Anomalous skin-effect in a metal in an inclined magnetic field

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We find an explicit solution of the kinetic equation for the electron distribution function in an inclined magnetic field for an arbitrary coefficient for the reflection of electrons from the metal surface. We evaluate the asymptotic value of the current density in the range of small angles of inclination. We obtain asymptotically exact solutions of the problem of the anomalous skin effect and evaluate the surface impedance in various limiting cases.

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1. INTRODUCTION

Reuter and Sondheimer[1] were the first, in 1948, to solve the problem of the penetration of an electromagnetic field into a metal under anomalous skin effect conditions. They obtained exact formulae for the field distribution and the surface impedance in two limiting cases—specular and diffuse reflection of the electrons from the metal surface. An asymptotic integral equation was obtained in [2] for the Fourier transform of the electromagnetic field in the metal for arbitrary coefficients for the reflection of electrons from the boundary. Subsequently Hartmann and Luttinger[3] used a Mellin transform to solve this equation. Fal'kovskiĭ[4] found small corrections to the surface impedance of the metal, necessitated by the scattering of electrons by a rough boundary with smooth inhomogeneities. For the normal skin effect these corrections turn out to be small because the mean free path l is appreciably shorter than the skin-layer thickness δ. However, in the case of an anomalous skin effect ($\delta_a \ll l$) the main role is played by glancing electrons for which the scattering by the surface is practically specular.

Hartmann and Luttinger[3] found also an exact solution of the problem of the anomalous skin effect in a magnetic field parallel to the metal surface for diffuse scattering of the electrons. The corresponding asymptotic integral equation was obtained in [5]. In [6–9] the influence was studied of the nature of the scattering of the electrons by the surface of the sample on the anomalous skin effect and the cyclotron resonance in a parallel magnetic field. This study showed that in the region of strong fields and close to cyclotron resonances the nature of the scattering of the electrons by the surface does not play an important role provided the reflection coefficient ρ is not close to unity. The case of specular reflection $\rho = 1$ is a singular one as then there is in the metal a group of so-called surface electrons which are grazing along the surface of the sample due to multiple collisions with the boundary. We must also note the work of Azbel' and Kaganov[10] who found the surface impedance of a metal in a normal magnetic field for $\rho = 0$ and $\rho = 1$. We emphasize that all these problems involved essentially an exact solution of the kinetic equation for the electron distribution function for arbitrary ρ.

In the case of an inclined magnetic field, apart from the difficulties of solving the electrodynamic problem, even finding the distribution function and calculating the current density is an extraordinarily complex problem. These difficulties are caused by the fact that every collision of an electron with the surface changes the form of the electron trajectory. This change continues until the electron moves sufficiently far into the metal where it does not collide at all with the surface. We consider the motion of an electron in an inclined magnetic field. Figure 1 illustrates the motion of an electron in velocity space, taking into account its scattering by the boundary. Let the first collision with the boundary occur at the point A_1 on the Fermi sphere. After the collision the electron goes to the point A_2 which is the mirror image of the point A_1 with respect to the equatorial plane $v_\eta = 0$ (the η axis is in the direction of the normal to the boundary). In the new cross section it rotates until it reaches the point A_3 in which it collides again with the metal surface. After this the electron goes into the state A_4, the mirror image of the point A_3. In the new orbit the electron collides for the third time with the boundary at the point A_5 and goes into the state A_6. In that orbit the electron does no longer collide at all with the boundary. If the number of collisions is large the nature of such a migration of the electron over the Fermi sphere turns out to be rather quaint and its analytical description is difficult.

From a mathematical point of view the difficulties of taking multiple collisions with the surface into account are caused by the non-conservation of the component of the electron momentum p_H along the magnetic field on reflection. In other words, the inclination of the magnetic field reduces the symmetry of the problem and as a consequence one of the integrals of motion (p_H) disappears. We note that for diffuse reflection it is sufficient for the determination of the distribution function to know only the moment of the last collision with the surface. For non-diffuse scattering the electron "remembers" all collisions, and the distribution function must be determined from a very complicated functional relation which takes this fact into account.

Azbel' and Rakhmanov[11] discussed the problem of the effect of the nature of the reflection of the electrons

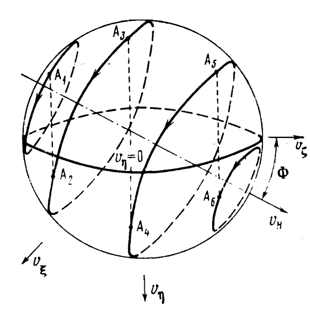

FIG. 1. Motion of an electron on the Fermi sphere, taking into account multiple collisions with the surface of the metal. The points A_1, A_3, and A_5 correspond to those values of the electron momentum at which it collides with the boundary.

by the boundary on the skin effect in a metal in an inclined magnetic field. However, the authors of [11] studied only the region of strong magnetic fields when $\delta \gg R$ (δ is the skin layer thickness and R the cyclotron radius). Shekter and we[12] considered the anomalous skin effect for $\delta \ll l$ in an inclined magnetic field for the case of a diffuse boundary. As far as we know the effect of the reflection of the electrons on the anomalous skin effect in an inclined magnetic field has not yet been studied. It is probable that this is due to the difficulties, mentioned above, of solving the kinetic problem.

In the present paper we find an exact solution of the kinetic equation for the distribution function of the electrons in a metal placed in a magnetic field oriented at an arbitrary angle Φ to the boundary. The scattering of the electrons by the boundary is characterized by a specularity coefficient ρ, with arbitrary magnitude and independent of the electron momentum **p**. Moreover, we obtain an asymptotically exact solution of the problem of the anomalous skin effect for small angles Φ. We study the contribution to the surface impedance from electrons from the vicinity of the central cross section of the Fermi surface which drift along the boundary.

2. STATEMENT OF THE PROBLEM AND SOLUTION OF THE KINETIC EQUATION

We consider a metal with a spherical Fermi surface. The magnetic field **H** is oriented at an angle Φ to the boundary. The $\xi\eta\zeta$ coordinate system is fixed to the metal surface $\eta = 0$; the η axis is parallel to the outward normal to the surface and the ζ axis is directed along the projection of the vector **H** on the $\eta = 0$ plane.

