Nonstationary acoustoelectric effects in propagation of transverse sound in superconductors

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Vortex currents with both normal and superconducting components appear in a superconductor when transverse sound is propagated in it. The presence of a superconducting component of the vortex current affects the order parameter of the superconductor. We consider two effects associated with this. The first effect is the appearance of a nonstationary phase shift of the order parameter in a suitably designed superconducting measuring circuit, when the latter is connected to the superconductor. In this case, the phase shift is proportional to the sound amplitude. The second effect involves the presence of a current produced in the superconductor by nonelectromagnetic processes; it is connected with the influence of the vortex currents on the modulus of the order parameter. The nonstationary phase difference recorded by the measuring circuit in this case is proportional to the square of the sound amplitude and varies at double the sound frequency.

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It has recently been shown in a number of papers that a phase difference in the order parameter can exist at the ends of a superconductor located in a nonequilibrium state. Such is the case, in particular, in the presence of a current of normal excitations, produced in a nonequilibrium state of the superconductor. The phase difference in this case is connected with the appearance of a volume current of the superconducting condensate, which compensates the current of normal excitations.[1,2] It is possible to measure such a phase difference either directly, by placing the sample in a superconducting circuit with weak coupling, or by an indirect method. Thus, if we include the sample with the "nonequilibrium" phase difference in a closed, non-singly-connected superconducting circuit, then an unquantized contribution to the magnetic flux passing through it is produced.[2–4] The generation of such a contribution in the case in which the normal current has a thermoelectric nature was studied experimentally by Zavaritskiĭ.[3] Thus the nonequilibrium phase difference is an observable quantity.

In a recently published paper,[5] the development of oscillations of the phase χ of the order parameter was predicted in the propagation of longitudinal sound along a thin superconducting conductor, the thickness of which is smaller than the penetration depth of the magnetic field. These oscillations are connected with the presence of a dependence of the modulus of the order parameter Δ on the relative change in the volume div $\mathbf{u}$ in deformation in the sound wave ($\mathbf{u}$ is the lattice displacement vector). As a consequence of this, the modulus of the order parameter and consequently, the effective number of "superconducting" electrons N_s[6] are modulated by the sound wave. The electric current in the superconductor is proportional to the product of N_s by the phase gradient of the order parameter, $\nabla\chi$. Therefore, if an extraneous current passes through the superconductor, by virtue of the equation of continuity, the phase gradient of the order parameter is also modulated by the sound disturbance. Consequently, between points of the sample separated by distances equal to an odd number of half wavelengths, a nonstationary phase difference is developed. It is not difficult to establish the fact that a similar effect should also occur in a bulky superconductor, and the order of this quantity should be about the same as in the thin wire.

The aim of our note is to consider two effects similar to that described above, which arise in the propagation, along a superconductor, of transverse sound, not accompanied by a relative change in the volume.

The first of these can be called the nonstationary acoustoelectric effect. In the propagation of transverse sound in a superconductor, vortex currents arise, directed into the interior of the superconductor, perpendicular to the sound wave vector and the surface of the sample (see [7]). These currents are closed in a thin near-surface layer of thickness of the order of the penetration depth. The vortex currents have both a normal and a superconducting component. The latter, as is well known,[6] is proportional to the so-called "superconducting" velocity $\mathbf{v}_s$:

$$\mathbf{v}_s=\frac{\hbar}{2m}\left(\nabla\chi-\frac{2e}{\hbar c}\mathbf{A}\right), \quad (1)$$

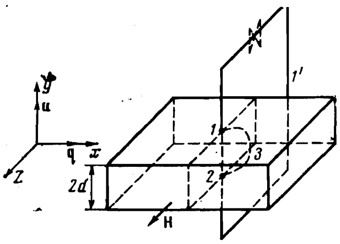
where e and m are the charge and effective mass of the electron, c is the velocity of light, and $\mathbf{A}$ is the vector potential of the electromagnetic field. However, since the superconducting currents that are generated are vortical, it is not generally possible to introduce the superconducting velocity potential and consequently a gauge-invariant phase difference. Actually, the quantity

$$\theta_{12}=\int_1^2 d\mathbf{r}\left(\nabla\chi-\frac{2e}{\hbar c}\mathbf{A}\right)$$

depends on the path of integration and in this sense there is no analogy with the case considered above. At the same time, we see that in the expression for the observed quantities, measured in corresponding fashion by the construction of a measuring circuit, an integral appears that is calculated along an entirely determined path.

