Electron dragging in inverse bremsstrahlung of light

M. Ya. Amus'ya and A. S. Baltenkov

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
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It is shown that the existence of a nonzero photon momentum leads, in the case of inverse bremsstrahlung of light in a plasma, to a translational motion of the electrons. The drag current produced when light of laser frequency is absorbed by electrons in a weakly-ionized gas is calculated. It is shown that dragging in inverse bremsstrahlung by neutral atoms predominates over Compton motion when the atom concentration exceeds 10^{15} cm^{-3}.

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1. In studies of the absorption of light in the low-energy part of the spectrum ($\hbar\omega < 0.5$ keV) it is customary to neglect the photon momentum and to consider this process in the dipole approximation only. It was shown in a number of papers[1-3] that the existence of a nonzero photon momentum leads to asymmetry in the angular distribution of the quantum-absorbing electrons relative to the propagation direction of the light. This in turn is the cause of the predominant motion of the electrons along the photon momentum direction, and consequently the cause of the current flowing in this direction.

It is known that absorption of light by an electron requires for its realization, in first-order perturbation theory, the presence of a "third body." In crystal structures, when light is absorbed by an electron situated in the conduction band, the role of the third body can be played by phonons (acoustic and optical), impurities, lattice defects, etc. In gas atoms, the light-absorption process accompanied by the release of an electron (the photoeffect) takes place in the field of the atomic residue. Light can also be absorbed in an ionized gas in the case when the free electron is the field of a neutral atom or ion. In each act of photon absorption the momentum of the quantum is dis-shared by the electron and the third body, and in the general case this leads[1-3] to translational motion of the electron. It is obvious that the distribution of the momentum depends significantly on the nature of the "recoil body."

We note here that the role of the third body in light-absorption processes can also be played by a photon. Electron dragging due to Compton scattering of light was investigated by Gurevich and Rumyantsev.[4]

The present paper is devoted to a study of the drag current produced when light is absorbed by electrons in the continuous spectrum of neutral atoms and molecules, i.e., in the process of inverse bremsstrahlung of light.

2. Let N_a be the concentration of the neutral atoms in the atomic gas considered by us, $N_i = N_e$ the concentrations of the ions and electrons (we assume for simplicity that all the ions are singly-charged), and let T be the temperature of the gas. Then in the case of thermodynamic equilibrium the average energy of the free electron is T (T is in energy units). In order to be able to consider the motion of each electron independently of the motion of the remaining ones, we stipulate that the plasma be ideal, i.e., that the energy of interaction of the electrons with one another, $e^2 N_e^{1/2}$, be much less then their thermal energy T. We assume furthermore that $\hbar\omega < I$, where I is the ionization potential of the gas atom, and shall use by way of example laser-emission quantum energies $E_L \approx 1.8$ eV (ruby laser) and $E_L \approx 0.12$ eV (CO_2 laser).[5]

We consider the absorption of light by an electron in the continuous spectrum of an ion of charge z = 1 (z = 0 for the neutral atom). The wave functions of the incident and scattered electrons are

$$\psi_{\mathbf{k}'}^{+}(\mathbf{r}) = \frac{(2\pi)^{3/2}}{k'}\left(\frac{m}{\hbar k'}\right)^{1/2} \times \sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'} i^{l'}\exp(i\delta_{l'})Y_{l'm'}^{*}(\theta_{k'},\varphi_{k'})Y_{l'm'}(\theta,\varphi)\frac{P_{k'l'}(r)}{r}, \quad (1)$$

$$\psi_{\mathbf{k}}^{-}(\mathbf{r}) = \frac{(2\pi)^{3/2}}{k}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^{l}\exp(-i\delta_{l})Y_{lm}^{*}(\theta_{k},\varphi_{k})Y_{lm}(\theta,\varphi)\frac{P_{kl}(r)}{r}. \quad (2)$$

Here $\mathbf{k}'$ and $\mathbf{k}$ are the wave vectors of the incident and scattered electrons, respectively, $\delta_{l'}(k')$ and $\delta_l(k)$ are the phases of the wave functions, $\theta_{k'}, \varphi_{k'}$ and $\theta_{k'}, \varphi_k$ are the angles that specify the directions of the electron momenta in a coordinate system with a polar axis coinciding with the photon momentum $\hbar\boldsymbol{\kappa}$.

