Particle with low binding energy in a circularly polarized field

N. L. Manakov and L. P. Rapoport

Voronezh State University
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We calculate the parameters of the quasistationary state of a particle with low binding energy E_0 in the field of a monochromatic circularly polarized electromagnetic wave of frequency ω. By changing to a coordinate frame rotating with the field frequency we obtain a transcendental equation for the complex energy E, the real and imaginary parts of which give the position and the width of the level. In the approximation $\omega \ll \omega_0$ and $F \ll F_0$ (F is the field amplitude and ω_0 and F_0 are the corresponding atomic values), the level shift is quadratic in F and the width coincides with the result of the adiabatic approximation. For arbitrary ω, we obtain an expansion of $\sqrt{E}$ in powers of the field intensity. We show that the condition for the applicability of perturbation theory to the level width Γ is of the form $\gamma^2 \ll 1/K_0^2$, where $\gamma = eF/\omega(2mE_0)^{1/2}$ and $K_0 = E_0/\hbar\omega$. We calculate the corrections to the first nonvanishing perturbation-theory order of the level shift and width. Results of numerical calculations are given for the values of the parameter $\gamma \approx 1$. The question of the applicability of the results to negative ions is discussed.

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1. INTRODUCTION

Interest in the behavior of a quantum system in the field of an intense electromagnetic wave has increased in recent years. For a sufficiently strong field, the problem is greatly complicated by the fact that the use of the first nonvanishing order of perturbation theory to take into account the interaction of the system with the field is insufficient in the calculation of concrete processes. Principal attention is paid therefore to the development of methods that make it possible to solve problems outside the framework of perturbation theory. Considerable progress in this direction was reached in the solution of the problem of the ionization of a bound level in a short-range potential by the field of a strong wave.[1-4] These studies, however, yielded only the width of the quasi-stationary state produced under the influence of the field, while the shift that determines the level position was not calculated.

In this paper we determine the shift and width of a bound level of a particle in a small-radius force field produced by a circular depolarized wave. The case of circular polarization is singled out because the wave intensity remains constant in time. As noted by Bunkin and Prokhorov[5], for systems having spherical symmetry this makes it possible to reduce the problem to a stationary one by changing over to a coordinate system that rotates at the frequency of the field. Confining ourselves for simplicity to the dipole approximation, we choose the operator of the interaction of the electrons with the field $\mathbf{F}(t)$ in the form

$$V(t) = eF(x\cos\omega t + y\sin\omega t) = eFr\sin\theta\cos(\omega t - \varphi).$$

The Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = (H_0 + V(t))\psi(t) \tag{1}$$

for the function $\varphi(\mathbf{r}, t) = \exp\{i\omega t\hat{L}_z/\hbar\}\psi(\mathbf{r}, t)$ takes the form

$$i\hbar\frac{\partial\varphi(\mathbf{r}, t)}{\partial t} = (H_0 - \omega\hat{L}_z + eFx)\varphi(\mathbf{r}, t) = \hat{Q}(\mathbf{r})\varphi(\mathbf{r}, t).$$

The operator Q is the quasi-energy operator for Eq. (1),[6] since the solutions of the equations

$$(\hat{Q}(\mathbf{r}) - E_n)\varphi_{E_n}(\mathbf{r}) = 0 \tag{2}$$

correspond to the functions

$$\psi_{E_n}(\mathbf{r}, t) = \exp\{-iE_nt/\hbar\}\Phi_{E_n}(\mathbf{r}, t), \quad \Phi_{E_n}(t + 2\pi/\omega) = \Phi_{E_n}(t),$$

which are solutions of (1) with a definite quasi-energy E_n.[7]

As follows from the form of the operator Q, for the Hamiltonians

$$H_0 = \mathbf{p}^2/2m + U(r) \quad \text{c} \quad U(r) \underset{r\to\infty}{\longrightarrow} -0$$

the quasi-energy spectrum is continuous and occupies the entire real axis. Therefore in a rigorous formulation the problem of calculating the parameters of the quasi-stationary state corresponding to the unperturbed level E_0 of the Hamiltonian H_0 reduces to a calculation of the coefficients C_E of the expansion of the unperturbed state in terms of the complete system of functions $\varphi_E(\mathbf{r})$.[5]

