Photoproduction of electron-positron pairs in a strong magnetic field and the field of a plane electromagnetic wave

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We consider in the framework of Dirac's hole theory, electron-positron pair production processes due to an external photon moving at right angles to the magnetic field and to the direction of propagation of a plane circularly polarized electromagnetic wave. Using the exact solution of the Dirac equation which describes the behavior of charged particles in such a field configuration we obtain formulae for the spectral and angular distributions of the pairs produced when the external photon interacts with "l photons" of the wave. We give a detailed analysis of single-quantum and two-quanta ($l=0, \pm 1$) pair production processes in all important ranges of the basic parameters of the problem, for instance, for magnetic fields comparable to the critical magnetic field $H \sim H_0 = m^2c^3/e\hbar = 4.4\times 10^{13}$ g.

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We have previously[1] studied the process where a strong electromagnetic wave was scattered by electrons in a static magnetic field for a special choice of the direction of propagation of the wave: the wave vector κ was oriented along the constant magnetic field strength vector $\mathbf{H}$.

It is well known[2] that one can for such a field configuration find an exact solution of the Dirac equation which in principle enables us to take the "photons" of the wave completely into account. Using this solution to analyze the interaction between the electrons and the photons leads, on the one hand, to the fact that the appropriate matrix elements of various processes are evaluated to a higher order in the fine-structure constant $\alpha_0 = e^2/\hbar c$ than in the usual perturbation theory, and, moreover, such an approach enables us to take into account effects connected with the wave intensity which are not present in the usual perturbation theory. A detailed discussion of the advantages and shortcomings of this approach is given in the well known papers by Nikishov, Ritus, and Gol'dman.[3,4]

In the present paper we consider the electron-positron pair production process due to an external photon which propagates at right angles to the direction of the vector $\mathbf{H}$ and hence also to κ. The external photon is taken into account in perturbation theory, and the whole process is considered in the framework of the Dirac hole theory and treated as the transition of an electron with absorption of the photon from a state with a negative quasi-energy to a state with a positive quasi-energy spectrum. We note here that Nikishov and Ritus[3] introduced the concept of an electron-four-quasimomentum in the field of an electro-magnetic wave.

Zel'dovich[5] and Ritus[6] developed the quasi-energy (the fourth component of the quasimomentum) method for applications to atomic systems. The quasi-energy method has also been applied to consider a number of problems in the field configuration studied by us.[1,7,8]

One should note that the process of the production of a pair by a photon in a magnetic field and the field of a weak electromagnetic wave has been considered before in[7,9-11]. The results obtained here by us for well-defined values of the parameters agree completely with those given in[7,9-11]. In addition to those results we obtain a number of new ones. In particular, we give a formula for the spectral and angular distribution of a pair produced by l photons of the wave and one external photon. We find the probabilities for the formation of a pair by two photons (external photon + wave photon) in all important ranges of the parameters of the problem ($\chi = H\kappa/H_0 k_0$, wave frequency ω, and cyclotron frequency $\omega_H = eH/mc$). We consider the process of the formation of a pair by two photons in a hyperstrong magnetic field $H \sim H_0 = m^2c^3/e\hbar$ and the field of the electromagnetic wave when one of the particles is produced in a state with large quantum numbers.

1. WAVE FUNCTIONS

Let the magnetic field $\mathbf{H} = (0, 0, H)$ be along the z-axis. The vector potential of a plane monochromatic circularly-polarized wave, propagating along the magnetic field, is given in the form

$$\mathbf{A}_1 = -\frac{cE_0}{\omega}(\mathbf{e}_1 \sin\omega\xi - g\mathbf{e}_2\cos\omega\xi),$$

and the total field is then $\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$, with $\mathbf{A}_2 = (0, -xH, 0)$; $\xi = t - z/c$, $g = \pm 1$ describes, respectively, right-hand and left-hand circular polarization. The electron wave functions in such a field are of the form[2]

$$\Psi = N\begin{pmatrix} ac_1\psi_{n-1} \\ bc_2\psi_n \\ ac_3\psi_{n-1} \\ bc_4\psi_n \end{pmatrix} + 2^{1/2}N(c_1 - c_3)\begin{pmatrix} bR^+\psi_n \\ -aR\psi_{n-1} \\ bR^+\psi_n \\ aR\psi_{n-1} \end{pmatrix}. \tag{1}$$

