Magnetic field at a μ^+ meson in a ferromagnet

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The magnetic fields B_μ at a μ^+ meson in iron, nickel, cobalt and gadolinium were investigated experimentally as a function of the intensity of the external magnetic field H. The field B_μ was measured in terms of the frequency of the Larmor precession of the μ^+ meson. From the measured functional dependences $B_\mu(H)$ the magnitudes and the directions were determined of the contact magnetic fields B_c of the polarized electrons at a μ^+ meson in the aforementioned ferromagnets; an esimate of the polarization of these electrons has been obtained. The induction B_{dom} in a domain of an unsaturated ferromagnet has been measured. It has been shown that the induction B_{dom} is equal to the saturation induction.

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1. INTRODUCTION

The study of the local magnetic field B_μ at a μ^+ meson represents a new method for the investigation of properties of ferromagnets. In a relatively small number of papers published on this subject rather varied experimental information has been obtained[1-6]. In the present work the magnetization curves $B_\mu(H)$ have been investigated, i.e., the dependence of the field B_μ on the intensity of the external field H in iron, nickel, cobalt and gadolinium. The experimental dependence $B_\mu(H)$ enables one to determine the contact magnetic field at a μ^+ meson due to the polarized electrons of the ferromagnetic, to estimate the magnitude and the direction of the polarization of these electrons, and also to measure the magnetic induction B_{dom} in a domain of an unsaturated ferromagnetic.

A part of the results of this work has been published previously[1-3]. The work has been carried out on the synchrocyclotron of the JINR in Dubna.

2. EXPERIMENTAL RESULTS

The magnetic field B_μ at a μ^+ meson in a ferromagnet was obtained from the expression for the frequency

$$\omega = eB_\mu/mc \tag{1}$$

of Larmor precession where m is the mass of the μ^+ meson. The frequency ω was obtained by the method of recording positrons from $\mu^+ \to e^+$ decay, which emerge primarily in the direction of spin of the μ^+ meson. The experimental arrangement is schematically shown in Fig. 1. A beam of longitudinally polarized μ^+ mesons was stopped in the target F made of the ferromagnetic under investigation. The target F was placed in a magnetic field H produced by an electromagnet with poles of 220 mm diameter and an 180 mm gap between the poles. The signals from the counters $123\bar{4}$ (coincidence of signals from 123 and an anticoincidence of the signal from 4) determined that the μ^+ meson stopped within the target F, the signals $456\bar{3}$ determined the emergence of the positron from the $\mu^+ \to e^+$ decay. The precession of the spin of the μ^+ meson was observed by a standard method[7].

The targets F in which the μ^+ mesons were stopped were made in the form of flat ellipsoids of revolution. The ellipsoidal shape was chosen in order to guarantee the homogeneity of the magnetic field over the whole volume of the sample when it is magnetized. The ellipsoids made of iron, nickel and cobalt were of 60 mm diameter and of 10 mm maximum thickness. The demagnetization factor for such an ellipsoid is $\gamma = 0.108$. The ellipsoid made of gadolinium had a diameter of 64 mm and a thickness of 12 mm and, consequently, had a demagnetization factor of $\gamma = 0.118$. The ellipsoids were made of polycrystalline material with an impurity content of less than 0.5% (Fe) and less than 0.1% (Ni, Co, Gd).

The direction of the beam of μ^+ mesons coincided, as can be seen from Fig. 1, with the direction of the minor axis of the ellipsoid. The external field H was directed along the major axis of the ellipsoid, perpendicular to the direction of the beam (and of the spin) of the μ^+ mesons.

Fig. 2 shows the precession of the spin of the μ^+ meson in iron, in the field H = 0. The experimentally observed precession N(t) is described by the relation

$$N_{theor}(t) = N_0 e^{-\Lambda_0 t}(1 - ae^{-\Lambda t}\cos\omega t). \tag{2}$$

Here $\Lambda_0 = 0.455 \times 10^6$ sec^{-1} is the rate constant for the $\mu^+ \to e^+$ decay, a is the amplitude of the precession at the initial instant of time, Λ is the rate of decay of the pre-

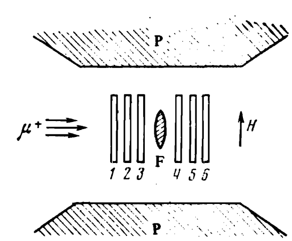

FIG. 1. Schematic experimental arrangement: F–ferromagnetic target, P–poles of electromagnet, 1-6-scintillation counters. The polarization of the μ^+-meson beam is in the direction opposite to the momentum.

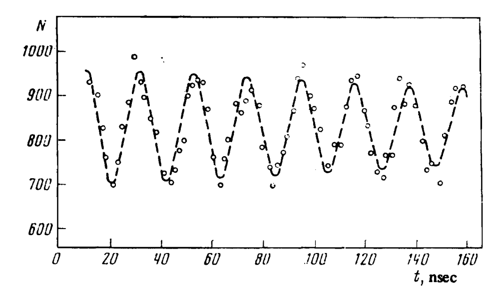

FIG. 2. Precession of μ^+ mesons in iron in an external field H = 0. N is the number of counts in the Δt = 2 nsec channel of a time analyzer. The smooth curve is the theoretical dependence (2) with parameters chosen by the best-fit method: a = 0.16 ± 0.01; Λ = 2.7 ± 0.6 μsec^{-1}; ω = 299.2 ± 0.6 sec^{-1}. The data given above have been corrected for the exponential $e^{-\Lambda_0 t}$ Characterizing the decay of μ^+ mesons.

cession or the rate of spin relaxation of the μ^+ meson in the ferromagnet, ω is the precession frequency. The parameters N_0, a, Λ, and ω were determined from the experimental dependence N(t) by the best-fit method. The precession amplitude a determined the polarization of the μ^+ meson in the ferromagnet at t = 0: $Q_\mu = a/a_{Cu}$, where $a_{Cu} = 0.310 \pm 0.010$ is the precession amplitude for a μ^+ meson in a copper target, the shape and the dimensions of which were the same as for the ferromagnetic targets. The frequency ω determines in accordance with relation (1) the magnetic field B_μ at a μ^+ meson.