We write down the Maxwell equations in the metal for the spatial Fourier components of the electrical field

$$k^2\mathscr{E}_\mu(k)+2E_\mu'(0)=4\pi i\omega c^{-2}j_\mu(k) \quad (\mu=\xi,\ \zeta), \tag{2.1}$$

$$j_\eta(k)=0. \tag{2.2}$$

We continue the field strength $\mathbf{E}(\eta)$ and the current density $\mathbf{j}(\eta)$ into the region $\eta < 0$ outside the metal in even fashion and introduce the following notation:

$$\mathscr{E}_\mu(k)=2\int_0^\infty d\eta\cos k\eta E_\mu(\eta),\quad E_\mu(\eta)=\pi^{-1}\int_0^\infty dk\cos k\eta\mathscr{E}_\mu(k), \tag{2.3}$$

ω is the wave frequency, k the wavenumber, and the prime on E_μ indicates the derivative $\partial/\partial\eta$.

The fact that it is possible to continue the current

$$\mathbf{j}(\eta)=-\frac{2e}{(2\pi\hbar)^3}\int d^3p\mathbf{v}f(\eta,\mathbf{v})$$

as an even function is caused by the fact that $\mathbf{j}(\eta)$ is in fact defined only antisymmetrically with respect to the velocity part of the distribution function

$$\Psi(\eta,\mathbf{v})=f(\eta,\mathbf{v})-f(\eta,-\mathbf{v}).$$

It has been shown earlier[5] that the kinetic equation for the function $\Psi(\eta,\mathbf{v})$ is a second order differential equation which is symmetric under the transformation $\eta\to-\eta$. Changing in that equation to the Fourier transforms with respect to the η coordinate we find easily (see [5,6])

$$\psi(k,\theta,\tau)=2\int_{-\infty}^{\tau}d\tau'\exp\gamma(\tau'-\tau)\cos[kR\alpha(\tau',\tau)]\Big\{g(k,\theta,\tau') - \frac{R|n_\eta(\theta,\tau')|}{1-\rho^2}[(1+\rho^2)\Psi(0,\theta,\tau')-2\rho\tilde\Psi(0,\theta,\tau')]\Big\}, \tag{2.4}$$

$$g(k,\theta,\tau)=\frac{e}{\Omega}\mathscr{E}_\mu(k)v_\mu(\theta,\tau)\frac{\partial f_0}{\partial\varepsilon},\quad \gamma=\frac{\nu-i\omega}{\Omega}.$$

Here $R = v/\Omega$ is the cyclotron radius, $\Omega = eH/mc$ the cyclotron frequency, m the mass, e the absolute magnitude of the conduction electron charge, v the Fermi velocity, $\mathbf{n} = \mathbf{v}/v$ the unit velocity vector, $f_0(\epsilon)$ the equilibrium Fermi distribution function, and ν the frequency of the collisions between electrons and scatterers.

The variables θ and τ are the polar and azimuthal angles in momentum space with polar axis p_H:

$$n_\xi=\sin\theta\cos\tau,\quad n_\eta=\cos\theta\sin\Phi+\sin\theta\cos\Phi\sin\tau,\quad n_\zeta=\cos\theta\cos\Phi-\sin\theta\sin\Phi\sin\tau. \tag{2.5}$$

The "phase" $\alpha(\tau',\tau)$ is given by the formula

$$\alpha(\tau',\tau)=\int_\tau^{\tau'}d\tau''n_\eta(\theta,\tau''). \tag{2.6}$$

The value of the function $\Psi(0,\mathbf{v})$ on the surface $\eta = 0$ is related to the Fourier-transform $\psi(k)$ through Eq. (2.3):

$$\Psi(0)=\pi^{-1}\int_0^\infty dk\,\psi(k). \tag{2.7}$$

The tilde on the function $\Psi(0,\mathbf{v})$ indicates a change in sign of the velocity component v_η:

$$\tilde\Psi(0,v_\xi,v_\eta,v_\zeta)\equiv\Psi(0,v_\xi,-v_\eta,v_\zeta), \tag{2.8}$$

Hence, to find $\psi(k)$ we must determine $\Psi(0)$, using (2.7). If we integrate Eq. (2.4) over k, $\delta[R\alpha(\tau',\tau)]$ appears in the integral over τ'. The argument of that δ-function vanishes when $\tau' = \tau, \lambda_1, \lambda_2, \ldots$, where the λ_n are the roots of the equation

$$\alpha(\lambda_n,\tau)\equiv\int_\tau^{\lambda_n}d\tau'\,n_\eta(\theta,\tau')=0. \tag{2.9}$$

The roots λ_n are numbered in order of decreasing value. If Eq. (2.9) does not have solutions for some θ and τ the corresponding root must be put equal to $-\infty$.

As an illustration we analyze the behavior of the first root $\lambda_1(\theta,\tau)$. In Fig. 2 we give the functions $n_\eta(\tau)$ and $\lambda_1(\tau)$ for $\bar n_\eta > 0$ when the electron drifts in the direction towards the metal surface. The points τ_1, τ_2, $\tau_1 - 2\pi$, and so on, are the zeroes of $n_\eta(\tau)$. In the hatched regions $\mu_1 - 2\pi n < \tau < \tau_1 - 2\pi n$ the root $\lambda_1(\tau) = -\infty$; the point μ_1 follows from the condition that the integral

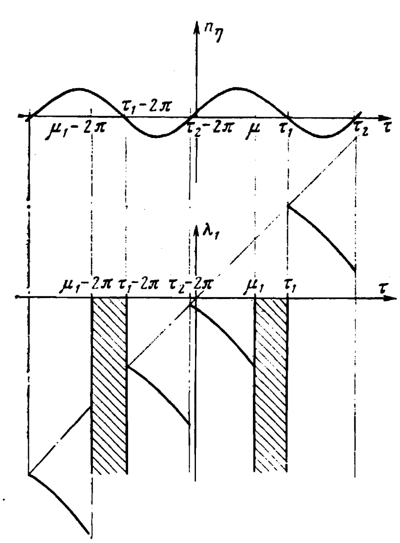

FIG. 2. The normal component n_η and the first root λ_1 as functions of τ; $\bar n_\eta = \cos\theta\sin\Phi > 0$. In the hatched regions $\lambda_1 = -\infty$.

$$\int_{\mu_1}^{\tau_1} d\tau' n_\eta(\tau')=0.$$

vanishes. In the other intervals $\lambda_1(\tau)$ takes on finite values, undergoing first-order discontinuities. This root $\lambda_1(\tau)$ behaves similarly also when $\bar{n}_\eta < 0$. By virtue of the definition (2.9) the roots have the following obvious properties:

$$\lambda_n(\theta,\lambda_1)=\lambda_1(\theta,\lambda_n)=\lambda_{n+1}(\theta,\tau),\quad \lambda_n(\theta,\tau+2\pi)=\lambda_n(\theta,\tau)+2\pi. \tag{2.10}$$