As an example, we consider the measuring circuit shown in the drawing, that is essentially analogous to that considered in [2,3]. We assume that the magnetic field, created by the vortex currents that accompany the sound wave, does not permeate the measurement circuit 1-1′-2. This can be achieved, for example, by locating this circuit in the yz plane, as is shown in the drawing. We can then write down the usual expression for the superconducting current density $\mathbf{j}_s$:

$$\frac{2m}{eN_s\hbar}\mathbf{j}_s=\left(\nabla\chi-\frac{2e}{\hbar c}\mathbf{A}\right). \quad (2)$$



Since $j_S = 0$ in the interior of the circuit outside the sample, by integrating this relation over the closed circuit 1-1′-2, we obtain

$$2\pi\frac{\Phi}{\Phi_0} = 2\pi n - \int_1^2 \frac{2m\mathbf{j}_s}{eN_s\hbar}\,d\mathbf{r}, \tag{3}$$

where Φ is the magnetic flux passing through the circuit and Φ_0 is the magnetic flux quantum. Thus, for example, by measuring the current in the measurement circuit (and, consequently, the magnetic flux), we determine the quantity

$$\theta_\perp = \int_1^2 \frac{2m\mathbf{j}_s}{eN_s\hbar}\,d\mathbf{r}.$$

The quantity $\theta_\perp$ is directly connected with the value of the superconducting current in the given cross section of the sample at a specified instant of time. One can conceive of other types of measurement schemes with the use of weak coupling, such as Clark galvanometer or the Zimmerman-Silver and Mercereau interferometers. It is important to note that for any type of measuring circuit in which: 1) the contacts are located in a plane perpendicular to the wave vector of the sound, and 2) the measuring circuit is not linked with the magnetic field of the sound wave, the result of the measurement will be expressed in terms of the quantity θ, which is directly connected with the superconducting current. As is easy to see from our discussions and Eq. (3), from the point of view of the measuring circuit, the sample behaves as a source of nonstationary phase difference[1)] $\theta_\perp$. In particular, if we insert in the measurement circuit a Josephson junction with a sufficiently small critical current, then a phase difference $2\pi n - \theta$ will exist at the junction (see the Appendix). In this case, there is an analogy with the effects considered in [1,2]. We see that, since the current lines are closed near the surface of the sample, there exist components of the superconducting current parallel to the surface. One can imagine a measuring circuit with contacts on the same face of the sample, in which is measured the quantity $\theta_\parallel$ connected with the parallel component of j_S just as the quantity $\theta_\perp$ is connected with the perpendicular component.

We now proceed to the second effect, which is in known degree similar to the effect suggested in [5]. It is well known ([8], p. 130) that the value of the modulus of the order parameter of the superconductor depends on the currents flowing in it. Therefore the vortex currents should bring about a change in the modulus of the order parameter. We now assume that a lateral current flows along the superconductor. This current can be described with the help of the phase, since the field of the velocities v_S corresponding to this current, is a potential one. In this case, just as in [5], oscillations should arise in the phase difference of the order parameter between two points which lie on the same side of the sample. The frequency of these oscillations, as we shall show, is equal to twice the sound frequency, 2ω.

Thus, our aim is to investigate the two effects noted and to compare them with the effect suggested in [5].