Table I shows magnitudes of $Q_\mu(0)$, $\Lambda(0)$ and $B_\mu(0)$, which characterize the precession of a μ^+ meson in different ferromagnets for a practically zero external magnetic field $H \lesssim 1$ Oe.

The functional dependences of $B_\mu(H)$ in iron, nickel, cobalt and gadolinium are shown in Figs. 3-5, and the functional dependences of $\Lambda(H)$ are shown in Fig. 6. It should be emphasized that the direction of the field $\mathbf{B}_\mu$ was not measured in this experiment. This is associated with the fact that the counter telescope $456\overline{3}$ was situated collinearly with the direction of the primary polarization of the μ^+ mesons (cf. Fig. 1) and recorded positrons from the $\mu^+ \to e^+$ decay in the same way for any direction of precession of the spin of the μ^+ meson. Thus, from the precession curves N(t) (cf., Fig. 2) only the absolute value was determined of the rate of precession and in accordance with expression (1) only the absolute value of the field $\mathbf{B}_\mu$. It is just the absolute values of B_μ as a function of H that are shown in Figs. 3-5.

TABLE I

Ferromagnet	T, K	$Q_\mu(0)$	$\Lambda(0)$, μsec^{-1}	$B_\mu(0)$, G
Iron	295	0.66±0,03	2.9±0.3	3509±4
Nickel	295	0.78±0,08	7.8±0.4	1341±7
Cobalt	295	0.63±0,10	6.0±1.0	200±10
Gadolinium	130	0.65±0,03	12.4±1.4	1679±17
Gadolinium	250	0.27±0,05	7.5±1.9	854±27

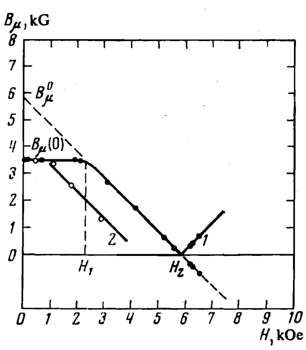

FIG. 3

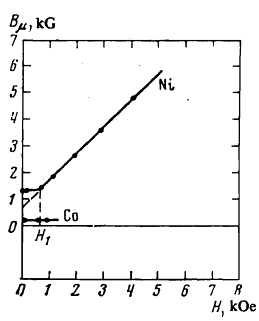

FIG. 4

FIG. 3. Dependence of the absolute value B_μ of the magnetic field at a μ^+ meson in iron on the intensity of the external field H. The dependence 1 has been obtained for an ellipsoid with a demagnetizing factor $\gamma = 0.108$. The dependence 2 characterizes a disk with $\gamma \approx 0.037$. The field H_1 separates the regions $H < H_1$, where B_μ is constant, and $H > H_1$, where $\Delta B_\mu = \Delta H$. The dotted line shows an extrapolation of the experimental dependence $B_\mu(H)$ for the ellipsoid in the range $H_1 < H < H_2$. The statistical errors $\delta(B_\mu)$ do not exceed $\delta(B_\mu) = 10$ G for the ellipsoid and $\delta(B_\mu) = 30$ G for the disk.

FIG. 4. Experimental curves for $B_\mu(H)$ in nickel and in cobalt. The statistical errors $\delta(B_\mu)$ are less than 1% in nickel and less than 20% in cobalt. The fields B_μ in cobalt were measured only for $H < 1$ kOe since for greater values of H the spin of the μ^+ meson relaxes rapidly in cobalt.

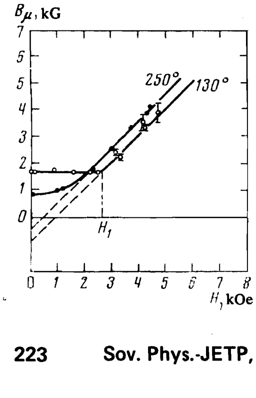

FIG. 5. Experimental curves for $B_\mu(H)$ in gadolinium at temperatures of T = 130 K and T = 250 K.

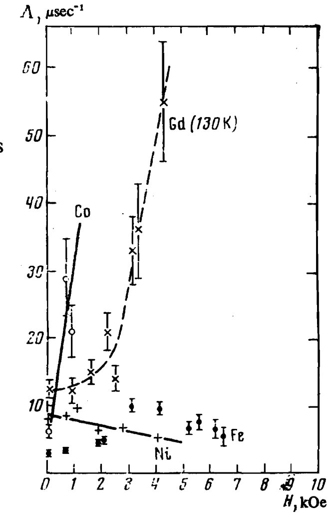

FIG. 6. Dependences of the rates of relaxation Λ of the spin of a μ^+ meson in iron, nickel, cobalt and gadolinium (T = 130 K) on the intensity of the external magnetic field H. The smooth curves have been drawn for graphic emphasis through the experimental points for the corresponding ferromagnetics. The errors $\delta(\Lambda)$ in nickel which have not been indicated on the diagram do not exceed 10%.