The integral over τ' is evaluated using $\delta[\alpha(\tau',\tau)]$. After this we obtain a functional equation to determine $\Psi(0,\tau)$; on the right-hand side of this there occurs a sum of terms containing $\Psi(0,\lambda_n)$ with all roots λ_n. We note that the term corresponding to the zeroth root $\tau'=\lambda_0=\tau$ is taken with a factor 1/2, since this root coincides with the limit of the integral (2.4). We can get rid of all roots in the sum if we replace τ by λ_1 in the functional equation which we obtained. Subtracting the two equations from one another we are led to the following relation for the function $\Psi(0)$:

$$[\Psi(0,\tau)-\rho\tilde{\Psi}(0,\tau)]e^{\gamma\tau}-\rho[\tilde{\Psi}(0,\lambda_1)-\rho\Psi(0,\lambda_1)]e^{\gamma\lambda_1}=(1-\rho^2)u_0(\tau). \tag{2.11}$$

We have introduced here the notation

$$u_n(\tau)=\frac{e}{\Omega}\frac{\partial f_0}{\partial\varepsilon}\int_{\lambda_{n+1}}^{\lambda_n} d\tau' \mathbf{E}[R\alpha(\tau',\tau)]\mathbf{v}(\theta,\tau')e^{\gamma\tau'}. \tag{2.12}$$

An important feature of Eq. (2.11), which enables us to find its exact solution, is the following fact: if we replace τ by λ_1 in the argument of the expressions which occur in the first square brackets of (2.11) and then perform the "tilde" operation, we get the expression which occurs in the second square bracket.

We can solve Eq. (2.11) relatively easily if we write it down for the values $\tau=\lambda_1, \lambda_2, \ldots, \lambda_N$, where λ_N is the last root of Eq. (2.9). The last two equations of this set for $\tau=\lambda_{N-1}$ and $\tau=\lambda_N$ take the following form:

$$[\Psi(0,\lambda_{N-1})-\rho\tilde{\Psi}(0,\lambda_{N-1})]e^{\gamma\lambda_{N-1}}-\rho[\tilde{\Psi}(0,\lambda_N)-\rho\Psi(0,\lambda_N)]e^{\gamma\lambda_N}=(1-\rho^2)u_{N-1},$$
$$[\Psi(0,\lambda_N)-\rho\tilde{\Psi}(0,\lambda_N)]e^{\gamma\lambda_N}=(1-\rho^2)u_N. \tag{2.13}$$

From the second Eq. (2.13) we find easily the expression occurring in the second square bracket of the preceding equation. After that we find in the same way the combination $\Psi(0,\lambda_{N-1})-\rho\tilde{\Psi}(0,\lambda_{N-1})$ which is substituted in the equation before that, and so on. As a result we get

$$[\Psi(0,\tau)-\rho\tilde{\Psi}(0,\tau)]e^{\gamma\tau}=(1-\rho^2)\sum_{n=0}^{N}\rho^n\hat{I}\hat{B}_{\lambda_1}\hat{B}_{\lambda_2}\ldots\hat{B}_{\lambda_n}u_n, \tag{2.14}$$

where $\hat{B}_{\lambda_0}=\hat{I}$ is the unit operator,

$$\hat{B}_{\lambda_n}=e^{\gamma\lambda_n}\hat{A}_{\lambda_n}e^{-\gamma\lambda_n}, \tag{2.15}$$

while the operator $\hat{A}_{\lambda_n}$ indicates the reflection (tilde) operation for the point (θ,λ_n) of the Fermi surface with respect to the equatorial plane $v_\eta=0$.

The physical meaning of the solution obtained consists in that the looked-for combination $\Psi-\rho\tilde{\Psi}$ is expressed in terms of partial contributions from separate sections of the electron trajectory between points of two consecutive collisions with the boundary. Each successive term of the sum differs from the preceding one by a factor ρ and an operator $\hat{B}$ which takes into account the change in p_H when the electron is scattered by the metal surface.

It is rather obvious that the "surface" part of the function $\psi(k,\theta,\tau)$ in (2.4) can be expressed in terms of the combination of functions $\Psi-\rho\tilde{\Psi}$. Using Eq. (2.14) we write the second term inside the braces in (2.4) in the form

$$\frac{1}{1-\rho^2}[(1+\rho^2)\Psi(0,\theta,\tau)-2\rho\tilde{\Psi}(0,\theta,\tau)]=\frac{ve}{\Omega}\frac{\partial f_0}{\partial\varepsilon}\frac{1}{\pi}\int_0^\infty dk'\chi_\mu(k',\theta,\tau)\mathscr{E}_\mu(k'), \tag{2.16}$$

Here

$$\chi(k',\theta,\tau)=(1-\rho\hat{A}_\tau)\sum_{s=0}^{N}\rho^s\exp\left\{\gamma\sum_{n=0}^{s}\{\lambda_1[(\tilde{\lambda}_1)^{(n)}]-(\tilde{\lambda}_1)^{(n)}\}\right\}w[(\tilde{\lambda}_1)^{(n)}], \tag{2.17}$$

$$w(\tau)=\int_{\lambda_1(\tau)}^{\tau} d\tau' e^{\gamma(\tau'-\tau)}\cos[k'R\alpha(\tau',\tau)]\mathbf{n}(\theta,\tau').$$

By $(\tilde{\lambda}_1)^{(n)}=\hat{A}_{\lambda_1}(\tilde{\lambda}_1)^{(n-1)}$ we have denoted the following quantity

$$(\tilde{\lambda}_1)^{(n)}=\hat{A}_{\lambda_1}\hat{A}_{\lambda_2}\ldots\hat{A}_{\lambda_n}\lambda_n=\tilde{\lambda}_1(\tilde{\lambda}_1(\tilde{\lambda}_1\ldots(\tilde{\lambda}_1))\ldots).$$

Equation (2.16) gives an explicit solution for the Fourier transform (2.4) of the distribution function for arbitrary angles of inclination Φ and values of the specularity parameter ρ.