Here $\psi_n = \exp(-iS)U_n(\rho)$ are the Hermite functions connected with the Hermite polynomials H_n through the relation

$$U_n(\rho) = (2^n n!\sqrt{\pi})^{-1/2}\exp\left(-\frac{\rho^2}{2}\right)H_n(\rho),$$

$$\rho = \gamma^{1/2}(x - c\gamma_1\Delta^{-1}\cos\omega\xi) + k_2/\gamma^{1/2}, \qquad \Delta = \alpha\omega - g\omega_H,$$
$$k_0\alpha = K - k_3, \quad K = (k_0^2 + 2\gamma n + k_3^2)^{1/2},$$

$\hbar k_0 = mc$, $mc\omega_H = eH$, $c\hbar\gamma = eH$, $\hbar k_2$ and $\hbar k_3$ are the momenta along the y- and x-axes, respectively. The function S is given by the expression

$$S = ck_0\alpha t + ck_3\xi + g\gamma_1\Delta^{-1}(k_0\omega_H x + ck_2)\sin\omega\xi - k_2 y + 0.5\Delta^{-1}k_0\gamma_1^2\omega c\xi$$
$$-0.25g\Delta^{-2}k_0\gamma_1^2 c\omega_H\sin 2\omega\xi.$$

The function R and the normalization constant N are given by the following equations:

$$2^{3/2}\Delta R = ig\omega_H \exp(ig\omega\xi), \qquad N = \left[1 + \frac{\gamma_1^2\alpha^2\omega^2}{\Delta^2(1+\alpha^2+2\gamma n k_0^{-2})}\right]^{-1/2}$$

a, b, and c_i are spin coefficients.[12] The electron and positron quasi-energies and quasimomenta are equal to

$$\tilde{K}^{\mp} = K^{\mp} \pm 0.5 k_0 \gamma_1^2 \omega \Delta_{\mp}^{-1}, \qquad \tilde{k}_3^{\mp} = \tilde{K}^{\mp} - \alpha^{\mp}, \qquad \Delta_{\mp} = \pm\alpha^{\mp}\omega - g\omega_H,$$

where the minus sign corresponds to an electron and the plus sign to a positron.

2. QUASICLASSICAL CASE

We shall assume that the momentum of the external photon is along the y-axis: $\hbar\boldsymbol{\kappa}' = (0, \hbar\kappa, 0)$. The calculation of the matrix elements and the probabilities for the pair production processes is performed along the same lines as in[1] with only this difference that the electron transition occurs from an initial state with a negative energy to a state with a positive energy. The probability for such a transition is for a given polarization of the external photon given by the formula[12]

$$W_j(\zeta) = \frac{\alpha_0 c k_0^2 H}{2\varkappa H_0} \sum_{n,n',l} \int_{-\infty}^{\infty} d\tilde{k}_3 |\alpha_j|^2 \delta(\tilde{K}^- + \tilde{K}^+ - \varkappa' - l\varkappa),$$

where $\kappa = \omega/c$; l is the number of wave photons participating in the reaction; j is the index determining the polarization of the photon.

It is convenient to introduce the invariant quantities[3]

$$\tilde{\varkappa} = \frac{2l\varkappa_\mu'\varkappa^\mu}{k_0^2}, \qquad u = \frac{(\varkappa_\mu'\varkappa^\mu)^2}{(\varkappa_\mu \tilde{k}^{-\mu})(\varkappa_\mu k^{+\mu})} = \frac{\varkappa'^2}{k_0^2\alpha^-\alpha^+},$$

$$\chi = \frac{e\hbar^2}{m^3c^4}\sqrt{-(F_{\mu\nu}\varkappa'^\nu)^2} = \frac{H\varkappa'}{H_0 k_0}, \qquad \nu, \mu = 0, 1, 2, 3$$

and after that evaluate the matrix elements of the Dirac matrices α_j.