We confine ourselves in the operator $H'(\mathbf{r}, t)$ of the interaction with the electromagnetic field to the terms proportional to κ, in which case the operator takes the form

$$H'(\mathbf{r},t) = -\frac{e}{mc}(\mathbf{A}\mathbf{p}) \approx -\frac{eA_0}{mc}(\mathbf{e}\mathbf{p})[1+i(\boldsymbol{\kappa}\mathbf{r})]e^{-i\omega t}, \quad (3)$$

where e and m are the charge and mass of the electron, c is the speed of light, $\mathbf{A}$ is the vector potential (div $\mathbf{A} = 0$), and $\mathbf{p}$ is the momentum operator.

The calculation of the matrix element of the operator (3) with wave functions (1) and (2) leads to a rather cumbersome expression. We present here only the final formula for the differential cross section of the inverse bremsstrahlung of light, obtained for unpolarized light, and integrated over all the directions of the vector $\mathbf{k}'$. It takes the form

$$\frac{d\sigma_k(\omega)}{d\Omega_k} = \sum_{l'=0}^{\infty}\frac{\sigma_k^{l'}(\omega)}{4\pi}\left\{1-\frac{1}{2}\beta^{l'}(k,k')P_2(\cos\theta_k)+\kappa\gamma^{l'}(k,k')P_1(\cos\theta_k) + \kappa\eta^{l'}(k,k')P_3(\cos\theta_k)\right\}, \quad (4)$$

$$\sigma_k^{l'}(\omega) = \frac{8\pi^4}{3}\frac{m^2e^2\kappa}{\hbar^3 kk'^3}\{l'R^2_{l',l'-1}+(l'+1)R^2_{l',l'+1}\}. \quad (5)$$

where $\sigma_k^{l'}$ [cm^4-sec] is the "partial" inverse-bremsstrahlung cross section. The sum

$$\sum_{l=0}^{\infty}\sigma_k^{l}(\omega) = \sigma_k(\omega)$$

is the effective inverse-bremsstrahlung cross section.[6] The functions $\beta^l(k, k')$, $\gamma^l(k, k')$, and $\eta^l(k, k')$ were written out in[2], and the phases contained in them should be taken at the energy $\epsilon = \hbar^2k^2/2m$. We present here only the expression that will be needed later on for $\gamma^l(k, k')$:

$$\gamma^l(k,k')=\frac{3}{10}[lR^2_{l,l-1}+(l+1)R^2_{l,l+1}]^{-1}\Big\{\frac{l+1}{2l+1}[3(l+2)D_{l,l+2}R_{l,l+1}\times\cos(\delta_{l+2}-\delta_{l+1})-lD_{l,l}R_{l,l+1}\cos(\delta_l-\delta_{l+1})]-\frac{l}{2l-1}[3(l-1)D_{l,l-2}R_{l,l-1}\times\cos(\delta_{l-2}-\delta_{l-1})-(l+1)D_{l,l}R_{l,l-1}\cos(\delta_l-\delta_{l-1})]\Big\}; \quad (6)$$

$R_{l,l\pm1}$ and $D_{l,l\pm2}$ are respectively the dipole and quadrupole matrix elements

$$R_{l,l\pm1}=\int_0^\infty P_{k,l\pm1}(r)rP_{k',l}(r)\,dr, \quad D_{l,l\pm2}=\int_0^\infty P_{k,l\pm2}(r)r^2P_{k'l}(r)\,dr. \quad (7)$$

The radial parts of the wave function of the continuous spectrum are normalized to $\delta(k - k')$ and have the following asymptotic form:

$$P_{kl}(r)\to\left(\frac{2}{\pi}\right)^{1/2}\sin\left(kr-\frac{z}{k}\ln 2kr-\frac{\pi l}{2}+\delta_l(k)\right) \quad (8)$$

(r and k are in atomic units). According to Sobel'man[6], the coefficient for inverse bremstrahlung of photons by ions is defined by

$$\alpha_k(\omega)=\sum_{l=0}^\infty\int_0^\infty N_iN_ef(v')v'\sigma_k{}^l(\omega)\,dv' \ [\text{cm}^{-1}], \quad (9)$$

where $f(v')$ is the distribution function of the electrons with respect to the velocities $v' = \hbar k'/m$, normalized by the condition

$$\int_0^\infty f(v)\,dv=1.$$

Using the method developed in[2], we can obtain the following expression for the current density due to the photon momentum and produced in the course of inverse bremsstrahlung of light by ions:

$$j^i(\omega)=-|e|WN_eN_i\int_0^\infty f(v')v'\tau(v)v\int_0^{2\pi}\int_0^\pi\frac{d\sigma_k{}^i(\omega)}{d\Omega}\cos\theta_k\sin\theta_k\,d\theta_k\,d\varphi_k\,dv' \ [\text{A-cm}^2], \quad (10)$$

where W is the flux density of the quanta $[\text{cm}^{-2}\text{-sec}^{-1}]$, $\tau(v)$ is the electron momentum relaxation time, $v = \hbar k/m$, and $m(v^2 - v'^2)/2 = \hbar\omega$. Since electrons that have absorbed quanta can relax both on the ions and on the neutral atoms, it follows that $\tau(v) = \tau_a(v)\tau_i(v)/\times[\tau_a(v) + \tau_i(v)]$, where $\tau_a(v) = [\sigma_a(v)N_a v]^{-1}$ and $\tau_i(v) = [\sigma_i(v)N_i v]^{-1}$ [7]; $\sigma_i(v)$ and $\sigma_a(v)$ are the cross sections for the elastic scattering of electrons by screened ions and by the gas atoms, respectively. Replacing in (10) N_i by N_a and assuming in the calculation of $R_{l,l\pm1}$ and $D_{l,l\pm2}$ that $z = 0$, we obtain an expression for the current $j^a(\omega)$ connected with the photon momentum and produced when the light is absorbed by electrons situated in the field of the neutral atoms. The total drag current $j(\omega)$ is the sum $j(\omega) = j^a(\omega) + j^i(\omega)$.

An investigation of the drag current formed when light is absorbed by neutral atoms is of particular interest. The reason is that the interaction of the electron with the neutral atom is much weaker than with the ions, and therefore the behavior of the current $j^a(\omega)$ depends essentially on the atomic structure, whereas the interaction with the ion is practically pure Coulomb. We shall therefore consider henceforth situations in which $j^a(\omega) \gg j^i(\omega)$. It is obvious that this relation can be obtained by decreasing the ion concentration, but then the electroneutrality requirement $N_e = N_i$ decreases also the concentration of the electrons and hence the value of the current. In some cases, however, the relation $j^a(\omega) \gg j^i(\omega)$ is satisfied and the ensuing current densities lend themselves to measurement. Let us consider one such cituation.

3. Let $N \approx 10^{12}$ cm^{-3} and $T \approx 300$°K, corresponding to room temperatures and ordinary pressures. We assume that the electron density is $N_e = N_a \approx 10^{14}$ cm^{-3} (the feasibility of producing such concentrations is discussed at the end of the section). Under these conditions the plasma is ideal. We consider the absorption of the emission of a ruby laser with quantum energy $E_L = 1.8$ eV by an electron situated in the field of a neutral atom. At $T \approx 300$°K the average energies of the free electrons before and after the absorption of the quantum does not exceed 1.9 eV. Therefore when the absorption process is considered we can confine ourselves to allowance for the s-phase only in the incident and scattered electron functions, and neglect all the remaining phases.

Since large r are of importance in the integrals (6), we replace the wave functions $P_{kl}(r)$ and $P_{k'l'}(r)$ by linear combinations of Bessel and Neumann functions[8]:

$$P_{k0}(r)=(2/\pi)^{1/2}kr[\cos\delta_0(k)j_0(kr)-\sin\delta_0(k)\eta_0(kr)],$$
$$P_{k1}(r)=(2/\pi)^{1/2}krj_1(kr), \quad P_{k2}(r)=(2/\pi)^{1/2}krj_2(kr). \quad (11)$$

The approximation (11) corresponds to the method developed in the article by Firsov and Chibisov.[9] Within the framework of this approximation, the dipole matrix elements of the $s \to p$ and $p \to s$ transitions take the form

$$R_{01}=\frac{2}{\pi}\int_0^\infty\sin(k'r+\delta_0(k'))\left(\cos kr-\frac{\sin kr}{kr}\right)r\,dr=\frac{4}{\pi}\frac{k^2\sin\delta_0(k')}{(k^2-k'^2)^2}, \quad (12)$$

$$R_{10}=\frac{2}{\pi}\int_0^\infty\sin(kr+\delta_0(k))\left(\cos k'r-\frac{\sin k'r}{k'r}\right)r\,dr=\frac{4}{\pi}\frac{k'^2\sin\delta_0(k)}{(k^2-k'^2)^2}. \quad (13)$$