We can thus finally write the current density in the form

$$j_\mu(k)=\sigma_{\mu\nu}(k)\mathscr{E}_\nu(k)-\pi^{-1}\int_0^\infty dk' Q_{\mu\nu}(k,k')\mathscr{E}_\nu(k'), \tag{2.18}$$

where

$$\sigma_{\mu\nu}(k)=\sigma_H\int_0^\pi d\theta\sin\theta\oint d\tau\, n_\mu(\theta,\tau)\times\int_{-\infty}^{\tau} d\tau' e^{\gamma(\tau'-\tau)}n_\nu(\theta,\tau)\cos[kR\alpha(\tau',\tau)], \tag{2.19}$$

$$Q_{\mu\nu}(k,k')=\sigma_H R\int_0^\pi d\theta\sin\theta\oint d\tau\, n_\mu(\theta,\tau)\int_{-\infty}^{\tau} d\tau' e^{\gamma(\tau'-\tau)}|n_\eta(\theta,\tau')|\times\cos[kR\alpha(\tau',\tau)]\chi_\nu(k',\theta,\tau'),\quad \sigma_H=\frac{3}{4\pi}\frac{Ne^2}{m\Omega}. \tag{2.20}$$

The tensor $\sigma_{\mu\nu}$ is the Fourier transform of the conductivity of a metal without boundaries, and the kernel $Q_{\mu\nu}$ is caused by the presence of the dividing boundary.

3. ASYMPTOTIC BEHAVIOR OF THE CURRENT DENSITY

We study the anomalous skin effect region

$$kR\gg 1. \tag{3.1}$$

The mean free path $l=v/\nu$ is assumed to be sufficiently large so that

$$|\gamma|=|\nu-i\omega|/\Omega\ll 1. \tag{3.2}$$

Finally, we restrict ourselves to the region of comparatively small angles of inclination Φ of the vector **H** with respect to the metal surface

$$(kl)^{-1}\ll\Phi\ll(kR)^{-1},\quad |\gamma|(kR)^{-1/2}. \tag{3.3}$$

We elucidate the physical meaning of these inequalities. The left-hand inequality in (3.3) means that after a mean flight time an electron moving on average along the magnetic field leaves the skin layer. The condition $kR\Phi \ll 1$ corresponds to the fact that after a cyclotron period the drift displacement $R\Phi$ of an electron along the normal to the surface is small compared to the skin-layer thickness k^{-1}. The inequality

$$w=|\gamma|/\Phi(kR)^{1/2}\gg 1 \tag{3.4}$$

expresses the requirement that the spread in orbit diameters $\Delta D \sim R(kl\Phi)^{-2}$ for electrons which stay in the skin layer during a mean free flight time is much larger than k^{-1}.[13]

All these conditions enable us to simplify the expressions for $\sigma_{\mu\nu}$ and $Q_{\mu\nu}$. Thanks to the inequality (3.1) the main contribution to the current comes from the neighborhoods of those points on the electron trajectories where the electron velocity is parallel to the surface $v_\eta = 0$. Close to the limiting points ($0 < \theta < \Phi$, $\pi - \Phi < \theta < \pi$) the projection of the velocity $v_\eta \neq 0$. Therefore, we restrict ourselves in the integration over θ to the section $\Phi \leq \theta \leq \pi - \Phi$.

The evaluation of the asymptotic behavior of $\sigma_{\mu\nu}$ is not difficult and has been done before:[12,13]

$$\sigma_{\xi\xi}(k) = 2\pi\sigma_H/(kR)^2\Phi. \tag{3.5}$$

We study only the ξ polarization as the remaining components of the current are appreciably less.

The most laborious and complicated part of the calculation of the asymptotic behavior of the current is the evaluation of the kernel $Q(k, k')$. The asymptotic behavior of Q is determined by contributions from different groups of electrons. First of all, we must take into account electrons which either do not collide at all with the surface and leave for the interior of the metal or collide a small number of times with the surface. The most important contribution to the current from these electrons comes from the electrons which are close to the central cross section $p_H = 0$ as they spend appreciably longer time in the skin layer than electrons with an appreciable drift velocity $\bar{v}_H$. The contribution to the current from slowly drifting electrons is different by the peculiar fact that in some regions of θ and τ the root λ_1 becomes $-\infty$, as a result of which resonant denominators of the type $(\gamma \pm ik'R\cos\theta\sin\Phi)^{-1}$ occur in the conductivity. For the sake of simplicity we shall call such contributions anomalous, since there is no electron drift into the interior of the metal when $\Phi = 0$. Apart from this group there are electrons which collide several times with the boundary before they leave for the interior of the metal. Such electrons also spend a prolonged time in the skin layer and play an important role in producing the screening current in the skin layer. We shall call the contribution from such electrons the normal one.

It is convenient to write the kernel $Q = Q_{\xi\xi}$ in the form

$$Q(k,k') = \sigma_H \int_{\Phi}^{\pi-\Phi} d\theta \sin\theta \frac{1}{4}\{M(k,k') + M(-k,k') + M(k,-k') + M(-k,-k')\}. \tag{3.6}$$

We replaced the cosines containing $kR\alpha$ and $k'R\alpha$ in (2.17) and (2.20) by half the sum of the appropriate exponentials

$$M(k,k',\theta) = \oint d\tau |n_\eta(\theta,\tau)| \chi_\xi^{(e)}(k',\theta,\tau) \exp[-ik_\perp R\cos\tau] \times \int_0^\infty dx \exp[-\Gamma(k)x + ik_\perp R\cos(\tau+x)] n_\xi(\tau+x), \tag{3.7}$$

where $\chi_\xi^{(e)}$ differs from (2.17) in that we have $\exp(ik'R\alpha)$ in w_ξ instead of $\cos(k'R\alpha)$,

$$k_\perp = k\sin\theta, \quad \Gamma(k) = \gamma + ikR\bar{\alpha} = \gamma + ikR\Phi\cos\theta.$$

Moreover, we have changed the order of integration over τ and $\tau' = \tau - x$.

To evaluate the anomalous contribution to the integral (3.7) we split off the section $\mu_1 < \tau < \tau_1$, where $\lambda_1(\tau) = -\infty$ ($\bar{n}_\eta > 0$, see Fig. 2). After this the integrals over the infinite interval can be folded, using the periodicity (with period 2π) of the integrands. As a result of the folding there appears in the denominator the difference $1 - e^{-2\pi\Gamma}$ which we replaced by $2\pi\Gamma$ by virtue of the inequalities (3.2) and (3.3). We note that when evaluating the anomalous contributions we can in (2.17) replace the operator $\hat{A}_\tau$ by the unit operator up to terms of order Φ^2. Moreover, we integrate the remaining integral over τ

$$M^{\text{anom}}(k,k',\theta) = \frac{1-\rho}{4\pi^2\Gamma(k)\Gamma(k')} \int_{\mu_1}^{\tau_1} d\tau\, n_\eta(\theta,\tau) \exp[-i(k_\perp - k_\perp')R\cos\tau] \times \int_0^{2\pi} dx \exp[-\Gamma(k)x + ik_\perp R\cos(\tau+x)] n_\xi(\tau+x) \int_0^{2\pi} dy \exp[-\Gamma(k)y - ik_\perp' R\cos(\tau-y)] n_\xi(\tau-y), \tag{3.7a}$$

by parts using the identity $\alpha(\tau_1, \mu_1) = 2\pi\bar{n}_\eta$. After that, wherever possible we put the angle Φ and the quantity γ equal to zero. We then get for $M^{\text{anom}}(k, k', \theta)$

$$M^{\text{anom}}(k,k',\theta) = \frac{\pi}{2}(1-\rho)\Phi\sin^2\theta|\cos\theta| \frac{J_1(k_\perp R) J_1(k_\perp' R)}{\Gamma(k)\Gamma(k')} \times \exp[-i(k_\perp - k_\perp')R\,\text{sign}\cos\theta]. \tag{3.8}$$