We shall assume that the intensity of the electromagnetic wave is small, i.e., $\gamma_1 = eE_0/mc\omega \ll 1$ (E_0 is the wave amplitude). Moreover, we assume that the energy of the external photon is much larger than the energy of the wave "photons" and also than the rest mass energy of the electron ($\kappa' \gg l\kappa$, $\kappa' \gg k_0$). In that case the energy of each of the particles produced is large and the main contribution goes into the "transverse motion" energy. Assuming that each of the ratios k_3^+/K^+, k_3^-/K^-, $k_0/K^\pm$ is much less than unity we can obtain, after some calculations, the following expressions for the probabilities:

$$dw_j(\zeta) = \sum_l \frac{\alpha_0 c\varkappa'^3}{\chi k_0^2} \frac{du\, dv}{u^2[u(u-4)]^{1/2}} |\alpha_j|^2, \tag{2}$$

where

$$\alpha_j = \left(\frac{2\chi}{u}\right)^{1/3} \frac{u^{1/2}}{2\pi^{1/2}} \frac{k_0^2}{\varkappa'^2} F_j, \tag{3}$$

$$\begin{aligned} F_1 = {} & \frac{1+\zeta^-\zeta^+}{2}[u(u-4)]^{1/2}\Big\{\gamma_2 f_1 \Phi J_l'(p) \\ & + \left[\zeta^+ \frac{u}{[u(u-4)]^{1/2}}\Phi - \left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_l(p) + \gamma_2 \frac{l}{p}\frac{2lu}{\tilde{\varkappa}} f_1 \left[\left(1+v^2-\frac{\varkappa}{u}\right)\Phi \right. \\ & \left. - \zeta^+ \frac{u}{[u(u-4)]^{1/2}}\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_l(p)\Big\} + \frac{1-\zeta^-\zeta^+}{2} u \Big\{\left(v + g\gamma_2 \frac{l}{p} f_1\right)\Phi J_l(p) \\ & - \gamma_2 v \frac{2lu}{\tilde{\varkappa}} f_1 \frac{l}{p}\left(\frac{2\chi}{u}\right)^{1/3} J_l(p)\Phi'\Big\}, \end{aligned} \tag{4}$$

$$\begin{aligned} F_2 = {} & \frac{1+\zeta^-\zeta^+}{2}[u(u-4)]^{1/2}\Big\{\left(v + g\gamma_2\frac{l}{p}f_1\right)\Phi J_l(p) \\ & - \gamma_2 v \frac{2lu}{\tilde{\varkappa}} f_1 \frac{l}{p}\left(\frac{2\chi}{u}\right)^{1/3} J_l(p)\Phi'\Big\} - \frac{1-\zeta^-\zeta^+}{2} u \Big\{\gamma_2 f_1 \Phi J_l'(p) \\ & + \left[\zeta^+\Phi - \left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_l(p) + \gamma_2 \frac{l}{p}\frac{2lu}{\tilde{\varkappa}} f_1 \left[\left(1+v^2-\frac{\tilde{\varkappa}}{u}\right)\Phi \right. \\ & \left. - \zeta^+\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_l(p)\Big\}, \end{aligned} \tag{5}$$

$$p = -g\gamma_2 \frac{2lu}{\tilde{\varkappa}} f_1 \left(v + 2g\frac{\chi}{\tilde{\varkappa}}\right),$$

$$f_1 = \left[\left(1 + g\frac{\omega_H}{\alpha^+\omega}\right)\left(1 - g\frac{\omega_H}{\alpha^-\omega}\right)\right]^{-1},$$

$$\gamma_2 = \gamma_1/(1+\gamma_1^2)^{1/2}, \qquad v = k_3/k_0,$$

$\Phi = \Phi(y)$ is the Airy function of the argument $y = (u/2\chi)^{2/3}(1 + v^2 - \tilde{\kappa}/u)$; $J_l(p)$ is a Bessel function $\zeta^- = \pm 1$ and $\zeta^+ = \pm 1$ determine the projections of the positron and electron spins, respectively, in the direction parallel (+) and antiparallel (−) to the magnetic field. A characteristic feature of Eq. (2) is the presence in it of the typical resonance factor f_1. The resonance in this case is of a purely classical nature and arises every time when the frequency of rotation along the orbit in the magnetic field (taking the Doppler shift along the z-axis into account) is the same for the produced particle as the wave frequency.