In the model a zero-radius potential, however, the problem becomes much simpler. In this case the boundary condition that determines the behavior of $\varphi_E(\mathbf{r})$ as $r \to 0$ (in the region where the potential acts) is

$$\varphi_E(\mathbf{r}) \longrightarrow \frac{1}{4\pi}\left(\frac{1}{r} - \frac{1}{a_0}\right), \tag{3}$$

where a_0 is the scattering length and is connected with the level energy in the absence of the field:

$$E_0 = -\hbar^2/2ma_0^2.$$

A solution of equation (2) satisfying the radiation condition as $r \to \infty$ coincides with the Green's function G_E of the operator $p^2/2m - \omega\hat{L}_z + eFx$, and the condition (3) makes it possible to determine the real and the imaginary parts of the complex energy E, which give the position and the width of the quasi-stationary state in the field $\mathbf{F}(t)$. This approach is analogous to the solution obtained by Demkov and Drukarov[8,9] for the problem of a particle with a small binding energy in a stationary electric and stationary magnetic field.

In Sec. 2 below we obtain the wave function and the equation for the determination of the energy. It is important to note that both the shift and the width of the level are determined in this approach in a unified manner, as the real and imaginary parts of the root of a

transcendental equation. For frequencies that are large in comparison with the ionization potential, the level shift is equal to the average vibrational energy of the electron in the field of the wave. In Sec. 3 we consider the case of low frequencies $\omega \ll \omega_0$, when the ionization process has a quasi-classical character. For the shift we obtain an expansion in an asymptotic series (as $F \to 0$); the width coincides with the result of the adiabatic approximation [4]. In Sec. 4 we consider the general case of arbitrary frequencies. For $F \ll F_0$ we obtain perturbation-theory series in F^2 for the shift and the width. The first two terms of the series are investigated in detail. Since an electron in a well has an orbital angular momentum $l = 0$, the results of the second-order perturbation theory are valid for a field of arbitrary elliptic polarization. It is shown that the condition for the applicability of the first nonvanishing order of perturbation theory for the level shift and width is smallness of the quantities F^2 and $(\gamma K_0)^2$. Thus, perturbation theory cannot be used for the level width at $\gamma^2 \sim 1/K_0^2$, and in this case the summation of the perturbation-theory series leads to expression (19) for Γ, which agrees at $F \ll F_0$ with the result obtained by Perelomov, Popov, and Terent'ev. [3]

To ascertain the accuracy of the different approximations, the equation for the energy was solved numerically in a wide interval of frequencies and field intensities all the way to $F \approx F_0$. We conclude with a discussion of the applicability of the results to negative ions.

2. WAVE FUNCTION AND EQUATION FOR THE ENERGY OF A QUASI-STATIONARY STATE IN A δ POTENTIAL

Inasmuch as the boundary condition that determines the behavior of the wave function as $\mathbf{r} \to 0$ takes the form (3) for an interaction having a zero radius, Eq. (2) assumes in this case the form:

$$(\hat{Q}-E)\varphi_E(r)=\left(\frac{\mathbf{p}^2}{2m}-\omega\hat{L}_z+eFx-E\right)\varphi_E(\mathbf{r})=\frac{\hbar^2}{2m}\delta(\mathbf{r}). \qquad (4)$$

Thus, in a rotating coordinate system, the wave function coincides with the Green's function of a free particle in a constant electric field.

If we know the retarded Green's function of the equation

$$\left\{i\hbar\frac{\partial}{\partial t}-\hat{Q}\right\}G(\mathbf{r},t;\mathbf{r}',t')=i\hbar\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'),$$

then

$$\varphi_E(\mathbf{r})=\frac{i\hbar}{2m}\int_0^\infty dt\exp\left\{\frac{i}{\hbar}(E+i\eta)t\right\}G(\mathbf{r},t;0), \quad \eta\to+0.$$

The Hamiltonian Q in (4) depends quadratically on the coordinates and momenta, and therefore G can be obtained by the method of Feynman path integrals. It is simpler, however, to calculate G(**r**, t; 0) directly from the spectral expansion

$$G(\mathbf{r},t;0)=\theta(t)\int d\mathbf{p}\,\varphi_{\mathbf{p}}(\mathbf{r},t)\varphi_{\mathbf{p}}^*(0),$$

where $\varphi_{\mathbf{p}}(\mathbf{r}, t)$ is the wave function of a free electron in a circular wave in a rotating coordinate system.