The quadrupole matrix element D_{02} of the $s \to d$ transition is given by

$$D_{02}=\frac{2}{\pi}\int_0^\infty\sin(k'r+\delta_0(k'))\left\{\left(\frac{3}{k^2r^2}-1\right)\sin kr-\frac{3}{kr}\cos kr\right\}r^2dr=\frac{8}{\pi}\frac{k^4+k'^4}{k(k^2-k'^2)^2}\sin\delta_0(k'). \quad (14)$$

(In the calculation of these integrals we used the prescription given in[10], p. 418.) According to (12) and (5), the "partial" cross section of the inverse bremsstrahlung of the $s \to p$ transition is (in cm^4-sec):

$$\sigma_k{}^0(\omega)=\frac{8\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{\hbar^2}{\omega^3m^2}\left(\frac{k}{k'}\right)^3\sin^2\delta_0(k'). \quad (15)$$

We shall show now that at the indicated concentrations of the ions and atoms the relaxation time is $\tau(v) = \tau_a(v) = [N_a\sigma_a(v)v]^{-1}$. To prove this it suffices to show that $[N_i\sigma_i(v)]/[N_a\sigma_a(v)] \ll 1$. It is known that in a plasma[7]

$$\sigma_i=2\pi\left(\frac{ze^2}{mv^2}\right)^2\ln\lambda, \quad (16)$$

where $\ln\lambda$ is the so-called Coulomb logarithm and $\lambda = r_D/R_0$, $r_D = (T/4\pi e^2N_e)^{1/2}$ is the Debye radius, and r_0 is the larger of the two quantities ze^2/mv^2 or $\hbar/mv$. Substituting $z = 1$ in (16) and the velocity v corresponding to the energy 1.9 eV, we obtain $\sigma_i(v) \approx 2 \times 10^{-16}$ cm^2, whereas $\sigma_a(v) \approx \pi a_0^2 \approx 0.9 \cdot 10^{-16}$ cm^2. Thus, we actually have $\tau(v) = [N_a\sigma_a(v)v]^{-1}$.

Substituting (12), (14), and (15) in (10), we obtain the following expression for the drag current produced when light is absorbed by an electron in the field of a neutral atom:

$$j^a(\omega) = -\frac{2\pi}{15}|e|W\left(\frac{e^2}{\hbar c}\right)\frac{N_e}{\omega^3 c}\int_0^\infty \frac{\sigma^s(v')}{\sigma_a(v)} f(v')(v'^4+v''^4)dv', \qquad (17)$$

where $\sigma^S(v') = 4\pi \sin^2\delta_0(k')/k'^2$.

At the temperature T, an important role in the integral of (17) is played only by the values v' corresponding to an energy $\epsilon' \sim T$. Since $T \approx 0.026$ eV, we can take outside the integral sign the cross sections $\sigma^S(v')$ and $\sigma_a(v)$, which vary little in this interval. If it is recognized that $\sigma^S(v') = \sigma^S(0)$ and $\sigma_a(v) = \sigma_a(E_L)$ with a high degree of accuracy, then we have

$$j^a(\omega) = -\frac{8\pi}{15}|e|W\left(\frac{e^2}{\hbar c}\right)\left[\frac{N_e\varkappa}{E_L^2}\right]_{AU}\frac{\sigma^s(0)}{\sigma_a(E_L)}\left\{1+2\left\langle\left(\frac{v'}{v^*}\right)^2\right\rangle+2\left\langle\left(\frac{v'}{v^*}\right)^4\right\rangle\right\}, \qquad (18)$$

where $mv^{*2}/2 = \hbar\omega$ and the angle brackets denote averaging over the function $f(v')$ (AU stands for atomic units). Since $\langle (v'/v^*)^2\rangle \sim T/E_L$ and $\langle (v'/v^*)^4\rangle \sim (T/E_L)^2$, they can be neglected in comparison with unity. The final form of (18) is

$$j^a(\omega) = -\frac{8\pi}{15}|e|W\left(\frac{e^2}{\hbar c}\right)\left[\frac{N_e\varkappa}{E_L^2}\right]_{AU}\frac{\sigma^s(0)}{\sigma_a(E_L)}. \qquad (19)$$

Substituting the values of N_e, κ, and E_L in (19) and assuming by way of estimate that $W = 10^{22}$ cm^{-2} sec^{-1} and $\sigma^S(0) \approx \sigma_a(E_L)$, we obtain $j^a(\omega) \approx 0.3 \times 10^{-10}$ A/cm^2.