The equation for the probability for pair production by a single external photon in a magnetic field and in the field of a plane circularly polarized wave follows from Eq. (2) for $l = 0$:

$$\begin{aligned} dw = {} & \frac{c\alpha_0 k_0^2}{\pi\varkappa'\chi}\left(\frac{2\chi}{u}\right)^{2/3} \frac{du\, dv}{[u(u-4)]^{1/2}} \Big\{\left[\Phi^2 - (1+v^2)\left(1-\frac{u}{2}\right)\left(\Phi^2 + \frac{\Phi'^2}{\eta}\right)\right] \\ & \times J_0^2(p) - \left(1-\frac{u}{2}\right)\gamma_2 f_1 \Phi J_0'(p)\left[\gamma_2 f_1 \Phi J_0'(p) - 2\left(\frac{2\chi}{u}\right)^{1/3}\Phi' J_0(p)\right]\Big\} \end{aligned} \tag{6}$$

where $\eta = (u/2\chi)^{2/3}(1 + v^2)$. It is important that the probability for the pair production by a single photon in a magnetic field and the field of the wave is smaller than the corresponding probability in a purely magnetic field ($J_0(p) \le J_0(0)$). If we put $\gamma_1 = 0$ in Eq. (6) we get Klepikov's result[13] (see also[12]).

We further consider the case $l = \pm 1$, i.e., when apart from the external photon one wave photon participates in the reaction. The case $l = 1$ corresponds to a two-photon pair production in a magnetic field, and $l = -1$ describes the process of the production of a pair by an external photon with the simultaneous absorption of a photon which is identical with a wave photon, i.e., the process

$$\gamma' \to e^+ + e^- + \gamma. \tag{7}$$

Zhukovskiĭ and Herrmann[10] indicated the possibility of such a process in an external magnetic field. We give here the formulae for the probabilities of the spectral distributions for the cases $l = \pm 1$ (see also[11]) in the range $\chi/\tilde{\kappa} < \kappa'/4k_0$:

$$\begin{aligned} \begin{bmatrix} W_1(\zeta) \\ W_2(\zeta) \end{bmatrix} = {} & \frac{\gamma_2^2\alpha_0 c k_0^2}{8\pi^{1/2}\tilde{\varkappa}'\varkappa^2}\int_4^\infty \frac{du}{[u(u-4)]^{1/2}}\Big\{\delta_{\zeta^-,\zeta^+}u^2\begin{bmatrix} R_1 - \zeta^+R_3 + R_2 \\ R_4 \end{bmatrix} \\ & + \delta_{\zeta^-,\zeta^+}u(u-4)\begin{bmatrix} R_4 \\ R_1 - \zeta^+\dfrac{u}{[u(u-4)]^{1/2}}R_3 + \dfrac{u^2}{u(u-4)}R_2 \end{bmatrix} \end{aligned} \tag{8}$$

$$\begin{aligned} R_1 = {} & F\left(1 \mp \frac{\tilde{\varkappa}}{u} + \frac{\tilde{\varkappa}^2}{2u^2} - \frac{4\chi^2}{\tilde{\varkappa}^2}\right) \\ & + \Phi\left(\frac{6\chi}{u} - \frac{8\chi}{\tilde{\varkappa}}\right)\left(\frac{\chi}{u}\right)^{1/3} - \Phi'\left(\frac{\chi}{u}\right)^{2/3}\left(1 + \frac{12\chi^2}{\tilde{\varkappa}^2}\right), \end{aligned} \tag{9}$$

$$R_2 = -2F\left(1 \mp \frac{\tilde{\varkappa}}{u} - 4\frac{\chi^2}{\tilde{\varkappa}^2}\right) - 8\Phi\frac{\chi}{\tilde{\varkappa}}\left(\frac{\chi}{u}\right)^{1/3} - 4\Phi'\left(\frac{\chi}{u}\right)^{2/3}, \tag{10}$$

$$R_3=-4\Phi\left(1\mp\frac{\tilde{\varkappa}}{2u}+4\frac{\chi^2}{\tilde{\varkappa}^2}\right)\left(\frac{\chi}{u}\right)^{1/3}-16\frac{\chi}{\tilde{\varkappa}}\left(\frac{\chi}{u}\right)^{2/3}\Phi', \qquad (11)$$

$$R_4=F\left(1\mp\frac{\tilde{\varkappa}}{u}+\frac{\tilde{\varkappa}^2}{2u^2}-\frac{4\chi^2}{\tilde{\varkappa}^2}\right)+2\Phi\left(\frac{\chi}{u}\right)^{1/3}+\left(\frac{\chi}{u}\right)^{2/3}\left(1-\frac{4\chi^2}{\tilde{\varkappa}^2}\right)\Phi' \qquad (12)$$

where

$$F(y)=\int_y^\infty \Phi(x)\,dx, \quad \Phi=\Phi(y), \quad y=\left(\frac{u}{\chi}\right)^{2/3}\left(1\mp\frac{\tilde{\varkappa}}{u}\right), \quad \tilde{\varkappa}=2\varkappa\varkappa'/k_0^2. \qquad (13)$$