From Table I and Figs. 3-5 it may be seen that the field B_μ at a μ^+ meson in a ferromagnet is much smaller than the saturation induction B_{sat}, which, for example, in the case of iron is equal to $(B_{sat})_{Fe} = 21.6$ kG. The structure of the field B_μ is examined in detail in Sec. 3. Now we note merely that a significant component of the field B_μ is the contact field of the polarized conduction electrons directed oppositely to the direction of magnetization.

The values $Q_\mu \approx 2/3$ or $a \approx 2a_{Cu}/3$ for H = 0 which follow from Table I are the natural result of the isotropic distribution of the magnetization vectors $\mathbf{M}_{dom}$ in the individual domains of the ferromagnetic, i.e., of the isotropic distribution of the vectors $\mathbf{B}_\mu$. The small value of $Q_\mu(0)$ in gadolinium at T = 250 K corresponds to the "anomalous" state of this ferromagnet[4] and is the result of more complicated processes.

From Figs. 3-5 it follows that the functional dependences $B_\mu(H)$ in iron, nickel, cobalt and gadolinium are characterized by a number of common properties. In all these ferromagnets one can indicate such a field H_1 that for $H < H_1$ the field B_μ is independent of H, while for $H > H_1$ the change ΔB_μ and the change ΔH corresponding to it coincide: $\Delta B_\mu = \Delta H$. Therefore the experimental dependences $B_\mu(H)$ can be schematically represented in the form of two intersecting straight lines which are characterized by B_μ = const for $H < H_1$ and $\Delta B_\mu = \Delta H$ for $H > H_1$. The difference from other ferromagnets of the functional dependence $B_\mu(H)$ in iron consists only of the relative direction of the fields $\mathbf{B}_\mu$ and $\mathbf{H}$: in iron for $H < H_2$ the directions of the fields $\mathbf{B}_\mu$ and $\mathbf{H}$ are opposite and, therefore, as H increases in this interval, B_μ decreases; in the other ferromagnets that have been investigated the directions of $\mathbf{B}_\mu$ and $\mathbf{H}$ always coincide.

The large errors $\delta(B_\mu)$ in gadolinium are associated with the sharp increase in the rate of spin relaxation of the μ^+ mesons Λ as H increases in the region $H > H_1$ (cf., Fig. 6). The functional dependence $B_\mu(H)$ in gadolinium at a temperature T = 250 K (cf., Fig. 5) differs by a smoother transition from the plateau for $H < H_1$ to the region $\Delta B_\mu = \Delta H$ for $H > H_1$, than is observed for other ferromagnets. As has been noted already, the value T = 250 K belongs to the temperature range ΔT = 230–280 K in which the magnetic properties of gadolinium differ significantly from the properties of ordinary ferromagnets[4].

The functional dependence $B_\mu(H)$ in cobalt has been studied experimentally, as can be seen from Fig. 4, only for such values of H when B_μ is constant. As H increases beyond $H \gtrsim 1$ kOe the precession of μ^+ mesons in cobalt becomes unobservable due to the rapid growth in the rate of spin relaxation of μ^+ mesons (cf., Fig. 6). Table II gives the values calculated by the best fit method of the coefficients $\kappa = \Delta B_\mu / \Delta H$ in ellipsoids made of different ferromagnets which quantitatively characterize the relationship $\Delta B_\mu = \Delta H$ for $H > H_1$. From these data it follows the coefficients κ are equal to unity within experimental error. Moreover, Table II also shows the numerical values of the fields H_1 obtained from the functional dependences $B_\mu(H)$ given in Figs. 3-5 for iron, nickel and gadolinium. These values of H_1 correspond to the point of intersection of the experimental straight lines corresponding to B_μ = const and $\Delta B_\mu = \Delta H$. The experimental values of H_1 given in Table II are compared with the maximum demagnetizing fields B_{demag}^{max} for the given ferromagnetic samples:

$$B_{demag}^{max} = 4\pi\gamma M_{sat} = \gamma B_{sat}. \quad (3)$$

Here γ is the demagnetizing factor, B_{sat} is the saturation induction.

From Table II it may be seen that for iron the field H_1 coincides with the demagnetizing field B_{demag}^{max}. For an additional experimental confirmation of the equality $H_1 = B_{demag}^{max}$ the functional dependence $B_\mu(H)$ was measured for an iron disk of 60 mm diameter and of 3 mm thickness, which simulated a thin ellipsoid with a demagnetizing factor $\gamma \approx 0.037$. From Fig. 3 it follows that also in this case $H_1 = \gamma B_{sat}$. In nickel and in gadolinium the field H_1 is smaller by approximately 10% than the demagnetizing field B_{demag}^{max}; for nickel this difference lies practically within experimental error.

The equality $H_1 = \gamma B_{sat}$ means that the fields $H < H_1$ and $H > H_1$ correspond respectively to the unsaturated, $M < M_{sat}$, and to the saturated, $M = M_{sat}$, states of the magnetic material. Thus, the constancy of B_μ observed experimentally for $H < H_1$ characterizes the unsaturated state of the magnetic material, while the relation $\Delta B_\mu = \Delta H$ observed for $H > H_1$ is satisfied for a saturated magnetic material. The somewhat smaller value of the field H_1 compared to γB_{sat} for gadolinium is, apparently, the result of an overestimate B_{sat} = 24 kG obtained by utilizing the theoretical Brillouin curve which is not sufficiently definite for gadolinium with its complicated magnetic structure. The relation $H_1 = \gamma B_{sat}$ can in principle be utilized to determine the saturation induction B_{sat} by the method of measuring the field H_1. The data of Table II illustrate the experimental possibilities of this method.