We consider a plate of a superconductor with an isotropic energy spectrum, in which transverse sound is propagated, polarized perpendicular to the surface of the sample (see the figure). Let

$$u \sim \exp i(qx - \omega t),$$

and we assume that the conditions

$$q\xi_0 \ll 1, \quad l \gg \xi_0,$$

are satisfied, where l is the mean free path of the electrons in the normal state and ξ_0 is the coherence length at zero temperature. If these conditions are satisfied we can obtain the following expression for the electric current density $\mathbf{j}$ (with the help of the kinetic equation, or by the density matrix method)

$$\mathbf{j}_q = \lambda e N_0 \dot{\mathbf{u}}_q\left(1 - \frac{\sigma_q^I}{\sigma_0}\right) - \frac{c}{4\pi}K_q\left[\tilde{\mathbf{A}}_q + \frac{mc}{e}\left(\lambda + \frac{m_0}{m}\right)\dot{\mathbf{u}}_q\right]. \tag{4}$$

Here N_0 is the total concentration of the electrons, m_0 is the mass of the free electron, λ is the dimensionless constant of the deformation potential (the transverse part of the tensor of the deformation potential is written in the form $\lambda p_i p_k/m$), $\tilde{\mathbf{A}}$ is the gauge-invariant combination

$$\tilde{\mathbf{A}} = \mathbf{A} - \frac{\hbar c}{2e}\nabla\chi,$$

K_q is the response function of the superconductor, determined by the expression[2)]:

$$K_q = \frac{1}{\delta^2}\left[\frac{N_s}{N_0} + i\frac{m\omega\sigma_q}{N_0 e^2}\right], \tag{5}$$

$\delta^2 = 4\pi e^2 N_0/mc^2$ is the London penetration depth for $T = 0$ (T is the temperature in energy units); the kinetic coefficients σ_q^I and σ_q are introduced in [9]. Near the transition point T_c they are of the order of the conductivity in the normal state σ_0 for $ql \ll 1$ and $\sim\sigma_0/ql$ for $ql \gg 1$, and in the case of a decrease in the temperature they fall off exponentially as $\exp(-\Delta/T)$. The second term in the square brackets in (5) in almost no case exceeds 10^{-3} (for $ql \gg 1$ it is of the order of w/v_F, and for $ql \ll 1$ it is of the order of $\omega\tau \ll w/v_F$; w is the sound velocity, v_F is the velocity of the electron on the Fermi surface, τ is the relaxation time of the momentum of the electron in the normal state). We shall assume that the relative closeness to the transition point is $\Theta = (T_c - T)/T_c \gg 10^{-3}$; consequently, the second term can be neglected.

The first term in the expression for the current (4) can be interpreted as the current associated with the incomplete dragging of the electrons by the moving periodic potential of the lattice. Actually, in the case of frequent collisions of the electrons with impurities moving together with the lattice, the mean velocity of the electrons relative to the lattice should be equal to zero. If we use the definition of the quantity σ^I from [9], it is easy to establish the fact that as $ql \to 0$ the first term in (4) also tends to zero. The second term in the expression for the current has the form of the response of the current to the electromagnetic field with the vector potential

$$\tilde{\mathbf{A}}_q + \frac{mc}{e}\left(\lambda + \frac{m_0}{m}\right)\dot{\mathbf{u}}_q.$$

According to [9,10], in the case of transverse sound, the role of the characteristic momentum of the superconducting condensate, which enters into the self-consistent equation, is played by the quantity $\mathbf{p}_S = -e\tilde{\mathbf{A}}/c - m_0\mathbf{u}$. The electric current $\mathbf{j}$ is connected with this quantity by the relation

$$\frac{m}{eN_0}\mathbf{j}_q=\lambda m\dot{\mathbf{u}}\left(1-\frac{\sigma_q^I}{\sigma_0}-\frac{N_s}{N_0}\right)+\frac{N_s}{N_0}\mathbf{p}_s. \quad (6)$$

If we use the expression for the ratio σ_q^I/σ_0 obtained in [9], the relation (6) can be rewritten in the form

$$\frac{m}{eN_0}\mathbf{j}_q=\lambda\dot{\mathbf{u}}mf(ql)(1-n)+n\mathbf{p}_s, \quad (7)$$

where for brevity the relative number of superconducting electrons N_S/N_0 is denoted by the letter n, and

$$f(x)=\frac{2x^3-3x-3(1+x^2)\operatorname{arctg}x}{2x^3}\begin{cases}\to 1 & \text{as } x\to\infty\\ \to {}^1/_3x^2 & \text{as } x\ll 1\end{cases} \quad (8)$$