If we now symmetrize $M^{\text{anom}}(k, k', \theta)$ according to (3.6) and use the well-known asymptotic properties of the Bessel functions for large arguments we get for $Q^{\text{anom}}(k, k')$ the following formula:

$$Q^{\text{anom}}(k,k') = \frac{(1-\rho)\sigma_H}{2(kk')^{1/2}R} \int_0^\pi d\theta \sin^2\theta \left(\frac{i\,\text{sign}\cos\theta}{k-k'} - \frac{1}{k+k'}\right) \times \left[\frac{1}{\gamma + ikR\Phi\cos\theta} - \frac{1}{\gamma + ik'R\Phi\cos\theta}\right] = \frac{(1-\rho)\sigma_H}{(kk')^{1/2}R^2\Phi}\left(k\frac{\ln(k/k')}{k-k'} - \ln kL - \pi\frac{k'}{k+k'}\right), \quad L = 2R\Phi\gamma^{-1}\exp(-1-\pi/2). \tag{3.9}$$

We now turn to the calculation of the normal contributions. To do this we estimate first of all the maximum number of roots of Eq. (2.9). For small Φ it has the form

$$\Phi\,\text{ctg}\,\theta(\tau - \lambda) = \cos\tau - \cos\lambda. \tag{3.10}$$

As we shall see in what follows, the main contribution to the asymptotic behavior comes from the region of θ-values close to $\theta \approx \pi/2$, where $kl\Phi|\cos\theta| \sim 1$, i.e., $\Phi|\cot\theta| \sim (kl)^{-1} \ll 1$. The maximum number N of roots λ_n of Eq. (3.10) turns then out to be of the order $(\Phi|\cot\theta|)^{-1} \sim kl$, i.e., much larger than unity. For normal contributions the lower limit of integration in the expression for $w_\xi(\tau)$ in (2.17) is a finite quantity (the anomalous contributions have been taken into account!). We can therefore in $w_\xi(\tau)$ put $\Phi = 0$, because $kR\Phi \ll 1$. It is rather obvious that, independently of n, the difference $(\tilde{\lambda}_1)^{(n)} - \lambda_1[(\tilde{\lambda}_1)^{(n)}]$ does not exceed 4π, i.e., the sum over n in the argument of the exponential with γ does not increase faster than s. This means that the normal contributions can essentially be calculated by the same method as for the case of a parallel field ($\Phi = 0$). In other words, we must for the normal contributions replace the roots λ_n by their values for $\Phi = 0$:

$$\lambda_1 = -\tau \quad (0 < \tau < \pi), \qquad \lambda_1 = 2\pi - \tau \quad (\pi < \tau < 2\pi).$$

In the case of a parallel field the action of the reflection operator $\hat{A}_\tau$ on a periodic function with period 2π is given by the formula

$$\hat{A}_\tau f(\tau) = f(\lambda_1) = f(-\tau).$$

What we said above enables us to obtain the following formula:

$$M^{\rm norm}(k,k',\theta)=\frac{2\sin^3\theta}{\pi\Gamma(k)}\int_0^{2\pi}dx\,e^{-\Gamma(k)x}\int_0^{\pi}d\tau\frac{\sin\tau}{e^{\gamma\tau}-\rho e^{-\gamma\tau}}\int_0^{\tau}d\mu\,\mathrm{ch}[\Gamma(k')\mu]\cos\mu$$
$$\times\{\cos(\tau+x)\cos[k_\perp R(\cos(\tau+x)-\cos\tau)-k_\perp' R(\cos\mu-\cos\tau)]$$
$$-\rho\cos(\tau-x)\cos[k_\perp R(\cos(\tau-x)-\cos\tau)-k_\perp' R(\cos\mu-\cos\tau)]\}. \qquad (3.11)$$

This formula differs from the similar expression for the case[7] $\Phi = 0$ only in that in the integral over x there occurs in the index of the exponential instead of the quantity γ, $\Gamma(k) = \gamma + ikR\bar{n}_\eta$ which takes into account the drift motion of the electrons into the interior of the metal.

We find the asymptotic behavior of $M^{\rm norm}$ in the limiting case

$$1-\rho\gg|\gamma|(kR)^{-1/2}. \qquad (3.12)$$

If inequality (3.12) is replaced by the opposite one we come to a situation which is completely analogous to the parallel magnetic field case. When $1-\rho \ll |\gamma|(kR)^{-1/2}$ we can neglect the anomalous contributions and the main term in the asymptotic expression of (3.11) is independent of Φ and is the same as the formulae given in [7].

The asymptotic behavior of (3.11) is determined by the contribution from the neighborhoods of the stationary-phase points $x = 0, \pi, 2\pi$; $\tau, \mu = 0, \pi$. Essentially simple, but cumbersome calculations lead to the following formula:

$$Q^{\rm norm}(k,k')=\frac{\pi\sigma_H}{1+b}\left\{\frac{1}{(kR)^2\Phi}\left[\pi\delta(k-k')+\frac{(k/k')^{1/2}}{k+k'}\right]\right.$$
$$\left.+2\frac{1+\rho}{1-\rho}(2+b)\frac{\ln(k/k')}{(k^2-k'^2)R}\right\},\quad b=\pi\gamma\frac{1+\rho}{1-\rho}. \qquad (3.13)$$

The first two terms are caused by the contribution from electrons drifting into the interior of the metal; this is shown by the factor $(kR\Phi)^{-1}$ in front of the square brackets. The last term is caused by electrons with small values of the velocity component v_η which graze along the surface due to multiple collisions with it. This fact manifests itself in that that term contains a factor $(1-\rho)^{-1}$. The total kernel (2.20) of the Fourier transform of the current density is the sum of the anomalous and the normal contributions, $Q = Q^{\rm anom} + Q^{\rm norm}$. As the complete asymptotic formula for the current turns out to be rather complicated we consider the following limiting cases in the framework of (3.1) to (3.3) and (3.12).

