega_3(t_3-r_{23})]}{(\omega_1-\omega_4-i\Gamma/2)(\omega_2-\omega_3-i\Gamma/2)}\theta(t_4-r_{14})\theta(t_3-r_{23})$$

and contains the common factor $\exp(-i\omega_3 t_3 - i\omega_4 t_4)$ that disappears from the expression for the probability $W(t_3, t_4)$, so that

$$W(t_3, t_4)\sim\left|\frac{\exp[i(\omega_3 r_{13}+\omega_4 r_{24})]}{(\omega_1-\omega_3-i\Gamma/2)(\omega_2-\omega_4-i\Gamma/2)}\theta(t_3-r_{13})\theta(t_4-r_{24}) + \frac{\exp[i(\omega_4 r_{14}+\omega_3 r_{23})]}{(\omega_1-\omega_4-i\Gamma/2)(\omega_2-\omega_3-i\Gamma/2)}\theta(t_3-r_{23})\theta(t_4-r_{14})\right|^2. \quad (19)$$

If the experiment continues long enough (the possibility of this is ensured by $\gamma \to 0$), the arguments of the θ functions become positive, and the θ functions themselves disappear, so that (19) takes the form

$$W\sim\left|\frac{\exp[i(\omega_3 r_{13}+\omega_4 r_{24})]}{(\omega_1-\omega_3-i\Gamma/2)(\omega_2-\omega_4-i\Gamma/2)}+\frac{\exp[i(\omega_4 r_{14}+\omega_3 r_{23})]}{(\omega_1-\omega_4-i\Gamma/2)(\omega_2-\omega_3-i\Gamma/2)}\right|^2 \quad (20)$$

i.e., we obtain a formula describing the momentum-energy variant of the interference experiment [when this is compared with (1), we must recall that, under the present conditions, $t_1 = t_2$).

The foregoing discussion shows that the space-time and momentum-energy interference two-particle correlations do, in fact, have a common character and are merely different limiting cases of the same physical phenomenon. We note that this conclusion does not depend on the specific features of the model of scatterers with variable width γ which we have adopted. The same result is obtained when arbitrary frequency filters tuned to frequencies ω_3 and ω_4 having variable transmission bandwidths are placed at points 3 and 4.

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[1)] In elementary-particle physics, the position of the detectors must be known before the momentum can be determined. However, the precision of their localization in space is always several orders lower than the limit imposed by the uncertainty relation $\Delta x\Delta p \sim 1$. Consequently, the formulation of the experiment discussed in this case must be classified as the momentum-energy variant.

[2)] Strictly speaking, the correlation function is $\langle g_j(t)g_j^*(t + \vartheta)\rangle - \langle g_j(t)\rangle|^2$ but we are assuming that $\langle g_j(t)\rangle = 0$ since $\langle A_j\rangle = 0$.

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