The integral with respect to **p** is Gaussian and can be easily calculated. As a result we obtain

$$\varphi_E(\mathbf{r})=\frac{1}{4\pi}\left(\frac{m}{2\pi i\hbar}\right)^{1/2}\int_0^\infty\frac{dt}{t^{3/2}}\exp\frac{i}{\hbar}\left\{Et+\frac{mr^2}{2t}+\frac{eFy}{\omega}\left(\frac{\sin\omega t}{\omega t}-1\right)\right. \qquad (5)$$
$$\left.-\frac{2eFx}{\omega^2 t}\sin^2\frac{\omega t}{2}-\frac{e^2F^2}{2m\omega^2}\left(t-\frac{1}{\omega^2 t}4\sin^2\frac{\omega t}{2}\right)\right\}.$$

In analogy with the solution of the problem in crossed field, [9] the equation for the complex energy E of the quasi-stationary state that results from E_0 when the field is turned on can be obtained by separating that part of $\varphi_E(\mathbf{r})$ which diverges as $r \to 0$, and then substituting this function in (3). The equation for the wave number $\alpha = \sqrt{-2mE/\hbar}$ takes the form

$$\alpha=\alpha_0-\left(\frac{m}{2\pi i\hbar}\right)^{1/2}\int_0^\infty\frac{dt}{t^{3/2}}\exp\left(\frac{i}{\hbar}(E+i\eta)t\right)\left\{1\right.$$
$$\left.-\exp\left(-i\frac{e^2F^2}{2m\hbar\omega^2}\left[\left(1-\frac{4}{\omega^2t^2}\sin^2\frac{\omega t}{2}\right]\right)\right)\right\}.$$

It will be convenient later on to change in the equation to the "Rydberg" system of units (the energies are measured in units of $I = \hbar^2\alpha_0^2/2m$ and the field intensities in units of $F_0 = \hbar^2\alpha_0^3/me$). Putting $\beta = \alpha/\alpha_0$, we have

$$\beta=1-(4\pi i)^{-1/2}\int_0^\infty\frac{dx}{x^{3/2}}\exp(-i\beta^2x)\left\{1-\exp\left(-i\gamma^2x\left[1-\frac{\sin^2\delta x}{\delta^2x^2}\right]\right)\right\}. \qquad (6)$$

Thus, the characteristic parameters of the problem are γ^2 and δ: $\gamma = eF/\omega\sqrt{2mI} = 2F/\omega$ is the adiabaticity parameter first introduced by Keldysh, [1] and $2\delta = \hbar\omega/I$ is a quantity connected with the number of photons K_0 needed to ionize a bound level in accordance with the energy conservation law, $K_0 = \langle I/\hbar\omega + 1\rangle$. We note that the quantity γ^2 in (6) has also the meaning of the ratio of the average vibrational energy in the field $E_F = e^2F^2/2m\omega^2$ to the ionization potential: $\gamma^2 = E_F/I$.

We consider first the cases of high and low frequencies. At $\omega \gg \omega_0$, the sine term in (6) can be left out, so that we get $E = E_0 + E_F$. The level width, which is governed by the photoionization probability, does not appear in this case, since it is of higher order of smallness in ω^{-1}:

$$\Gamma\approx\omega^{-5/2}.$$

3. LEVEL SHIFT AND WIDTH IN THE ADIABATIC APPROXIMATION ($\omega \ll \omega_0 = I/\hbar$)

Let us investigate the level position in a well when the field frequency satisfies the inequality $\omega \ll \omega_0$. For fields that are sufficiently small in comparison with F_0, the ionization level is determined by the "leaking" of the particle through the barrier and has a quasiclassical character. [4] The integral (6) for β can then be calculated by the saddle-point methods.

The equation for the saddle points in (6) is of the form

$$\sin t+(\cos t-1)/t=\tfrac{1}{2}t(1+\gamma^{-2}), \quad t=2\delta x. \qquad (7)$$

If we introduce the variable $\tau_0 = it$, then (7) coincides with the equation considered by Popov, Kuznetsov, and Perelomov [4] for the quantity τ_0 that determines the total time $t_0 = \tau_0/\omega$ of the particle motion below the barrier. They also give the numerical values of τ_0 for $4 \geq \gamma \geq 0.01$.