The lower sign refers to the process (7). We can obtain from Eqs. (8) to (13) the probabilities for pair production for different ratios of the parameter $\chi/\tilde{\kappa}$. For instance, if $\chi/\tilde{\kappa} \ll 1$ (weak magnetic field) we find by integrating over u from Eqs. (8) to (13):

$$W=\gamma_2^2\frac{\alpha_0 c k_0^2}{4\varkappa'}\left\{\left(2+\frac{8}{\tilde{\varkappa}}-\frac{16}{\tilde{\varkappa}^2}\right)\ln\left[\left(\frac{\tilde{\varkappa}}{4}-1\right)^{1/2}+\left(\frac{\tilde{\varkappa}}{4}\right)^{1/2}\right] -\left(1+\frac{4}{\tilde{\varkappa}}\right)\left(1-\frac{4}{\tilde{\varkappa}}\right)^{1/2}-\frac{16\chi^2}{\tilde{\varkappa}^2}\left(\frac{3}{4}+\frac{2}{\tilde{\varkappa}^2}-\frac{2}{\tilde{\varkappa}^2}\ln\tilde{\varkappa}\right)\right\}. \qquad (14)$$

The first two terms, independent of χ in Eq. (14), give the well-known Breit-Wigner formula and the last term a correction to it introduced by the magnetic field. The probability for process (7) is in this case exponentially small. This result follows from the properties of the Airy functions. When $l = -1$ its argument is always positive and the condition $\chi \to 0$ entails $y \gg 1$, but in that region the Airy function is exponentially small.

In the region $1 \ll 2\chi/\tilde{\kappa} \ll \kappa'/2k_0$, which corresponds approximately to resonance, the probabilities for single-quantum processes ($l = \pm 1$) turn out to be equal and can as follows be expressed in terms of the probability $w^{(0)}(l = 0)$, which, as we noted above, characterizes the pair production process in a purely magnetic field:

$$dw_{1,2}^{(\pm1)}=\gamma_2^2\left(\frac{2u}{\chi}\right)^2\frac{\chi^4}{\tilde{\varkappa}^4}dw^{(0)}. \qquad (15)$$

In the range $2\chi/\tilde{\kappa} > \kappa'/2k_0$ (above-resonance region) the single-quantum probabilities ($l = \pm 1$) are also the same and the following relation holds:

$$dw_{1,2}^{(\pm1)}=\gamma_2^2\frac{\varkappa'^4}{4\chi^2 k_0^4}dw^{(0)}. \qquad (16)$$

We draw attention to the fact that the way the pair production probabilities depend on the spins of the particles is determined by the same functional behavior as for the case of pair production in a purely magnetic field. This is clear from Eqs. (15) and (16) and can be obtained from Eqs. (8) to (13) for the range $\chi/\tilde{\kappa} \ll 1$. However, this is not a necessary consequence as there is continuous transition from this region to the region $1 \ll 2\chi/\tilde{\kappa} \ll \kappa'/2k_0$.

When the external photon interacts with l wave photons (l-quanta process) the pair production probability will depend strongly on the intensity of the wave:

$$dw_{1,2}^{(l)}\sim J_l^2(p)\,dw_{1,2}^{(0)}.$$

We note here that for a given polarization of the electromagnetic wave resonance can occur for only one of the particles which is explained by the different nature of the motion of an electron or a positron in a magnetic field (right-hand or left-hand circular motion).

Oleĭnik[7] has considered the electron-positron pair production in fields with the given configuration near the resonance point. An important peculiarity of the process is the fact that the production phenomenon occurs for energies of the external photon which are appreciably lower than the threshold energy (in the present case $\kappa' \ll 2k_0$). If, for instance, the wave frequency $\omega \sim 10^{15}$ Hz, wave photons contribute little to the total energy of the pair in single-quantum processes $l = \pm 1$ and its energy is mainly determined by the external photon. Generally speaking, it is apparently incorrect to consider the strict limit $\alpha\omega = g\omega_H$ as in that case the spectra of the quasi-energy states (QES) of the electrons and the positrons are mixed up and the single-particle approximation becomes inadmissible. We note also that such a situation with respect to the QES spectra arises only in the quasiclassical approach and when damping effects are neglect. A strictly quantal approach removes this difficulty with QES.[14]