The condition for electric neutrality div $\mathbf{j}$ = 0 takes in our notation the form

$$\operatorname{div}\mathbf{p}_s=-(\mathbf{p}_s\nabla\ln n)+\lambda mfn(\dot{\mathbf{u}}\nabla\ln n). \quad (9)$$

The distribution of the "superconducting" momentum p_S over the sample is described by the Maxwell equations. With account of (9), these equations can be rewritten in the form

$$-\nabla^2\mathbf{p}_s-\nabla(\mathbf{p}_s\nabla\ln n)+\frac{n}{\delta^2}\mathbf{p}_s=m\lambda f\left[\frac{n-1}{\delta^2}\dot{\mathbf{u}}-\nabla n(\dot{\mathbf{u}}\nabla\ln n)\right]+m_0\nabla^2\dot{\mathbf{u}}. \quad (10)$$

The second terms on the left and right sides of Eq. (10) have an essentially nonlinear nature. They are connected with the dependence of the relative number of superconducting electrons n on the coordinates and, in the linear approximation in the sound amplitude, they are absent.

The boundary condition for Eq. (10) is the vanishing of the y component of the total current on the boundaries. For the variable p_S, this condition takes the form

$$p_{s,y}\Big|_{y=\pm d}=\lambda m\dot{u}_yf\frac{n-1}{n}\Big|_{y=\pm d}. \quad (11)$$

We first consider the case in which there is no extraneous current. For reasonable values of the sound intensity, the value of $p_S v_F/\Delta$, which determines the effect of the current on the modulus of the order parameter, turns out as a rule to be small. In the lowest, linear approximation in this ratio, we can assume the quantity n to be constant, equal to the thermodynamic equilibrium value. In this case, the solution of the system (9) and (10) with boundary conditions (11) gives

$$p_{s,y}^{\,0}=m\dot{u}\left\{\lambda f\frac{n-1}{n}-(q\Lambda)^2\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right)\left(1-\frac{\operatorname{ch}(y/\Lambda)}{\operatorname{ch}(d/\Lambda)}\right)\right\}, \quad (12)$$

$$p_{s,x}^{\,0}=im\dot{u}(q\Lambda)\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right)\frac{\operatorname{sh}(y/\Lambda)}{\operatorname{ch}(d/\Lambda)}, \quad (13)$$

where

$$\Lambda=(q^2+n/\delta^2)^{-1/2}.$$

For $q\Lambda \ll 1$ ($q\delta \ll \sqrt{n}$) we have $p_{Sx}^0 \ll p_{Sy}^0$ and the quantity p_{Sy}^0 can be assumed to be independent of the coordinate y. Such a solution means the following: at each point of the crystal the total current is equal to zero—the drag current of normal excitations is entirely compensated by the superconducting current. In this connection, we note the following circumstance. Equations (12) and (13) were obtained without account of the spatial dispersion N_S and therefore, generally speaking, are valid only for type II superconductors. However, the expression for the part of the superconducting current which compensates the volume current of the normal excitations (the first term in (12)), remains valid in the case of type I superconductors. At finite values of $q\Lambda$, the currents are not fully compensated, and a nonvanishing current exists in the interior of the superconductor:[3)]

$$j_y=eN_0\dot{u}(q\Lambda)^2\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right)\left(\frac{\operatorname{ch}(y/\Lambda)}{\operatorname{ch}(d/\Lambda)}-1\right),$$

$$j_x=eN_0\dot{u}i(q\Lambda)\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right)\frac{\operatorname{sh}(y/\Lambda)}{\operatorname{ch}(d/\Lambda)}. \quad (14)$$

Such a current leads to the appearance of a magnetic field[4)] H:

$$H_z=iq\Lambda^2\frac{4\pi eN_0\dot{u}}{c}\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right). \quad (15)$$

In the interior of a bulky superconductor, far from the transition point, and at q ~ 10^3 cm^{-1}, and at a sound intensity S ~ 1 W/cm^2, the amplitude of this field is of the order of 5×10^{-3} Oe. It seems that such a magnetic field lends itself to experimental observation. We note that the presence of a magnetic field in the interior of a superconductor in no way contradicts the Meissner effect, and is a consequence of the appreciable inhomogeneity of the normal current.