For $\gamma \gg 1$, Eq. (7) can be solved analytically and we have for the root lying in the lower half of the t plane

$$t_1=-\frac{i}{\gamma}\left(1-\frac{1}{9\gamma^2}\right).$$

Changing over in (6) to integration along a contour lying in the lower half-plane and passing through the point t_1, we use the saddle-point method. The condition for the applicability of the method is $\tau\delta \gg 1$, i.e., $F \ll F_0$. The calculation of the integral (6) is analogous in this case to the calculations carried out by Drukarev and Monozon. [9]

As a result we obtain for the energy of the quasi-stationary state resulting from E_0, in a low-frequency field,

$$E=E_0-\frac{1}{4}\left(\frac{F}{F_0}\right)^2\left\{1+\frac{7}{24}\omega^2\right\}-i\Gamma,$$
$$\Gamma=\frac{F}{2F_0}\exp\left\{-\frac{2F_0}{3F}\left(1-\frac{1}{15\gamma^2}\right)\right\}, \quad (8)$$

where Γ is the level width and is connected with the tunneling probability. The expression obtained for Γ coincides with the results of [2,3] and with the adiabatic-approximation formula [4]. As follows from (8), an alternating field increases the level width in comparison with a constant field having the same amplitude. The dependence of the pre-exponential factor on the frequency appears only when terms $\sim\gamma^{-4}$ are taken into account. At $\omega = 0$, formula (8) goes over into the expression for the energy of a particle in a constant electric field, first obtained by Demkov and Drukarev. [8]

The level shift in (8) gives a quadratic Stark effect and can be calculated by perturbation theory. In the approximation $\omega \ll \omega_0$ it is easy to calculate also the higher-order field-dependent corrections to the level shift. For this purpose it is necessary to expand the sine term in (6) and retain the first two terms. Expanding then the exponential in a series and the integrating term by term, we obtain an equation for β, and by iterating this equation we obtain β in the desired order F^{2n} with allowance for terms $\sim\omega^2F^{2n}$.

The level width does not appear in this calculation, since it is a non-analytic function of F and cannot be obtained by perturbation theory. When the first three terms are taken into account the level shift $\Delta E = E - E_0$ is given by

$$\Delta E=-\frac{1}{4}F^2\left(1+\frac{7}{24}\omega^2\right)-\frac{3}{2}F^4\left(1+\frac{91}{36}\omega^2\right)-\frac{147}{32}F^6\left(1+\frac{7031}{56}\omega^2\right)-\ldots \quad (9)$$

As is well known, the perturbation-theory series for the level shift in a constant field is asymptotic and at $\omega = 0$ the coefficients of F^{2n} increased rapidly with increasing n.

At $\omega \neq 0$ the situation becomes somewhat more complicated. As will be shown in the next section, at $n > N \approx 1/\omega$ the coefficients of F^{2n} in (9) are oscillating functions of ω and cannot be expanded in powers of ω^2. At $\omega > 0$ the expression (9) therefore determines the asymptotic expansion of the shift ΔE as $F \to 0$, accurate to the N-th term.

As noted by Demkov and Drukarev [8], the shift ΔE for atoms can be much larger than in the δ-potential model. Thus, for the hydrogen atom in the ground state the expansion analogous to (9) takes the form [10]

$$\Delta E=-\frac{9}{2}F^2(1+1.4768\omega^2)-\frac{3555}{64}F^4(1+38.653\omega^2)-\ldots$$

4. CASE OF ARBITRARY FREQUENCIES ω

When the condition $\omega \ll \omega_0$ is not satisfied, the reasoning of the preceding section does not hold. We consider below the general case of arbitrary values of the parameter ω, assuming that the condition $F < F_0$ is satisfied. It is convenient in this case to write down the equation for the energy in the form

$$\sqrt{\varepsilon}=1-(4\pi i)^{-1/2}\int_0^\infty\frac{dt}{t^{3/2}}e^{-i\varepsilon t}\left\{1-\exp\left(i\frac{\gamma^2}{\delta^2 t}\sin^2\delta t\right)\right\}, \quad (10)$$

where

$$\varepsilon=(E_F-E)/I.$$

Expression (10) can be obtained from (6) by adding and subtracting the term $\exp(-i\gamma^2 t)$ in the curly brackets. Expanding the exponential with the sine in (10) in a power series and integrating term by term, we obtain for $\sqrt{\epsilon}$ the expansion

$$\sqrt{\varepsilon}=1-\sum_{n=1}^{\infty}\left(\frac{\gamma}{\delta}\right)^{2n}R_n, \quad R_n=\frac{1}{2n+1}\sum_{m=-n}^{n}\frac{(-1)^m(\varepsilon+2m\delta)^{n+1/2}}{(n-m)!(n+m)!}. \quad (11)$$