TABLE II

Ferromagnet	T, K	γ	B_{sat}, kG	B_{demag}^{max}, G	H_1, G	κ
Iron	295	0.108	21.6	2330	2330±34	0.99±0.02
Nickel	295	0,108	6.08	655	606±21	1.00±0.02
Gadolinium	130	0.118	24.0	2830	2542±70	1.02±0.11
Gadolinium	250	0.118	12.7	1500	1322±60	1.06±0.03
Cobalt	295	0.108	17.9	1930	—	—

Note. The value of $B_{sat}(130\ K)$ for gadolinium equal to 24.0 kG was obtained from the experimental value of $B_{sat}(0\ K) = 25.4$ kG in agreement with the theoretical Brillouin curve $B_{sat}(T)$: $B_{sat}(130\ K) = 0.95\ B_{sat}(0\ K)$. The value $B_{sat}(250\ K) = 12.7$ kG was obtained from the experimental dependence $B_\mu(T)$ in gadolinium [4]; $B_{sat}(250\ K) = B_{sat}(130\ K)\ B_\mu(250\ K)/B_\mu(130\ K)$.

We now consider the field $\mathbf{B}_\mu$ in somewhat greater detail. The experimentally observed precession frequency ω of the μ^+ mesons determines only the average value of the absolute magnitude of the field $\mathbf{B}_\mu$, which in fact is not homogeneous. This follows from the relatively large value of Λ (cf., Table I and Fig. 6), which significantly exceeds the rates of spin relaxation for μ^+ mesons in nonmagnetic metals and depends on the field H. The rate of relaxation Λ enables one to make an estimate of the average inhomogeneity δB_μ of the field B_μ, which is equivalent to the broadening of the resonance line in the NMR method. From Fig. 6 it can be seen that for small fields $\Lambda \lesssim 10\ \mu sec^{-1}$ in all the ferromagnetics that have been investigated. Such values of Λ lead to the estimate $\delta B_\mu < 0.2$ kG. As the intensity of the field H increases the values of Λ, and consequently also of δB_μ, in iron and in nickel remain within the same limits. In gadolinium as the field H increases the rate of relaxation Λ rises rapidly; thus for H = 4 kOe we have $\Lambda \approx 50\ \mu sec^{-1}$, which corresponds to $\delta B_\mu = 1.2$ kG. The rate of spin relaxation of μ^+ mesons in cobalt increases even more rapidly with increasing H. We shall see below that the inhomogeneity δB_μ is associated with the inhomogeneity of the contact magnetic field B_c at a μ^+ meson due to the polarized conduction electrons in a polycrystalline ferromagnet.

It is more difficult to make an estimate of the scatter $\delta \mathbf{B}_\mu$ in the directions of the fields. In the following calculations we shall assume that the directions of B_μ and $\mathbf{M}$ are practically collinear. Evidence concerning the validity of such an assumption is provided by an increase in the amplitude a of the precession of μ^+ mesons as the ferromagnet is magnetized which is associated with the orientation of the vectors $\mathbf{M}_{dom}$ along the direction of the external field $\mathbf{H}$. The collinearity of the vectors $\mathbf{B}_\mu$ and $\mathbf{H}$ is also confirmed by the relation $\Delta B_\mu = \Delta H$ for a saturated ferromagnet which holds well for $H \gtrsim H_1$. In the case of iron where the fields $\mathbf{B}_\mu$ and $\mathbf{H}$ are oppositely directed for $H_1 < H < H_2$ there exists a method for a sufficiently accurate estimate of the component of the field $(B_\mu)_\perp$, perpendicular to the direction of $\mathbf{H}$ or of $\mathbf{M}$. This estimate is based on the fact that perpendicular component $(B_\mu)_\perp$ cannot be compensated by the field $\mathbf{H}$ and must become evident as $H \to H_2$ when the field B_μ reverses its direction. From Fig. 3 it can be seen that the experimentally measured field in iron $B_\mu = 0.26$ kG for H = 5.6 kOe does not show any deviation from the relation $\Delta B_\mu = \Delta H$. This result leads to the estimate $(B_\mu)_\perp < 0.2$ kG or $(B_\mu)_\perp / B_\mu(0) < 0.2/3.5 \approx 0.06$. The field $\langle \mathbf{B}_\mu \rangle$ averaged over the different orientations of the polycrystalline ferromagnet is naturally collinear with $\mathbf{M}$. For the collinear vectors $\mathbf{B}_\mu$ and $\mathbf{M}$ the direction of field $\mathbf{B}_\mu$ is determined by its component $(B_\mu)_Z$ along the direction of $\mathbf{M}$: $(B_\mu)_Z > 0$, if the vectors $\mathbf{B}_\mu$ and $\mathbf{M}$ are parallel, and $(B_\mu)_Z < 0$, if $\mathbf{B}_\mu$ and $\mathbf{M}$ are antiparallel. In order to determine the sign of $(B_\mu)_Z$ we utilize the experimental dependence $B_\mu(H)$ for the completely magnetized magnetic material ($H > H_1$) when the directions of $\mathbf{M}$ and $\mathbf{H}$ coincide. The direction of $\mathbf{B}_\mu$ or the sign of $(B_\mu)_Z$ are determined from the condition that as H is increased the algebraic value of $(B_\mu)_Z$ must increase. Thus, for example, in iron for $H_1 < H < H_2$ the component $(B_\mu)_Z < 0$, i.e., the directions of the field $\mathbf{B}_\mu$ and $\mathbf{H}$, as has already been noted above, are opposite to each other. The experimental functional dependences $\mathbf{B}_\mu(H)$, constructed according to the data of Figs. 3-5 and taking into account the direction of $\mathbf{B}_\mu$ in iron, nickel and gadolinium, are shown in Fig. 7. The direction of the field $\mathbf{B}_\mu$ in this diagram is determined by the sign of $(B_\mu)_Z$, while the absolute value of $(B_\mu)_Z$ is taken to be equal to the absolute value of B_μ determined from relation (1). The dependence $\mathbf{B}_\mu(H)$ for cobalt is not shown in Fig. 7, since the region of magnetic saturation in this ferromagnetic is unobservable and the determination of the sign of $(B_\mu)_Z$ is impossible.