It can be shown that the drag current produced when light is absorbed by ions is much smaller than $j^a(\omega)$ defined by Eq. (19). To prove this, we estimate first the effective cross section $\sigma^i_{k'}(\omega)$ of the inverse bremsstrahlung by ions and $\sigma^a_{k'}(\omega)$ by atoms. To estimate $\sigma^i_{k'}(\omega)$ we use the classical expression for the bremsstrahlung cross section[11] (z = 1):

$$\frac{d\sigma}{d\omega} = \frac{16\pi}{3\sqrt{3}}\frac{e^6}{m^2c^3v^2\hbar\omega}\ [\text{cm}^2\ \text{sec}], \quad \frac{me^4}{\hbar^2} \gg \hbar\omega \gg \frac{mv^3}{e^2}\hbar. \qquad (20)$$

Since the quantum energy is $\hbar\omega = 1.8$ eV, we have according to (20): 2 Ry $\gg$ 1.8 eV > 1.12 eV.

Thus, expression (20) can be used to estimate the order of magnitude of the bremsstrahlung cross section. According to the detailed-balancing principle[6]

$$\sigma_{k'}{}^i(\omega) = \frac{\pi^2}{\varkappa^2}\left(\frac{v}{v'}\right)^2\frac{d\sigma}{d\omega} = \frac{16\pi^3}{3\sqrt{3}}\frac{e^6}{m^2cv'^2\hbar\omega^3}\ [\text{cm}^4\ \text{sec}]. \qquad (21)$$

Dividing (21) by (15) we have

$$\sigma_{k'}{}^i(\omega)/\sigma_{k'}{}^a(\omega) = 2\pi[k'/k^3]_{AU}/\sqrt{3} \approx 5, \qquad (22)$$

i.e., the cross section for inverse bremsstrahlung by an ion with z = 1 is ~5 times larger than the cross section for inverse bremsstrahlung by a neutral atom. Taking into account the fact that $\gamma \sim D/R \leq (10$ to $20)a_0$, we easily see that at the indicated concentrations of the ions and of the neutral atoms $j^i(\omega)$ is less than $j^a(\omega)$ by 10^2–10^3 times. The decrease of the concentration N_e, of course, enhances the inequality $j^a(\omega) \gg j^i(\omega)$, and $N_e = 10^{14}$ cm^{-3} is the maximum density of the electrons at which the plasma remains ideal at room temperatures.

We call attention to the following: According to the Saha relation[7], under the conditions of thermodynamic equilibrium, we have

$$\frac{N_eN_i}{N_a} = \frac{g_eg_i}{g_a}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}e^{-I/T}, \qquad (23)$$

where g_i, g_e, and g_a are the statistical weights of the ions, electrons, and atoms, respectively. It is easy to see from (23) that if $I \sim 1$ Ry, then at $T \approx 300$°K the gas is practically not ionized. To produce concentrations $N_e = N_i = 10^{14}$ cm^{-3} we can, as indicated by Zel'dovich and Raĭzer,[12] introduce into the gas an impurity with a small ionization potential (e.g., Cs with I = 3.8 eV), which will be strongly ionized in the laser beam, and will yield the necessary electron density N_e.

Thus, in the situation considered in Sec. 3, the drag current reaches $\sim 10^{-10}$ A/cm^2 and consequently is perfectly accessible to experimental investigation.

4. We consider one more situation, in which $j^a(\omega) \gg j^i(\omega)$ and $j^a(\omega)$ can be measured. Let us determine the drag current produced in a helium or neon plasma heated to T = 1 eV = 11606°K (I ≈ 22–25 eV) under the influence of laser radiation with $E_L = 0.12$ eV (CO_2 laser). According to relation (23), at $N_a \approx 10^{19}$ cm^{-3} we have $N_i = N_e \approx 10^{15}$ cm^{-3}. We compare $\sigma^i_k(\omega)$ and $\sigma^a_k(\omega)$ for the electron energy ~1 eV and the quantum energy $E_L = 0.12$ eV. In this case $\sigma^i_k(\omega)$ cannot be estimated from formula (21), since $\omega \ll mv'^3/e^2$. To determine the cross section $\sigma^i_k(\omega)$ near the long-wave boundary we shall use the classical expression for the bremsstrahlung of an electron in the infrared region of the spectrum[11] (z = 1):

$$\frac{d\sigma}{d\omega} = \frac{16e^6}{3v'^2c^3m^2\hbar\omega}\ln\left(\frac{2mv'^3}{\gamma e^2\omega}\right)\ [\text{cm}^2\ \text{sec}], \qquad mv'^3/e^2 \gg \omega, \qquad (24)$$