1. Let the reflection of the electrons from the boundary be sufficiently far from specular so that we can neglect the last terms in (3.13) from the grazing electrons,

$$kR\Phi\ll1-\rho. \qquad (3.14)$$

The formula for the current density then has the following form:

$$j(k)=\frac{\pi\sigma_H}{(kR)^2\Phi}\left\{\mathscr{E}(k)-\frac{1}{\pi}\int_0^\infty\frac{dk'}{k}\left(\frac{k}{k'}\right)^{3/2}\left[\left(k\frac{\ln(k/k')}{k-k'}-\ln kL\right)\frac{1-\rho}{\pi}\right.\right. \qquad (3.15)$$
$$\left.\left.+\rho\frac{k'}{k+k'}\right]\mathscr{E}(k')\right\}.$$

2. If the coefficient of reflection of the electrons from the surface is close to unity and satisfies the conditions

$$|\gamma|/\sqrt{kR}\ll|\gamma|,\quad 1-\rho\ll kR\Phi, \qquad (3.16)$$

the grazing electrons play the main role in the current

$$j(k)=-2\frac{2+b}{1+b}\frac{1+\rho}{1-\rho}\frac{\sigma_H}{R}\int_0^\infty dk'\frac{\ln k/k'}{k^2-k'^2}\mathscr{E}(k'). \qquad (3.17)$$

The relation between $|\gamma|$ and $1-\rho$ can then be arbitrary.

4. SOLUTION OF THE MAXWELL EQUATIONS. SURFACE IMPEDANCE

1. We turn to the solution of the Maxwell equation (2.1) for the ξ polarization in the limiting case (3.14). We introduce dimensionless variables

$$q=kL,\quad q'=k'L,\quad \mathscr{E}(k)=-2E'(0)L^2\mathscr{F}(q). \qquad (4.1)$$

The integral Eq. (2.1) together with (3.15) can in these variables be written in the form

$$\left(q^2-\frac{i\beta}{q^2}\right)\mathscr{F}(q)+\frac{i\beta}{\pi q^2}\int_0^\infty\frac{dq'}{q}\left(\frac{q}{q'}\right)^{3/2}\left[\left(q\frac{\ln(q/q')}{q-q'}-\ln q\right)\frac{1-\rho}{\pi}\right.$$
$$\left.+\rho\frac{q'}{q+q'}\right]\mathscr{F}(q')=1, \qquad (4.2)$$
$$\beta=L^4/\delta_1^4,\quad \delta_1=(\Phi R^2c^2/4\pi^2\omega\sigma_H)^{1/4}\sim(\delta_a^3R\Phi)^{1/4}.$$

The quantity δ_1 is the effective penetration depth of the electromagnetic wave into the metal in the case considered: $\delta_a = (4c^2v/3\pi\omega\omega_0^2)^{1/3}$, ω_0 is the plasma frequency. We note that part of the kernel—the second term within the square brackets in (4.2)—is a degenerate kernel.

One can solve Eq. (4.2) exactly using a two-sided Laplace transformation. The method of solution is similar to the one proposed by Hartmann and Luttinger[3]. We substitute in (4.2) the variables

$$q=\exp t,\quad q'=\exp\tau,\quad \mathscr{F}(q)=g(t).$$

We get then instead of (4.2)

$$(e^{4t}-i\beta)g(t)+i\beta\int_{-\infty}^{\infty}d\tau\,\Lambda(t-\tau)g(\tau)=e^{2t}+Cte^{t/2}, \qquad (4.3)$$

where

$$\Lambda(x)=\frac{(1-\rho)xe^x}{2\pi^2\,\mathrm{sh}(x/2)}+\frac{\rho}{2\pi\,\mathrm{ch}(x/2)}.$$

The constant C is determined by the integral of the required function:

$$C=i\beta\frac{1-\rho}{\pi^2}\int_{-\infty}^{\infty}d\tau\,e^{-\tau/2}g(\tau). \qquad (4.4)$$

We shall look for the solution of (4.3) in the form

$$g(t)=g_1(t)+Cg_2(t). \qquad (4.5)$$

By virtue of the linearity of the original Eq. (4.3) the equations for g_1 and g_2 are the same as (4.3) with that difference that the right-hand side occur the functions e^{2t} and $te^{t/2}$, respectively,

$$(e^{4t}-i\beta)g_1(t)+i\beta\int_{-\infty}^{\infty}d\tau\,\Lambda(t-\tau)g_1(\tau)=e^{2t}. \qquad (4.6)$$

$$(e^{4t}-i\beta)g_2(t)+i\beta\int_{-\infty}^{\infty}d\tau\,\Lambda(t-\tau)g_2(\tau)=te^{t/2}. \qquad (4.7)$$

We introduce the Laplace transform

$$T(z)=\int_{-\infty}^{\infty}dt\,e^{-zt}g(t),\quad g(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dz\,e^{zt}T(z),\quad c=\mathrm{Re}\,z. \qquad (4.8)$$

The constant c is chosen inside the band where the function T(z) is regular.

If we apply the Laplace transformation to the integral equation, there occurs, in general, an inhomogeneous difference equation for the function T(z). We can obtain instead of such an inhomogeneous equation a homogeneous one, if we require that the function have an isolated singularity such as a pole of the appropriate order, such that the integral over z in (4.8) over a small neighborhood around the pole is the same as the right-hand side of the equation. The regularity band of the function must be found from the condition that the integral over z in (4.8) must converge as $z \to \pm i\infty$ and the presence inside it of the required singularity; the width of the band is determined by the behavior of the original Eq. (4.3) as $t \to +\infty$ and $t \to -\infty$. It is clear for both Eqs. (4.6) and (4.7) that the width of the band equals 4. The singularity for $T_1(z)$ for Eq. (4.6) is a simple pole at the point $z = -2$ with a residue equal to unity. For Eq. (4.7) the function must have near $z = -(7/2)$ a second order pole of the kind $(z + (7/2))^{-2}$. For the Laplace transform $T_1(z)$ of the function $g_1(t)$ we choose the regularity band between 1 and $-(7/2)$, viz.

$$S_1 = -{}^7/_2+\Delta < \operatorname{Re} z < {}^1/_2+\Delta, \quad 0<\Delta<{}^1/_2.$$

This band satisfies all conditions formulated above. For the function $T_2(z)$ the regularity band is displaced to the left by 1/2, i.e.,

$$S_2 = -4+\Delta < \operatorname{Re} z < \Delta.$$

The functions $T_\alpha(z)$ satisfy one and the same difference equation which we obtain easily from (4.6) and (4.7) ($\alpha = 1, 2$),

$$T_\alpha(z-4) = i\beta\theta(z)T_\alpha(z), \quad \theta(z) = 1 - \frac{1-\rho}{\cos^2\pi z} - \frac{\rho}{\cos\pi z}. \tag{4.9}$$

The differences between the functions T_1 and T_2 are caused by the differences in the singularities and the position of the bands S_1 and S_2. It is clear that the general solution of Eq. (4.9) contains an arbitrary periodic function of period 4. The regularity conditions and the requirement of the existence of the appropriate singularities enable us to determine uniquely this arbitrary periodic function.