3. THE CONTACT FIELD AT A μ^+ MESON IN A FERROMAGNETIC

For a further description of the experimental functional dependence $\mathbf{B}_\mu(H)$ we consider the structure of the field $\mathbf{B}_\mu$ in a ferromagnet. This field can be represented in the form of a sum:

$$\mathbf{B}_\mu = \mathbf{B}_c + \mathbf{B}_d + \mathbf{H}. \quad (4)$$

Here $\mathbf{B}_c$ is the contact field which is produced at the μ^+ meson by the polarized electrons of the ferromagnetic, basically the conduction electrons; $\mathbf{B}_d$ is the dipole field of the magnetized atoms of the ferromagnetic. The contact field can be written in the form

$$\mathbf{B}_c = {}^8/_3\pi\beta_0\rho_\mu\mathbf{P}. \quad (5)$$

Here β_0 is the magnetic moment of the electron, $\mathbf{P}$ and ρ_μ are respectively the polarization and the density of the wave function for the electrons at a μ^+ meson. The dipole field $\mathbf{B}_d$ can be represented as a sum of two fields:

$$\mathbf{B}_d = \mathbf{B}_d' + \mathbf{B}_d'', \quad (6)$$

where $\mathbf{B}_d' = 4\pi\mathbf{M}/3 + \mathbf{B}_{demag}$ is the field at the centre of a small hollow sphere, referred to as the Lorentz sphere, which represents the dipole field at a μ^+ meson due to the distant atoms of the magnetic material, when one can neglect the discrete atomic structure of matter; $\mathbf{B}_d''$ is the dipole field due to the atoms closest to the μ^+ meson situated within the Lorentz sphere.

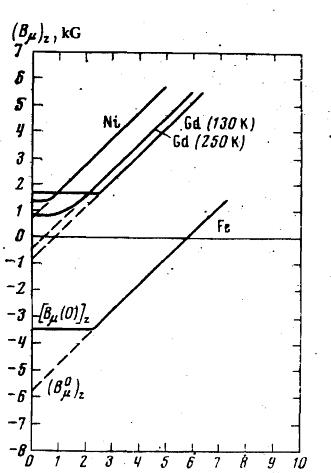

FIG. 7. Dependences $B_\mu(H)$ in iron, nickel and gadolinium showing the direction of the field B. Along the vertical axis is plotted the component $(B_\mu)_Z$ of the field B_μ along the direction of H. The values of $(B_\mu)_Z > 0$ correspond to the case when the directions of the fields B_μ and H coincide, the values of $(B_\mu)_Z < 0$ correspond to the case when B_μ and H are oppositely directed. Only the smooth curves are given for the dependences $B_\mu(H)$ constructed according to the experimental points of Figs. 3-5. The components $[B_\mu(0)]_Z$ and $(B^0_\mu)_Z$ characterizing the experimental dependences $B_\mu(H)$ are indicated on the diagram only for iron. Numerical values of the components $[B_\mu(0)]_Z$ and $(B^0_\mu)_Z$ for all the ferromagnetics are given in Table III.

The field $\mathbf{B}_d''$ can be calculated only after making an assumption as to the point of the crystalline lattice at which the μ^+ meson is situated. It is usually assumed that the impurity particle (in the present case the μ^+ meson) is situated interstitially or in pores corresponding to a minimum in the Coulomb potential energy. A calculation of the fields $\mathbf{B}_d''$ in the pores of ferromagnetic crystals is carried out in [8] where it is shown that the fields $\mathbf{B}_d''$ are equal to zero in nickel are small in cobalt (~0.1 kG), are great in iron (~10 kG), and in all the ferromagnetics the field $\langle\mathbf{B}_d''\rangle$, averaged over the possible positions of the μ^+ meson (pores) in the crystal, is equal to zero for any orientation of the crystal with respect to the direction of the magnetization. The equation $\langle\mathbf{B}_d''\rangle = 0$ means that the fields B_d'' can lead only to spin relaxation of the μ^+ meson in a ferromagnetic and do not make a contribution to the observed value (4) of $\mathbf{B}_\mu$. Therefore expression (4) can be rewritten in the following manner:

$$\mathbf{B}_\mu = \mathbf{B}_c + \mathbf{B}_d' + \mathbf{H} = \mathbf{B}_c + {}^4/_3\pi\mathbf{M} + \mathbf{B}_{demag} + \mathbf{H}. \quad (7)$$

In making an estimate of the rate of spin relaxation of μ^+ mesons due to the fields $\mathbf{B}_d''$ one should take into account the diffusion of the μ^+ meson in the crystal of the ferromagnet. In the case of a sufficiently rapid diffusion of the μ^+ meson the fields $\mathbf{B}_d''$ acting on it become variable with time and only the field $\langle\mathbf{B}_d''\rangle$ which is equal to zero turns out to be significant. Therefore the diffusion of μ^+ mesons diminishes the rate of relaxation of the spin of μ^+ mesons. The rapid diffusion of μ^+ mesons in iron was confirmed experimentally in studying the temperature dependence[1] of the rate of relaxation of the spin of μ^+ mesons. In this experiment it was shown that as the temperature is lowered, in which case the diffusion is slowed down, the rate of damping Λ of the precession of μ^+ mesons increases. The large fields $\mathbf{B}_d''$ acting on a μ^+ meson in iron must lead to just as such an effect. In [1] it is also shown that the value of Λ in nickel does not depend on the temperature, as should be the case for $\mathbf{B}_d'' = 0$.