where $\gamma = e^C$, C is Euler's constant, and $\gamma = 1.78\ldots$

Using the principle of detailed balancing, we can determine $\sigma^i_k(\omega)$ with the aid of (24). To calculate $\sigma^a_k(\omega)$, i.e., the effective cross section of the inverse bremsstrahlung by a neutral atom, we use the fact that $\epsilon' \approx \epsilon \gg E_L$. In this case, according to Johnston[13], the inverse-bremsstrahlung cross section can be expressed in terms of the transport cross section of elastic scattering of an electron by a neutral atom

$$\sigma^{tr}(k') = \int(1-\cos\theta_{k'})|f(\theta_{k'})|^2d\Omega_{k'} = \frac{4\pi}{k'^2}\sum_{l'=0}^\infty(l'+1)\sin^2(\delta_{l'}(k')-\delta_{l'+1}(k')). \qquad (25)$$

In this case $\sigma^a_k(\omega)$ takes the form

$$\sigma_{k'}{}^a(\omega) = \frac{16\pi^2}{3}\left(\frac{e^2}{\hbar c}\right)\frac{a_0^5}{v'}\left[\frac{k'}{E^3}\right]\sum_{l'=0}^\infty(l'+1)\sin^2(\delta_{l'}(k')-\delta_{l'+1}(k')). \qquad (26)$$

Here a_0 is the Bohr radius. Comparison of $\sigma^i_k(\omega)$ obtained from (24) with $\sigma^a_k(\omega)$ from (26) shows that $j^a(\omega) \gg j^i(\omega)$ at the indicated concentrations.

We note that (26) was obtained from formula (5) in which, in the calculation of the dipole matrix elements, the wave functions $P_{kl}(r)$ were replaced by their proper asymptotic forms (8). In this approximation,

$$R_{l,l\pm1} = \mp\frac{\hbar^4k^2}{\pi m^2E_L^2}\sin(\delta_{l\pm1}-\delta_l), \qquad (27)$$

$$D_{l,l\pm2} = -\frac{2\hbar^6k^3}{\pi m^3E_L^3}\sin(\delta_{l\pm2}-\delta_l), \quad D_{l,l}=0. \qquad (28)$$

Substituting (27) and (28) in (10), we obtain the following expression for the drag current:

$$j^a(\omega)=-\frac{8\pi^2}{5}\left(\frac{e^2}{\hbar c}\right)|e|W\int_0^\infty f(v)\left[\frac{v^2\varkappa N_e}{\sigma_a(v)E_L^4}\right]_{AU}\sum_{l=0}^\infty\frac{(l+1)(l+2)}{(2l+3)}$$
$$\times\sin^2(\delta_{l+2}(v)+\delta_l(v)). \quad (29)$$

To estimate the current in this case we confine ourselves in (29) to allowance for the s-phase only. Then (29) goes over into formula (17), in which $v = v'$. Assuming by way of estimate $\sigma_a(v) \approx \sigma^s(v)$ and recognizing that $\langle v^4 \rangle = 15T^2/m^2$. We obtain at $W = 10^{22}$ cm^{-2} sec^{-1} the value $j^a(\omega) \approx 2.5 \times 10^{-6}$ A/cm^2.

Thus, in this case, too, the drag current is perfectly accessible to experimental investigation.

5. In conclusion, let us compare the drag current produced in the case of inverse bremsstrahlung of light, with the current due to the Compton scattering. It is obvious that their values will be of the same order if

$$\sigma_{cl}N_e\approx\sigma^{ff}N_eN_a, \quad (30)$$

where $\sigma_{cl} = (e^2/mc^2)^2$ and σ^{ff}[cm^5] is the cross section of the free-free transition. Recognizing that $\sigma^{ff} \approx 10^{-40}$ cm^5, we obtain $N_a \approx 10^{15}$ cm^{-3}. Thus, the Compton dragging begins to compete with the dragging in inverse bremsstrahlung at $N_a \lesssim 10^{15}$ cm^{-3}.

We call attention to the following circumstance. According to (19) and (29), the drag current is inversely proportional to the cross sections for elastic scattering of electrons by the gas atoms, and consequently the experimental study of the current makes it possible not only to verify the correctness of the employed models, but also to study the electron scattering lengths at low energies, and to obtain information on the phases and the matrix elements in free-free transitions.

We note that formula (10) admits of a reversal of the sign of the current, but in the approximations considered here the drag current does not reverse sign and flows in the direction opposite that of the photon momentum.

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