We find the function $T_1(z)$. We seek it in the form

$$T_1(z) = \frac{\pi\exp[\pi i(z+2)/8]}{4\sin\pi(z+2)/4}\beta^{-(z+2)/4}\mu_1(z). \tag{4.10}$$

The function $\mu_1(z)$ satisfies the equation

$$\mu_1(z-4) = \theta(z)\mu_1(z)$$

which is regular in the band S_1 and equal to unity in the point $z = -2$. We put

$$\mu_1(z) = \exp[D_1(z) - D_1(-2)] = \exp\left\{\int_{-2}^{z} dz' D_1'(z_1)\right]. \tag{4.11}$$

The function $D_1'(z)$ satisfies the following difference equation:

$$D_1'(z-4) - D_1'(z) = \pi\left(2\operatorname{tg}\pi z + \operatorname{ctg}\frac{\pi z}{2} - \frac{\sin\pi z}{\cos\pi z+1-\rho}\right).$$

The general solution of this equation has the form

$$D_1'(z) = -\frac{\pi z}{4}\left(2\operatorname{tg}\pi z + \operatorname{ctg}\frac{\pi z}{2} - \frac{\sin\pi z}{\cos\pi z+1-\rho}\right) + V_1(z). \tag{4.12}$$

The periodic function $(V_1(z))$ of period 4 must be chosen such that there are no singularities of $D_1'(z)$ in the band S_1. Moreover, the average value of the function $V_1(z)$ must equal zero.

In the table we give the position of the poles and the magnitude of the residues for the first term in (4.12):

Position of the singularities of $D_1'(z)$ in the band S_1.	$^1/_2$	$-^1/_2$	$-^3/_2$	$-^5/_2$	-2	$-z_1$	$-z_1-2$	z_1-2	z_1
Residue	$^1/_4$	$-^1/_4$	$-^3/_4$	$-^5/_4$	1	$\frac{z_1}{4}$	$\frac{z_1+2}{4}$	$-\frac{z_1-2}{4}$	$-\frac{z_1}{4}$

The periodic function $V_1(z)$ which cancels all these singularities in the band S_1 has the form

$$V_1(z) = \frac{\pi}{16}\Big[-\operatorname{ctg}\frac{\pi}{4}\left(z-\frac{1}{2}\right) + \operatorname{ctg}\frac{\pi}{4}\left(z+\frac{1}{2}\right) + 3\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{3}{2}\right)$$
$$+5\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{5}{2}\right) - 4\operatorname{ctg}\frac{\pi}{4}(z+2) - z_1\operatorname{ctg}\frac{\pi}{4}(z+z_1) \tag{4.12$'$}$$
$$-(z_1+2)\operatorname{ctg}\frac{\pi}{4}(z+z_1+2) + (z_1-2)\operatorname{ctg}\frac{\pi}{4}(z-z_1+2) + z_1\operatorname{ctg}\frac{\pi}{4}(z-z_1)\Big].$$

Equations (4.10) to (4.12) give the explicit solution for $T_1(z)$.

We turn to the solution of Eq. (4.9). In accordance with the above-formulated conditions on $T_2(z)$ we seek it in the form

$$T_2(z) = \frac{\pi^2\exp[-\pi i(z+{}^7/_2)/8]}{16\sin^2\pi(z+{}^7/_2)/4}\beta^{-(z+{}^7/_2)/4}\mu_2(z). \tag{4.13}$$

The function $\mu_2(z)$ satisfies the same equation as $\mu_1(z)$, but differs from μ_1 in the regularity band and the condition $\mu_2(-(7/2)) = 1$. We introduce

$$\mu_2(z) = \exp[D_2(z) - D_2(-{}^7/_2)] = \exp\left[\int_{-{}^7/_2}^{z} dz' D_2'(z')\right]. \tag{4.14}$$

The function $D_2'(z)$ satisfies the same equation as $D_1'(z)$, the general solution is the same as (4.12), differing only in the periodic function $V_2(z)$. The difference in the bands S_1 and S_2 leads to the fact that instead of the point $z = 1/2$ there is in the band S_2 the point $z = 7/2$ with residue $-(7/4)$, and instead of $z = z_1$ there occurs $z = z_1 - 4$ with residue $1 - z_1/4$ ($z_1 = 1 - (1/\pi)\arccos(1-\rho)$) so that we get for $D_2'(z)$

$$D_2'(z) = -\frac{\pi z}{4}\left(2\operatorname{tg}\pi z + \operatorname{ctg}\frac{\pi z}{2} - \frac{\sin\pi z}{\cos\pi z+1-\rho}\right)$$
$$+\frac{\pi}{16}\Big[\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{1}{2}\right) + 3\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{3}{2}\right) + 5\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{5}{2}\right)$$
$$+7\operatorname{ctg}\frac{\pi}{4}\left(z+\frac{7}{2}\right) - 4\operatorname{ctg}\frac{\pi}{4}(z+2) - z_1\operatorname{ctg}\frac{\pi}{4}(z+z_1) \tag{4.15}$$
$$-(z_1+2)\operatorname{ctg}\frac{\pi}{4}(z+z_1+2) + (z_1-2)\operatorname{ctg}\frac{\pi}{4}(z-z_1+2) + (z_1-4)\operatorname{ctg}\frac{\pi}{4}(z-z_1)\Big].$$