We now continue the investigation of the experimental functional dependences $\mathbf{B}_\mu(H)$ shown in Figs. 3-5 and 7. We consider two problems: 1) the determination of the contact field $\mathbf{B}_c$ and of the polarization $\mathbf{P}$ of the electrons of the ferromagnetic at a μ^+ meson—in this paragraph, and 2) the determination of the induction B_{dom} in a domain of an unsaturated ferromagnetic—in Sec. 4.

The contact field $\mathbf{B}_c$ is determined from expression (7) for $\mathbf{B}_\mu$. When the polycrystalline ellipsoid has its maximum magnetization ($H > H_1$)

$${}^4/_3\pi\mathbf{M} = {}^4/_3\pi\mathbf{M}_{sat} = {}^1/_3\mathbf{B}_{sat}, \quad \mathbf{B}_{demag} = -\gamma\mathbf{B}_{sat}$$

and expression (7) can be rewritten in the form

$$\mathbf{B}_\mu = \mathbf{B}_c + ({}^1/_3 - \gamma)\mathbf{B}_{sat} + \mathbf{H}, \quad H > H_1, \quad (8)$$

or, in terms of the components along the direction of $\mathbf{H}$,

$$(B_\mu)_Z = (B_c)_Z + ({}^1/_3 - \gamma)B_{sat} + H. \quad (9)$$

In obtaining Eq. (9) it was taken into account that in the ellipsoid where $\gamma < 1/3$ the directions of the fields $(1/3-\gamma)\mathbf{B}_{sat}$ and $\mathbf{H}$ coincide. Practically it is convenient to extend the dependence (9) $(B_\mu)_Z = f(H)$ which is valid only for $H > H_1$ to its intersection with the vertical axis at the point $(B^0_\mu)_Z$ (cf., Fig. 7). In this case in accordance with (9) we have

$$(B_\mu^0)_Z = (B_c)_Z + ({}^1/_3 - \gamma)B_{sat}. \quad (10)$$

The equations $\Delta B_\mu = \Delta H$ for $H > H_1$ and $H_1 = B_{demag}$

$= \gamma B_{sat}$ indicated above lead to the relation (cf., Fig. 7)

$$(B_\mu^0)_z = [B_\mu(0)]_z - \gamma B_{sat}, \qquad (11)$$

which enables us to rewrite expression (10) in the form

$$[B_\mu(0)]_z = (B_c)_z + {}^1/_3 B_{sat} \qquad (12)$$

and to obtain, in this manner, one more relation for the determination of B_c. Here $[B_\mu(0)]_Z$ is the component of the field $\mathbf{B}_\mu(H_1)$ numerically equal to $B_\mu(0)$, along the direction of $\mathbf{H}$ (cf., Fig. 7). One can eliminate B_{sat} from relations (10) and (12) and obtain an expression for the evaluation of $(B_c)_Z$ only from the experimental values of $[B_\mu(0)]_Z$ and $(B_\mu^0)_Z$:

$$(B_c)_z = \frac{1}{3\gamma}\{(B_\mu^0)_z - (1-3\gamma)[B_\mu(0)]_z\}. \qquad (13)$$

The components $(B_\mu^0)_Z$ appearing in expressions (10) and (13) were calculated by the best fit method in terms of the experimental values of $\mathbf{B}_\mu(H)$ for $H > H_1$. The experimental values of $B_\mu(0)$ are shown in Table I. The direction of the field $\mathbf{B}_\mu(0)$ or the sign of the component $[B_\mu(0)]_Z$ as well as the sign of the component $(B_\mu^0)_Z$ follow from the dependence $\mathbf{B}_\mu(H)$ shown in Fig. 7. The saturation induction B_{sat} is shown in Table II. The values of $(B_c)_Z$ for the ferromagnets that have been investigated calculated with the aid of relations (10) and (12) are given in Table III. The errors in $(B_c)_Z$ are not shown in Table III, since they are equal to the experimental errors in $(B_\mu^0)_Z$ and $[B_\mu(0)]_Z$, as can be seen respectively from expressions (10) and (12). Due to the relatively large statistical errors Table III does not show values of $(B_c)_Z$ calculated with the aid of relation (13).

The contact field $(B_c)_Z$ in cobalt has been calculated only by means of formula (12) on the assumption that $[B_\mu(0)]_Z = 0$., i.e., $(B_c)_Z = -B_{sat}/3$, since the direction of the field $\mathbf{B}_\mu$ or the sign of the component $[B_\mu(0)]_Z$ are not known in the case of cobalt. The error $\delta(B_c) = B_\mu(0)/B_c \approx 3\%$ obtained in the course of this is not great, since the field $B_\mu(0)$ in cobalt is small.