However, for values of $\alpha\omega$ close to ω_H the energy threshold for the external photon is, indeed, less than $2k_0$,[7] while the pair production occurs with a very small probability. The lowering of the threshold follows from the fact that the quantities α^+ and α^-, which are equal to the quasi-energy and quasimomentum differences along the z-axis, occur in the invariant quantity u, rather than the pair energy. Because of this the limiting value u = 4 near the point $\alpha\omega \sim g\omega_H$ may correspond to values $\kappa' < 2k_0$. As the quantity $\alpha\omega$ must be small, in the case considered pairs are produced with large quasi-momenta along the z-axis and with small transverse momenta.

3. PAIR PRODUCTION IN A STRONG MAGNETIC FIELD

We now consider the range $H/H_0 \sim 1$, $k_0/\kappa' \ll 1$. As was noted in[15], the main feature in this case is that the contribution to the total probability for the production of particles with greatly differing quantum numbers ($n' \sim n(k_0/\kappa')^2$) is no longer small. In this region ($H/H_0 \sim 1$) the generalized Laguerre functions $I_{nn'}(r)$ can be approximated by parabolic cylinder functions:[15]

$$I_{nn'}(r)=(-1)^{n'}(n'!\sqrt{2\pi n})^{-1/2}D_{n'}(r), \quad r=\left(\frac{2H_0}{H}\right)^{1/2}\frac{K^+}{k_0}. \qquad (17)$$

In the region $\omega > \omega_H$, $\hbar\omega > mc^2$ the probabilities for single-quantum processes are given by the expression

$$W(\zeta)=\frac{\gamma_1^2\alpha_0 c k_0}{2\pi^{1/2}\varkappa'}\left(\frac{H}{H_0}\right)^{1/2}\sum_{n'}\int dk_3^+G_1, \quad K_0^+=(2\gamma n'+k_0^2)^{1/2},$$
$$G_1=\frac{1}{16n'!}\left[\left(1-\zeta^+\frac{k_0}{K_0^+}\right)(n'D_{n'-1}^2-D_{n'}^2)+2D_{n'}^2\right]. \qquad (18)$$

Integrating in Eq. (18) over k_3^+ we get easily

$$W(\zeta)=\frac{\alpha_0 c k_0^2}{4\varkappa'}\frac{H}{H_0}\gamma_1^2\sum_{n'}(-1)^{n'}\int_\mu^\infty e^{-y/2}\left\{L_{n'}(y) -\frac{1}{2}\left(1-\zeta^+\frac{k_0}{K_0^+}\right)[L_{n'}(y)+L_{n'-1}(y)]\right\}dy, \quad \mu=4n'+\frac{2H_0}{H}, \qquad (19)$$

where L_n' is a Laguerre polynomial. In particular, the probability for pair production in which the positron is in the ground state ($n' = 0$) can be estimated from the formula

$$W\sim\frac{\alpha_0 c k_0^2}{2\varkappa'}\gamma_1^2\frac{H}{H_0}e^{-H_0/H}. \qquad (20)$$

If the conditions are such that $1 \ll n' \ll n$, the single-quantum probabilities ($l = \pm 1$) decrease as $(n')^{-2/3}$ with increasing n':

$$W_{nn'}=\frac{\Gamma(2/3)3^{1/6}}{\pi 2^{14/3}}\frac{\alpha_0 c k_0^2 H}{\varkappa' H_0}\frac{1}{n'^{2/3}}. \qquad (21)$$

For l-quanta processes we can get the following expression:

$$dw^{(l)} \sim J_{l-1}^2(p)\,dw^{(1)} \tag{22}$$

In the frequency range $\alpha\omega < \omega_H$ the expressions for the probabilities for the various processes are somewhat changed

$$dw^{(l)} \sim J_l^2(p)\,dw^{(0)}, \tag{23}$$

where

$$dw^{(0)} = \sum_{n'} \frac{dk_3^+}{2\pi^{1/2}} \frac{\alpha_0 c k_0}{\varkappa'} \left(\frac{H}{H_0}\right)^{1/2} G_{2n},$$
$$G_2 = \frac{1}{4n'!}\left[\left(1-\zeta^+\frac{k_0}{K_0^+}\right)(n'D_{n'-1}^2 - D_{n'}^2) - \frac{2}{r}\frac{d}{dr}D_{n'}^2\right]. \tag{24}$$