We now estimate the quantity θ, which has the meaning of the phase difference for the external (measuring) circuit. Using (12) and (13), we have the following estimates for $\theta_\perp$ and for the value of $\theta_\parallel$ corresponding to points on one face of the sample, separated by a distance equal to an odd number of half wavelengths:

$$\theta_\perp=\frac{2md\dot{u}}{\hbar}\lambda f\frac{n-1}{n},$$

$$\theta_\parallel=\frac{m\Lambda\dot{u}}{\hbar}\left(\lambda f\frac{n-1}{n}+\frac{m_0}{m}\right). \quad (16)$$

In the derivation of (16), we assume $d \gg \Lambda$. Taking $\Lambda \sim 10^{-5}$ cm, d ~ 1 cm, λ = 3, n ~ 1, S ~ 1 W/cm^2, and also, assuming $ql \gtrsim 1$, we get: $\theta_\perp \sim 10$, $\theta_\parallel \sim 10^{-4}$. Near the transition point, for $\Theta \ll 1$, the value of $\theta_\perp$ increases by a factor Θ^{-1}, and $\theta_\parallel$ by a factor of $\Theta^{-1/2}$. Thus, the effect is rather large, especially in the transverse direction. Thus, for $\Theta = 10^{-2}$, a phase difference of several degrees appears in the longitudinal direction for a sound intensity 10^{-6} W/cm^2, and in the transverse direction, the value of θ can evidently serve as a very sensitive method for the determination of the transverse sound intensity. We again recall that $\theta_\perp$ depends periodically on time with the frequency of the sound frequency.

We now consider the case of the presence of an external current. Here

$$\mathbf{p}_s=\mathbf{P}+\mathbf{p}_s^{(1)}+\mathbf{p}_s^{(2)}. \quad (17)$$

The "superconducting" momentum $\mathbf{P}$ here is created by the external current (this current can be of the order of the critical current), $p_S^{(1)}$ and $p_S^{(2)}$ are the linear and quadratic terms, respectively, of the expansion in the sound amplitude. The momentum $\mathbf{P}$ is parallel to the sound wave vector $\mathbf{q}$. It is easy to see that for $q\delta \ll \sqrt{n}$ (which we shall assume to be satisfied) we can assume that $p_S^{(1)} \perp q$ (since $p_{Sx}^{(1)} \ll p_{Sy}^{(1)}$). In order to calculate the phase change associated with the presence of the external current, we must integrate the system (9) and (10) up to second order in the amplitude of the sound wave. It should be taken into account here that the quantity n depends on the square of the modulus of the vector p_S or, more precisely, on the dimensionless quantity

$$\alpha=\frac{v_F^2}{\Delta^2}|p_s|^2=\frac{v_F^2}{\Delta^2}[P^2+(p_s^{(1)})^2+2Pp_s^{(2)}]. \quad (18)$$

We shall assume that the modulus of the order parameter follows adiabatically the change in this quantity.[5)]

It is simplest to obtain a solution in the case of a thin film, when all the derivatives with respect to the y coordinate can be omitted. In this case, iteration of the system (9) and (10), as is easy to verify, gives the following result:[6)]

$$p_{s_x}^{(2)}=-\frac{n'/n_0}{1+2(Pv_F)^2n'/(n_0\Delta^2)}P\left[\left(\frac{p_F\lambda f(n-1)}{\Delta n}\right)^2(\dot{u}^2-\langle\dot{u}^2\rangle)\right] \quad (19)$$

(the angle brackets denote averaging over the time). A result of the same order of magnitude is obtained in the case of a bulky sample.