From Table III it follows that the fields $(B_c)_Z$ calculated from expressions (10) and (12) either coincide or are close to each other. This is a result of the equation $H_1 = \gamma B_{sat}$ and, thus, once again confirms it. We note that in calculating $\mathbf{B}_c$ using formulas (10) and (12) we assumed that the directions of the vectors $\mathbf{M}$, $\mathbf{B}_\mu$ and $\mathbf{B}_c$ are collinear. In Sec. 2 it was shown that the vectors $\mathbf{B}_\mu$ and $\mathbf{M}$ are close to being collinear; in the case of iron an estimate was made of the transverse component $(B_\mu)_\perp < 0.2$ kG. Collinearity of $\mathbf{B}_\mu$ and $\mathbf{M}$ means that the vectors $\mathbf{B}_c$ and $\mathbf{M}$ are also collinear. The relatively large values of the fields $\mathbf{B}_{sat}/3$ and $\mathbf{B}_c$ in comparison with $\mathbf{B}_\mu$ lead to the fact that the possible noncollinearity of $\mathbf{B}_c$ and $\mathbf{M}$ (or $\mathbf{B}_{sat}$) is even smaller than that of the vectors $\mathbf{B}_\mu$ and $\mathbf{M}$.

The contact field $\mathbf{B}_c$ in a polycrystalline sample is inhomogeneous. The values of $\mathbf{B}_c$ given in Table III are averaged over crystals of different orientation. As has been noted in Sec. 2, it is just the inhomogeneity of the field B_c that leads to the experimentally observed scatter δB_μ of the local fields B_μ at the position of a μ^+ meson in a ferromagnet. This conclusion follows from expression (7) from which it can be seen that $\delta B_\mu = \delta B_c$. From Table III it can be seen that in all the derromagnets that have been investigated the component $(B_c)_Z < 0$, i.e., the contact field $\mathbf{B}_c$ at a μ^+ meson is directed oppositely to the magnetization $\mathbf{M}$. In iron this is clearly seen from the experimental dependence $B_\mu(H)$ (cf., Fig. 3), from which it follows that for $H_1 < H < H_2$ the field B_μ decreases as H increases. The "inverse" dependence $B_\mu(H)$ means that the fields $\mathbf{B}_\mu$ and $\mathbf{H}$ (or $\mathbf{M}$) in iron are oppositely directed and, consequently, the fields $\mathbf{B}_c$ and $\mathbf{M}$ are also oppositely directed. The absolute values of the contact fields B_c in iron, cobalt and gadolinium are significantly greater than in nickel.

The negative value of $(B_c)_Z$ shows that the electrons of the ferromagnetic producing the contact field $\mathbf{B}_c$ at a μ^+ meson, i.e., basically the collectivized conduction electrons, are polarized oppositely to the direction of magnetization $\mathbf{M}$. The degree of polarization P of these electrons depends on the field B_c in accordance with expression (5) and can be calculated if one knows the density ρ_μ of the electron wave function at the μ^+ meson. The possible values of ρ_μ lie within the limits $\bar{\rho} < \rho_\mu < \rho_{Mu}$. Here $\bar{\rho}$ is the average density of conduction electrons in a metal; $\rho_{Mu} = 2.1 \times 10^{24}\ \mathrm{cm}^{-3}$ is the electron density at a μ^+ meson in a free muonium atom corresponding to a contact field $(B_c)_{Mu} = 164$ kG.

It should be noted that the range of possible values of ρ_μ is not so very wide: in the case of iron, nickel and cobalt $\bar{\rho} \approx 0.08\rho_{Mu}$, in the case of gadolinium $\bar{\rho} \approx 0.04\rho_{Mu}$. One can obtain a very approximate estimate of ρ_μ by utilizing the results of the work of Pathak[9] in which the density of the electron gas is calculated on electrically charged inclusions. In the present case a μ^+ meson is such an inclusion. The values of ρ_μ obtained in accordance with these calculations for iron, nickel and cobalt are approximately the same and amount to $\rho_\mu \approx \rho_{Mu}/3$. However, we shall not make use of Pathak's calculations, but state the minimally possible values of $P_{min} = B_c/(B_c)_{Mu}$, obtained for a maximal density $\rho_\mu = \rho_{Mu}$. It is just these values of P_{min} that are shown in Table III, where the minus sign shows the negative polarization of the electrons of the ferromagnetic at a μ^+ meson.

The polarization of the conduction electrons of a ferromagnetic was also measured in a number of other experiments. The most informative of them are experiments on the scattering of polarized neutrons. In Table IV are given values obtained by this method of the average magnetic moments β of the localized (β_{loc}) and of the collectivized (β_{coll}) electrons in iron, nickel, cobalt and gadolinium[10-15]. The values of β were obtained from experimentally determined neutron form factors

TABLE III. Contact fields and minimum polarization of electrons at a μ^+ meson in ferromagnetics

Ferromagnet	T, K	(10)		(12)		P_{min}, %
		$(B_\mu^0)_z$, G	$(B_c)_z$, G	$[B_\mu(0)]_z$, G	$(B_c)_z$, G	
Iron	295	—5839±30	—10690	—3509±4	—10710	—6.5
Nickel	295	735±14	—635	1341±7	—689	—0.42
Gadolinium	130	—863±53	—6020	1679±17	—6320	—3.85
Gadolinium	250	—468±33	—3200	854±27	—3390	—2.07
Cobalt	295	—	—	±(200±10)	—5970	—3.65

TABLE IV

Ferromagnet	β_{loc}, β_0/atom	β coll, β_0/atom	$\bar{P}$, %	(B_c) coll. kG	References
Iron	2.39	—0.21	—10.5	—1.39	[10-12]
Nickel	0.711	—0.105	—5.2	—0.75	[13]
Cobalt	1.99	—0.28	—14.0	—1.98	[14]
Gadolinium (96 K)	6.42	—1.21 *	—40 *	—2.88 *	[15]

*Maximum negative value, cf., text.

for iron, nickel and cobalt on the assumption that the distribution of the spin density of the collectivized electrons in these ferromagnets is isotropic. For gadolinium such an interpretation is unsatisfactory, and for it in Table IV there is shown the maximum negative value (in an interstitial pore) of the oscillating density of the magnetic moment of collectivized electrons. Table IV also shows the values corresponding to the experimental values of β_{coll} of the average polarization $\overline{P} = \beta_{coll}/\beta_n$ of the collectivized electrons and of the contact field $(B_c)_{coll} = 8\pi\beta_0\overline{\rho}\,\overline{P}/3 = 8\pi M_{coll}/3$. Here β_n is the maximum possible magnetic moment of n valence (collectivized) electrons of one atom of the ferromagnetic, M_{coll} is the magnetic moment per unit volume produced by the collectivized electrons. The negative values of these quantities denote, as in Table III, the negative polarization of the collectivized electrons of the ferromagnetic.