Since the parameter γ is proportional to F, it follows that (11) is in fact the Brillouin-Wigner perturbation-theory series for the energy ϵ in terms of the field intensity. The coefficient of F^{2n} contains, as it should, terms of the type $\epsilon \pm 2n\delta$, which correspond to interaction with n photons. By successive iteration it is possible to obtain from (11) an expression for the real and imaginary parts of E in the form of a series in powers of F^2. For a given frequency ω, the width corresponding to multiphoton ionization appears when account is taken of terms with $n \geq K_0$, where the minimum number of the quanta K_0 needed for the ionization is $K_0 = (I + E_F)/\hbar\omega$. The addition of E_F to I is explained by the fact that the electron, after absorbing n photons, can be regarded as free if its energy exceeds the average vibrational energy in the wave field. The series for the level shift begins with terms of order F^2, corresponding to the quadratic Stark effect.

We confine ourselves first to terms $\sim F^2$. For the level energy we have in this approximation

$$\Delta E=E-E_0=-\alpha_2(\omega)F^2,$$
$$\alpha_2(\omega)=-\frac{4}{\omega^2}-\frac{32}{3\omega^3}+\frac{16}{3\omega^4}\{(1+\omega)^{3/2}+(1-\omega)^{3/2}\}, \quad (12)$$

where α is the dynamic polarizability and determines the quadratic Stark effect of the E_0 level. At $\omega < 1$, the polarizability α_2 is expanded in a converging power series:

$$\alpha_2(\omega)=\frac{8}{\sqrt{\pi}}\sum_{K=0}^{\infty}\frac{\Gamma(2K+5/2)}{\Gamma(2K+5)}\omega^{2K}.$$

Inclusion of the first two terms of the series yields for ΔE an expression that agrees with the energy shift (8) in the tunnel case. At $\omega > 1$, the one-photon ionization channel is opened and α_2 has an imaginary part which gives the level width Γ_1 connected with the photoionization probability

$$\Gamma_1(\omega)=16F^2(\omega-1)^{3/2}/3\omega^5. \quad (13)$$

As follows from (13), the width has a root singularity at the threshold, reaches a maximum at $\omega \sim 1.6$, and decreases rapidly with increasing frequency. Re $\alpha_2(\omega)$ increases with increasing ω, reaches a maximum near the ionization threshold, goes through zero at $\omega \sim 2.3$, and tends to the asymptotic value $\alpha_2(\omega) = -4/\omega^2$. This corresponds to the fact that $\Delta E = E_F$ at high frequencies.

As already noted in the Introduction, the polarizability α_2 takes the form (12), and for a field with arbitrary elliptic polarization we have

$$\mathbf{F}(t)=\{x\cos\omega t,\ \varepsilon y\sin\omega t,\ 0\}, \quad 0\leqslant\varepsilon\leqslant 1.$$

In this case the term with F^2 in expression (12) for ΔE has an additional factor $(1 + \epsilon^2)/2$ corresponding to the fact that in an elliptically polarized field the average vibrational energy of the electron is $(1/2)(1 + \epsilon^2)E_F$. The dynamic polarizability $\alpha_2(\omega)$ determines in the dipole approximation the total cross section of the Rayleigh scattering of the radiation from a particle with small binding energy:

$$\sigma(\omega) = {}^8/_3\pi |\alpha_2(\omega)|^2\omega^4.$$

In this connection, an expression of the type (12) was obtained for α_2 by another method by Bethe and Peierls [11] in an investigation of the elastic scattering of γ quanta by a deuteron. According to the optical theorem, Im $\alpha_2(\omega)$ determines at $\omega > 1$ the cross section for the photodisintegration of the deuteron.