The result (19) has a lucid physical meaning. The expression with the square bracket determines the relative change in the order parameter. It follows from the equation of continuity that the change in the "superconducting" velocity should be proportional to the product of the external momentum P and the relative change in the order parameter. In essence, this effect is analogous to that predicted in [5]; however, in our case, it is quadratic in the sound amplitude and varies with time at twice the sound frequency.

We now estimate the characteristic phase change over a distance equal to an odd number of half waves. Assuming $n'/n_0 \sim 1$ and $Pv_F/\Delta \sim 1$, we have

$$\theta_{\parallel}^{(2)}\sim\frac{\Delta}{\hbar qv_F}\left[\frac{p_F\lambda f(n-1)}{\Delta n}\dot{u}\right]^2=\frac{p_F}{\hbar q}\frac{m\dot{u}^2}{\Delta}\left(\frac{\lambda f(n-1)}{n}\right)^2. \quad (20)$$

The case of low frequencies and a pure superconductor is the most favorable (the condition $ql \gtrsim 1$ is necessary so that the factor f will not be small). Under these assumptions, assuming $q \sim 10^{-1}$ cm and $S \sim 10$ W/cm^2, far from the transition point ($n \sim 1$) we have $\theta_{\parallel}^{(2)} \sim 3\times10^{-3}$. This effect is approximately of the same order as that predicted in [5] (for sound of the same frequency). However, the effects have different temperature dependences near T_c. Thus the effect predicted in [5] is proportional to $\theta^{-1/2}$ near T_c, while the effect predicted in the present paper is proportional to $\theta^{-5/2}$. It is therefore natural to expect that near T_c the transverse sound modulates the phase of the order parameter more effectively than the longitudinal sound.

If the extraneous current changes with time, then, as was noted in [5], the phase difference will contain combination frequencies. In particular, a constant component of the phase difference exists when the frequency of change of the current equals twice the sound frequency.

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APPENDIX

Let a Josephson junction be connected in the circuit 1-1'-2 (it is shown as a dashed line in the figure). It is then easy to verify that in place of (3) we have (ρ is the phase difference at the junction):

$$2\pi\frac{\Phi}{\Phi_0}=2\pi n-\rho-\int_1^2\frac{2m\mathbf{j}_s}{eN_s\hbar}d\mathbf{r}. \quad (A.1)$$

On the other hand, if the magnetic field of the vortex currents does not pass through the circuit,

$$\Phi=\frac{1}{c}LI_c\sin\rho,$$

where L is the inductance of the circuit and I_c is the critical transition current. Thus we have

$$\frac{2\pi LI_c}{\Phi_0}\sin\rho+\rho=2\pi n-\theta_{\perp}. \quad (A.2)$$

If the dimensionless parameter $2\pi LI_c/\Phi_0$ is small, then the phase difference at the junction is $2\pi n - \theta_{\perp}$.

We note that the result would not change if we were to choose another path of integration inside the sample, for example, the circuit 1-3-2. Actually, the integral over the closed path

$$\int_{1-3-2-1}\frac{2m\mathbf{j}_s}{eN_s\hbar}d\mathbf{r}=-\frac{2e}{\hbar}\oint_{1-3-2-1}\mathbf{A}\,d\mathbf{r}$$

is completely compensated by the change in the magnetic flux through the circuit 1-1'-2-3-1 in comparison with the flux through the circuit 1-1'-2-1.

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1)We note that this picture is very similar to the measurement of the voltage at the transverse boundaries of a normal metal in a similar situation.

2)We shall discuss below the problem of the necessity to account for spatial dispersion of the real part of the response function.

3)We note that these formulas do not admit of limiting transition to the case of a normal metal since we have neglected the second term in (5) which is important as $n \to 0$.

4)We emphasize again that Eqs. (14) and (15) are exact for a type II superconductor and have the character of estimates for type I superconductors.

5)Such an assumption is valid when the sound frequency is less than the reciprocal of the relaxation time of the order parameter, associated with the presence of inelastic processes. [11]

6)We note that $p_{sx}^{(1)} = 0$ in the zeroth approximation in the parameter $q\Lambda$.

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