From (4.4), (4.5), and (4.8) we get a linear equation to find C, and solving it we get

$$C = \frac{T_1({}^1/_2)}{\pi^2/i(1-\rho)\beta - T_2({}^1/_2)}. \tag{4.16}$$

As the point $z = 1/2$ lies outside the regularity band S_2 we must use Eq. (4.9) to obtain the value $T_2(1/2)$, expressing $T_2(1/2)$ in terms of $T_2(-(7/2))$. As the functions $T_2(z-4)$ and $\theta(z)$ have second order poles as $z \to 1/2$, we have $T_2(1/2) = i\pi^2/\beta(1-\rho)$, i.e.,

$$C = \frac{i\beta(1-\rho)}{2\pi^2}T_1({}^1/_2).$$

We have thus obtained an explicit solution of the Maxwell equations (2.1) in the limiting case (3.14). The surface impedance is expressed by the following formulae:

$$Z = \frac{4i\omega}{c^2E'(0)}\int_0^\infty dk\,\mathscr{E}(k) = -8i\omega Lc^{-2}T(-1) = \frac{4\pi}{c}\left(\Phi\frac{V^2\omega^3}{c^2\sigma_H\Omega^2}\right)^{1/4}u(\rho), \tag{4.17}$$

where

$$u(\rho)=\frac{\sqrt{2}+1}{\sqrt{2\pi}}(\sqrt{2}+\sqrt{\rho})^{1/2}\left[e^{-3\pi i/8}+\frac{(1-\rho)^3\sin\pi(1/2-z_2)/4}{16\sqrt{2}\sin 3\pi/8\cdot\sin\pi(1+z_1)/4}\right]. \quad (4.18)$$

The function $u(\rho)$ changes smoothly from the value[1)] $u(0) = 1.146(e^{-3\pi i/8} + 0.0259)$ to $u(1) = 1.496\, e^{-3\pi i/8}$.

2. In the limiting case (3.16) when the reflection of the electrons by the surface is nearly specular the equation for the Fourier component $\mathscr{E}(k)$ takes the form

$$k^2\mathscr{E}(k)+\frac{2i}{\pi^2}\delta_2^{-3}\int_0^\infty dk'\,\frac{\ln k/k'}{k^2-k'^2}\mathscr{E}(k')=-2E'(0);$$
$$\delta_2^3=\frac{1-\rho}{1+\rho}\frac{1+b}{2+b}\frac{c^2R}{4\pi^3\sigma_H\omega}=\frac{1-\rho}{1+\rho}\frac{1+b}{2+b}\delta_a^3,\quad \delta_a=\left(\frac{4c^2v}{3\pi\omega\omega_0^2}\right)^{1/3}. \quad (4.19)$$

This equation is a particular case of the analogous and more general equation in the theory of the anomalous skin effect.[2,3,14] Equation (4.19) differs in that on its left-hand side we do not have the term $-i\,\mathscr{E}(k)/k\delta_a^3$ which is small compared to the integral term due to the condition (3.16).

Indeed, $(\delta_2/\delta_a)^3 \sim 1-\rho \ll kR\Phi \ll 1$. Denoting the quantity δ_a^{-3} in that small term by d, we shall take the solution of Eq. (4.19) as the limit as $d \to 0$ of the solution of the more general equation. The quantity δ_2^{-3} is then assumed to be finite. The necessity of such a definition of the solution is caused also by the fact that a straight application of the method described above leads to an impossibility to regularize the Mellin transform T(z) at infinity ($z \to \pm i\infty$) inside the regularity bands. This means that the behavior of T(z) as $z \to \pm i\infty$ and $d \to 0$ depends in an essential way on the order of taking the limits.

Using the results of [3,14] we can easily write down the final expression for the surface impedance:

$$Z=\frac{4\pi\sqrt{3}}{2^{1/3}}\frac{\omega\delta_2}{c^2}=\frac{4\pi\omega\delta_a}{c^2}\left(\frac{3\sqrt{3}}{2}\frac{1-\rho}{1+\rho}\frac{1+b}{2+b}\right)^{1/3}. \quad (4.20)$$

The surface impedance (4.20) changes smoothly with the magnetic field in accord with the dependence of the parameter $b \approx 2\pi(\nu - i\omega)/\Omega(1-\rho)$ on H. The numerical coefficient in the first Eq. (4.20) is the same as the one found by Meĭerovich.[8] The second Eq. (4.20) differs from Meĭerovich's results by the factor $\{(b+1)/(b+2)\}^{1/3}$. The imaginary part of Z has an appreciable magnitude when compared to Re Z when $|b| \sim 1$ and $\omega \geq \nu$.

3. For completeness we also give the expression for the impedance in the case of specular reflection, when

$$1-\rho\ll|\gamma|(kR)^{1/2}\ll|\gamma|\ll kR\Phi. \quad (4.21)$$

The asymptotic behavior of the current then turns out to be the same as in the parallel magnetic field case, and according to [7]

$$Z_{\xi\xi}=4.1\left(\frac{\omega^3v}{\Omega c^6\sigma^2}\right)^{1/5}\exp\left(-\frac{3\pi i}{10}\right),\quad \sigma=\frac{Ne^2}{m(\nu-i\omega)}. \quad (4.22)$$

Here Z decreases with increasing field as $H^{-1/5}$.

4. We discuss briefly the results. First, we consider the angular dependence of Z. In the region of very small angles, $\Phi \ll |kl|^{-1}$ (the quantity $\delta = k^{-1}$, δ is the effective skin-layer depth in a parallel field) Z is independent of Φ. The corresponding formulae for the impedance when $\Phi = 0$ are known in the case of non-specular reflection ($1-\rho \gg |\gamma|$)[2,3], for reflection close to specular ($1-\rho \ll |\gamma|$)[8] and for $\rho = 1$.[7]. In the range of angles $|kl|^{-1} < \Phi < (1-\rho)|kR|^{-1}$ the impedance increases in proportion to $\Phi^{1/4}$ (see (4.17)). When the angle Φ increases further in the region (3.16), the impedance ceases to depend on the angle. Finally, in the range $\Phi \gtrsim |kR|^{-1}$ the impedance grows in magnitude, independent of the magnetic field, and is equal to the impedance for H = 0 as far as order of magnitude is concerned.

The impedance as function of the reflection coefficient changes smoothly in accordance with (4.17) when ρ changes from zero up to $1-\rho \sim |kR\Phi|$. In the region $1-\rho < |kR\Phi|$ the impedance falls steeply $\sim(1-\rho)^{1/3}$ to the values given by Eq. (4.22).

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1)We use this occasion to rectify an earlier error [12] in the solution of the integral equation (4.8) for the case $\rho = 0$. In that paper we did not take into account the fact that the kernel of the integral Eq. (4.2) contains a degenerate part and that the solution must be written as a sum of two functions that are regular in different bands. As a result it turned out that the solution for T(z) obtained in [12], first of all, contains non-regularized singularities of the kind z ln z and, secondly, the impedance differs from (4.18) by the absence of the second term inside the square brackets and an additional factor $2^{5/4} = 2.38$.

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66