Allowance for terms of higher order in F^2 in the iterations of (11) yields the corrections of next order in the field to the level shift and width. In this sense, the coefficients α_{2n} in the expansion

$$\Delta E = -\sum_{n=1}^{\infty} \alpha_{2n}(\omega) F^{2n}$$

are the dynamic hyperpolarizabilities of the corresponding orders. Thus, for n = 2 we have

$$\alpha_4(\omega) = \frac{128}{15\omega^8}\{f(\omega) + f(-\omega)\}, \tag{14}$$

$$f(\omega) = 32 + 15\omega^2 + \frac{5}{3}(7-\omega^2)\sqrt{1-\omega^2} + \frac{3\omega^2 - 14\omega - 32}{12}\sqrt{1-\omega} - (1+2\omega)^{5/2}.$$

At $\omega \ll 1$ the expression in the curly brackets is $\sim\omega^8$ and $\alpha_4(\omega)$ coincides with the result of the adiabatic approximation (9). The expansion of $\alpha_4(\omega)$ in a power series converges at $\omega < 1/2$. At $1/2 \leq \omega \leq 1$ the imaginary part of $\alpha_4 F^4$ yields the level width

$$\Gamma_2(\omega) = \frac{128F^4}{15\omega^8}(2\omega - 1)^{5/2},$$

which corresponds to the two-photon ionization probability. At $\omega > 1$ the quantity Im $\alpha_4 F^4$ gives the corrections $\sim F^2$ to the one-photon width $\Gamma_1(\omega)$.

When the condition $n\omega < 1$ is satisfied, the energy change ΔE is real in all orders in the field up to order n inclusive. The first n iterations of series (11) then yield n terms of the asymptotic expansion of the level shifts in powers of F^2. The coefficients $\alpha_{2n}(\omega)$ can be expanded in the frequency region $\omega \sim 1/n$ in a converging series in powers of ω^2, and at $\omega > 1/n$ the quantity Re α_{2n} oscillates, n oscillations being produced in the n-th order. A comparison of the first two terms of the series shows that the corrections of higher order in the field to the Stark shift of the level can be neglected up to fields $F \sim 0.05\ F_0$.

At $1/(n-1) > \omega > 1/n$, the n-photon ionization channel is opened. The corresponding n-photon width follows from (11):

$$\Gamma_n(\omega) = \frac{1}{(2n+1)!}\left(\frac{4F}{\omega^2}\right)^{2n}(n\omega - \varepsilon)^{n+1/2}. \tag{15}$$

In the zeroth approximation, ϵ in this formula is equal to $1 + \gamma^2$.

The presence of root singularities of the type (15) at the threshold for ionization by circular light was noted also by Nikishov and Ritus. [2] The quantity $E_n = n\omega - \epsilon$ determines the energy of the photoelectron. Since the s-electron acquires an orbital angular momentum $l = n$ after absorbing n circularly polarized photons, it follows from the solution of the classical equation of motion of an electron in a circular field that the probability of small values of E_n is greatly suppressed with increasing n, in contrast to the case of linear polarization, where the threshold behavior is proportional to $E_n^{1/2}$ or $E_n^{3/2}$ for even or odd n, respectively.

It should be noted that as long as the n-photon ionization channel is closed ($\omega < 1/n$), the function $\Delta_n(\omega) = -i(-1)^n\Gamma_n(\omega)$ is real and yields a correction $\sim F^{2n}$ to the level shift, its contribution decreasing to zero as ω approaches $1/n$. As $\omega \to 0$, the terms $\sim\omega^k$ ($k \leq 4n - 1$), which arise when $\Delta_n(\omega)$ is expanded in powers of ω^2, are cancelled by the corresponding terms that appear in the n-th iteration of (11). As a result, as $\omega \to 0$, the contribution of the width of the closed n-photon ionization channel to the level shift is determined by the quantity

$$(-1)^n\frac{(4n^2F)^{2n}}{(2n+1)!}\{C_{4n}^{n+1/2} + (n\omega)^2 C_{4n+2}^{n+1/2}\}.$$

For experiments performed to date on multiphoton ionization of atoms, the most interesting case is $\gamma \lesssim 1$, and the question of the limits of applicability of the first nonvanishing order of perturbation theory to the description of the ionization is quite vital. The level width $\Gamma_n^{(2)}$, with allowance for the correction F^2, takes in our problem the form (we assume $\epsilon = 1$ for $\gamma \ll 1$)

$$\Gamma_n^{(2)}(\omega) = \Gamma_n(\omega)\left\{1 + \frac{1}{4n+6}\left(\frac{4F}{\omega^2}\right)^2\left[\frac{n\omega - 1}{n+1}\left\{\frac{(n+1)\omega - 1}{n\omega - 1}\right\}^{n+3/2} + \frac{(n+1/2)\omega^2 - (n\omega-1)^2}{2(n\omega-1)}\right]\right\}. \tag{16}$$