The values of the magnetic moments of the localized and of the collectivized electrons in Table IV are shown in units of β_0/atom (Bohr magnetons per atom).

From Table IV it can be seen that experiments on the scattering of polarized neutrons show a negative polarization of the collectivized electrons in all the ferromagnets noted above. This is in agreement with the results of the present work.

The absolute values of $(B_c)_{coll}$ are significantly smaller than the contact fields B_c at a μ^+ meson in all the ferromagnets with the exception of nickel. An increase in the contact field B_c at a μ^+ meson in comparison with the contact field $(B_c)_{coll}$ due to the collectivized electrons measured in neutron experiments is the result of an increase in the density ρ of the polarized electrons at a μ^+ meson of a ferromagnet. However, it should be noted that this increase in ρ cannot be attributed entirely to an increase in electron charge density, since changes in the spin and the charge densities of the collectivized electrons at a μ^+ meson in a ferromagnet could occur without being proportional to each other[16]. We now turn our attention to other experiments.

The negative sign of the polarization of the conduction electrons in iron was discovered in [17] by the method of hyperfine splitting of the Mössbauer spectrum of tin dusted onto iron. The polarization of the electrons of a ferromagnetic can also be measured by the method of recording γ quanta produced as a result of annihilation of polarized positrons. However, appropriate experiments[18–22] so far have not led to sufficiently well defined results on the determination of the polarization of the collectivized electrons of a ferromagnet. It should be noted that investigations of the polarization of collectivized electrons by the positron and the μ^+ meson methods have much in common, since in both cases the test particle is a charged one. It is obvious that in the course of further development of methodology and theory all the experiments described above will effectively complement each other.

4. INDUCTION IN A DOMAIN OF AN UNSATURATED FERROMAGNETIC

From Table I and Figs. 2-5 it follows that the precession of a μ^+ meson in a ferromagnet is also observed in the absence of an external magnetic field. The field $B_\mu(0)$ corresponding to the precession frequency for H = 0 represents a magnetic field at a μ^+ meson in a spontaneously magnetized domain of a ferromagnetic. The relatively low rate of relaxation $\Lambda(0)$ for H = 0 (cf., Table I and Fig. 6) means that the fields $B_\mu(0)$ in different domains of a multidomain ferromagnet are close in value to each other. The values $\Lambda(0) \lesssim 10\ \mu\text{sec}^{-1}$ observed for H = 0 enable us to estimate the possible scatter $\delta B_\mu(0) < 0.2$ kG, which, as we can see, is small and therefore the field $B_\mu(0)$ in a domain is an entirely definite quantity. Consequently, the local magnetic induction B_{dom} in separate domains of an unmagnetized ferromagnetic is also a definite quantity. We shall show that the induction B_{dom} in a domain of an unsaturated ($H < H_1$) ferromagnetic is equal to the saturation induction; $B_{dom} = B_{sat}$.

The equation $B_{dom} = B_{sat}$ for H = 0 follows graphically from Fig. 7, from which it can be seen that the field $B_\mu(0)$ exceeds B_μ^0 by the amount $H_1 = \gamma B_{sat}$. Remembering that B_μ^0 represents the field at a μ^+ meson in a completely magnetized ellipsoid with a demagnetizing factor γ at H = 0, we can define $B_\mu(0)$ as the field at a μ^+ meson in a completely magnetized sample (domain) with a demagnetizing factor $\gamma = 0$. This corresponds to an induction in the domain $B_{dom} = B_{sat}$.

The same result is also obtained from relation (12) from which it follows that the field $B_\mu(0)$ can be written in the form

$$B_\mu(0) = B_c + {}^1/_3 B_{sat}, \qquad (12')$$

i.e., represents the sum of the contact field $\mathbf{B}_c$ and the field $\mathbf{B}_{sat}/3$ in the Lorentz sphere of a saturated ferromagnetic with a demagnetizing factor $\gamma = 0$. The latter means that the induction in a domain with an external field H = 0 is equal to the maximum possible value:

$$B_{dom} = 4\pi M_{sat} = B_{sat}. \qquad (14)$$

Relations (12′) and (14) are a consequence of the experimental equality $H_1 = \gamma B_{sat}$ which, as can be seen from Table II, is satisfied for all the ferromagnetics.

The constancy of the field B_μ as H varies in the interval $H < H_1$ (cf., Figs. 3-5) means further that Eq. (14) is satisfied also when the ferromagnet is being magnetized, i.e., when the domains are being lined up in the direction of the external field, since the external field H for $H < H_1$ is completely compensated by the demagnetizing field $B_{demag} = \gamma B(H)$. Here B(H) is the induction in an ellipsoid with a demagnetizing factor γ.

Thus, it has been shown experimentally that the induction of the internal domain magnetic field B_{dom} in an unsaturated (in a particular case unmagnetized) ferromagnet is equal to the saturation induction. Just such a picture must correspond to a minimum in the magnetic energy.

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