Integrating over k_3^+ in Eq. (24) we find

$$W^{(l)} = \frac{\alpha_0 c k_0^2}{4\varkappa'}\frac{H}{H_0}\sum_{n'} J_l^2(p)(-1)^{n'}\Big\{e^{-\mu/2}L_{n'}(\mu)$$
$$-\frac{1}{4}\left(1-\zeta^+\frac{k_0}{K_0^+}\right)\int_\mu^\infty e^{-x/2}[L_{n'}(x)+L_{n'-1}(x)]\,dx\Big\}. \tag{25}$$

For a single-quantum process $(l = \pm 1)$ we get from (25) for $\gamma_1 \ll 1$ and $n' = 0$

$$W \approx \frac{\alpha_0 c k_0^2}{16\varkappa'}\frac{H}{H_0}p^2 e^{-H_0/H}. \tag{26}$$

We note here that the laser intensities which are at present attainable correspond, for instance, in the visible range of frequencies to a value of the parameter $\gamma_1 \sim 0.1$. Existing pulsed magnetic fields reach values $\sim 10^6$ to 10^7g. Hence, for laboratory studies only the results of single-quantum and two-quanta processes $(l = 0, \pm 1)$ are of practical interest. The situation is, however, changed when we analyze similar processes near astrophysical objects, for instance, near pulsars where one assumes that the above-mentioned parameters are very large:[16] $\gamma_1 \sim 10^{11}$, $H \sim 10^{12}$ to 10^{13}g. Under such conditions the analysis of many-quanta processes which we gave above may also turn out to be useful.

[1] I. M. Ternov, V. G. Bagrov, V. R. Khalilov, and V. N. Rodionov, Yad. Fiz. 22, 1040 (1975) [Sov. J. Nucl. Phys. 22, 542 (1976)].
[2] P. J. Redmond, J. Math. Phys. 6, 1163 (1965).
[3] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 776 (1964); 47, 1130 (1964) [Sov. Phys.-JETP 19, 529 (1964); 20, 757 (1965)].
[4] I. M. Gol'dman, Zh. Eksp. Teor. Fiz. 46, 1412 (1964) [Sov. Phys.-JETP 19, 954 (1964)].
[5] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 51, 1492 (1966); Usp. Fiz. Nauk 110, 139 (1973) [Sov. Phys.-JETP 24, 1006 (1967); Sov. Phys.-Uspekhi 16, 427 (1973)].
[6] V. I. Ritus, Zh. Eksp. Teor. Fiz. 51, 1544 (1966) [Sov. Phys.-JETP 24, 1041 (1967)].
[7] V. I. Oleĭnik, Zh. Eksp. Teor. Fiz. 61, 27 (1971) [Sov. Phys.-JETP 34, 14 (1972)].
[8] I. M. Ternov and V. R. Khalilov, Yad. Fiz. 16, 174 (1972) [Sov. J. Nucl. Phys. 16, 94 (1973)].
[9] V. P. Leventuev, O. S. Pavlova, and V. R. Khalilov, Vestnik Moscow Univ., Phys.-Astron. 4, 89 (1969).
[10] V. Ch. Zhukovskiĭ and I. Herrmann, Yad. Fiz. 14, 1014 (1971) [Sov. J. Nucl. Phys. 14, 569 (1972)].
[11] V. Ch. Zhukovskiĭ and N. S. Nikitina, Yad. Fiz. 19, 148 (1974) [Sov. J. Nucl. Phys. 19, 77 (1974)].
[12] A. A. Sokolov and I. M. Ternov, Relyativistskiĭ élektron (The Relativistic Electron) Nauka, Moscow, 1974.
[13] N. P. Klepikov, Zh. Eksp. Teor. Fiz. 26, 19 (1954).
[14] A. E. Kazakov and M. V. Fedorov, Short Reports on Physics, Acad. Sc. USSR, Lebedev Physical Institute, 6, 3 (1973).
[15] A. A. Sokolov, I. M. Ternov, A. V. Borisov, and V. Ch. Zhukovski, Phys. Lett. 49A, 9 (1974).
[16] J. P. Ostriker and J. E. Gunn, Astroph. J. 157, 1395 (1969).

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