The corrections to the first nonvanishing order in weak fields consist of terms of two types. First, the correction connected with the direct (n + 1)-photon ionization (the first term in the square bracket). It is essential near the threshold, where $\Gamma_n(\omega) \approx 0$, and changes the energy distribution of the photoelectrons. Second, corrections to Γ_n come from processes with reradiation of photons (second term in the square bracket), which do not change the energy spectrum of the photoelectrons.

An analysis of (16) shows that processes with photon reradiation make a small contribution in the entire frequency range, and the corrections to the first nonvanishing order are determined by processes with absorption of a large number of photons. The applicability of perturbation theory is determined mainly by the smallness of the parameter $(4F/\omega^2)^2$. It is more convenient to write this condition in the form

$$\gamma^2 \ll K_0^{-2} \text{ or } E_F \ll I/K_0^2. \tag{17}$$

We note that a similar condition of applicability of perturbation theory was obtained for the problem of multiphoton excitation in a two-level system by Zaretskiĭ and Kraĭnov [12]. It should be borne in mind that the estimate (17) is quite crude, since the first term in the square brackets of (16) has a strong frequency dependence. In particular, it is equal to $3^{n+1}n^{-2}$ for the frequency ω corresponding to the center of the interval $(1/n, 1/(n-1))$.

Thus, the conditions for the applicability of perturbation theory for the level shift and width in an alternating field are greatly different. Therefore formula (15) for the level width is not valid if $\omega \ll \omega_0$, when many photons are required for the ionization. To calculate the width in this case we use the easily verified relation

$$\sum_{n=1}^{\infty}\left(\frac{\gamma}{\delta}\right)^{2n}\frac{1}{2n+1}\sum_{m>K_0}^{n}\frac{(-1)^m(e-2m\delta)^{n+1/2}}{(n-m)!(n+m)!}$$
$$=\sum_{m>K_0}^{\infty}(-1)^m\int_0^{(e-2m\delta)^{1/2}}dx\,I_{2m}\left(\frac{2\gamma}{\delta}x\right).$$

Recognizing that at $m > K_0$ the upper limit in the integral with respect to x is pure imaginary, and making the change of variable

$$x=-im\delta y/\gamma,$$

we rewrite expression (11) in the form

$$\sqrt{\varepsilon}-1=-\sum_{n=1}^{\infty}\left(\frac{\gamma}{\delta}\right)^{2n}R_n^{K_0}+\frac{i}{2}\Gamma, \qquad (18)$$

where $R_n^{K_0}$ differs from R_n in that at $n > K_0$ the summation over m begins with $m = -K_0$. The sum in (18) is real and determines the level shift. The width is given by the second term

$$\Gamma(\omega)=\frac{2\delta}{\gamma}\sum_{n>K_0}^{\infty}\int_0^{(\gamma/m\delta)(2m\delta-\varepsilon)^{1/2}}I_n(2ny)\,dy. \qquad (19)$$

If we disregard the level shift ($\epsilon = 1 + \gamma^2$), then expression (19) accurate to a factor that takes into account the difference between the asymptotic form of the wave function of the particle in a well of finite radius and the function of the particle in a δ-potential, coincides with formula (67) of the paper of Perelomov, Popov, and Terent'ev[3] for the ionization potential of an S level in the short-range potential of a circularly polarized wave. Since the integral in (19) at $\omega \ll \omega_0$ was investigated by us in detail[3], we shall not present here the final formulas. We note only that at $\gamma \gg 1$ the result coincides with (8).

To assess the accuracy of the various approximations, we have solved Eq. (10) for $\sqrt{\epsilon}$ numerically by an iteration method. As the zeroth iteration we assumed $\epsilon = 1$ in the right-hand side of (10). The results for the level shift in weak fields coincides with the quadratic shift given by formula (12), and as F increases the deviation from the quadratic dependence becomes appreciable.

Figure 1 shows the dependence of ΔE on F for two values of the frequency ω. At low frequencies, the inclusion of fourth-order terms is justified so long as the correction due to them is smaller than the quadratic term. For the width, the deviation from the power-law dependence F^{2K_0} sets in at much smaller field intensities F_{cr}, with F_{cr} decreasing with decreasing ω in accordance with (1). The dependence of the level width on the parameter ω at fixed frequency is given for a number of values of ω in Fig. 2. As follows from the figure, with increasing F ($F = \gamma\omega/2$), a transition from the multiphoton to the tunnel mechanism of the ionization process takes place. Figure 3 shows plots of $\Gamma(\omega)$ against the frequency ω for a number of fixed values of γ. (The field intensity F for each curve decreases in proportion to γ with decreasing ω.) A characteristic feature is the tendency of the frequency dependence to become smoother with decreasing ω. Thus, at $\omega \ll \omega_0$ and $\gamma \lesssim 1$ the level width is determined only by the parameter γ.

It is important to note that at $\gamma \lesssim 1$ it is necessary, when solving (10), to take into account several iterations in ε in order to obtain the correct value of Γ. The exact value of Γ is smaller here than that calculated in the zeroth approximation. Thus, for example, for $\omega = 0.2$

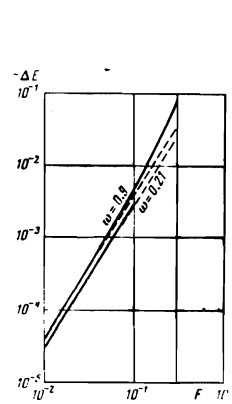

FIG. 1

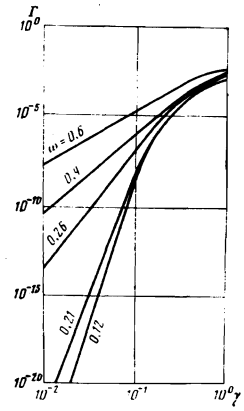

FIG. 2

FIG. 1. Dependence of the level shift ΔE on the field intensity F. The dashed lines correspond to the quadratic Stark effect.
FIG. 2. Dependence of the level width on the parameter γ.

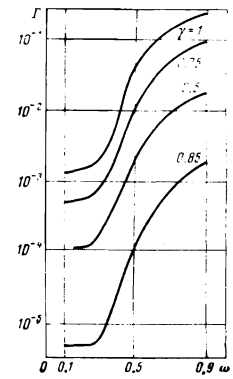

FIG. 3. Frequency dependence of the level width at fixed values of γ : reading downwards, γ = 1, 0.75, 0.5, and 0.25.

and $\gamma = 0.8$ the results differ by a factor of 1.5, which means that for strong fields the level width cannot be calculated separately from the shifts. The reason can be clearly understood from a classical consideration, if it is recognized that the shift ΔE leads to a lowering of the level E_0, increasing by the same token the width of the barrier and decreasing the tunneling probability. We recall in this connection that the formulas obtained in[1-4] for the ionization probability give the zeroth approximation in the sense indicated above, and therefore then can give results that are too high in the case of sufficiently strong fields.

In conclusion, we consider the application of the results to negative ions. It must be recognized in this case that the zero-radius potential approximation gives the minimum value of ΔE for the class of short-range potentials with one bound state.[8] The asymptotic behavior of the wave function of an S electron in a negative ion

$$\psi(\mathbf{r})\underset{r\to\infty}{\longrightarrow}B\left(\frac{\alpha_0}{2\pi}\right)^{1/2}\frac{1}{r}e^{-\alpha_0 r}$$

differs by a factor B from the wave function of the particle in a δ-potential. The interaction with the field at $F \ll F_0$ is determined by the matrix elements of the operator $\mathbf{F}\cdot\mathbf{r}$, for which distances r far from the nucleus are significant, and therefore, taking into account the same considerations as in the use of the quantum-defect method for the calculation of multiphoton processes in atoms[13], it is necessary to introduce the factor B^2 in the formulas for $\alpha_{2n}(\omega)$ and $\Gamma_n(\omega)$. Thus, for example, for the H^- ion we have $B^2 = 2.8$. When this factor is

taken into account we obtain for static polarization $\alpha_2(0)$ = 226 at. un. The most accurate calculations, using a variational wave function with 96 parameters, yield $\alpha_2(0)$ = 206.[14] The cross section for the photodecay of H^-, calculated from formula (13) with B^2 taken into account, also agrees well with experiment and with the exact calculations (the corresponding comparison is given at [14]).

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