Quantum waves in a homogeneous electron plasma

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We use the quantum dielectric tensor to obtain dispersion relations for arbitrary nonpotential waves of a collisionless electron plasma in a quantized magnetic field. We study the frequency spectra and damping rates of essentially quantum waves. We compare the quantum spectra with the classical ones.

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1. INTRODUCTION

We study in the present paper electromagnetic waves in a homogeneous collisionless ($\omega \gg \nu_e$) electron plasma in a strong quantized magnetic field B_0 when

$$\hbar\Omega_e \gg T_e,$$

where T_e is the temperature of the electron gas in energy units and $\Omega_e = |e|B_0/mc$ is the electron Larmor (cyclotron) frequency. Electromagnetic waves in a plasma in quantized magnetic fields have been called quantum waves.

Potential quantum waves in single-component uniform and two-component non-uniform Maxwellian plasmas have been studied in[1,2]. Here we shall consider arbitrary non-potential quantum waves. To do this we need an expression for the dielectric tensor of a quantum plasma. Zyryanov and Kalashnikov[3] were the first to develop the quantum theory of the tensor $\epsilon_{ij}(\omega, \mathbf{q})$. Unfortunately, an error slipped into the calculations of that paper and the quantum tensor $\epsilon_{ij}(\omega, \mathbf{q})$ obtained does not reduce in the limit as $h \to 0$ and $T_e \to 0$ to the well-known classical result. Later[4,5] a correct quantum tensor $\epsilon_{ij}(\omega, \mathbf{q})$ was obtained for a Maxwellian plasma by a somewhat different method. However, when analyzing the electromagnetic wave spectra Korneev and Starostin[4] restricted their discussion to the limit $\alpha_{qu} = \lambda^2 q_\perp^2/2 \ll 1$ (where $\lambda = (\hbar c/|e|B_0)^{1/2}$ is the so-called magnetic length while q_z and $q_\perp$ are the components of the wave vector $\mathbf{q}$ along and at right angles to the magnetic field) which strongly restricted the most interesting quantum wave spectra. As the role of quantum effects increases with increasing parameter α_{qu} we shall assume in what follows that $\alpha_{qu} > 1$, and hence, that $q^2c^2 \gg \omega^2$, i.e., the quantum waves studied are in the region of large plasma refractive indices $n^2 = q^2c^2/\omega^2 = c^2/v_{ph}^2 \gg 1$. It was shown in[4] that quantum effects in the small n^2 region, when $\alpha_{qu} \ll 1$, are practically always unimportant (except in the case when the waves propagate strictly at right angles to the magnetic field).

In a uniform collisionless electron plasma the quantum effects turn out to be important in the cyclotron frequency region $\omega \approx s\Omega$,

$$|\omega - s\Omega| \gg |q_z| v_T. \tag{1.1}$$

We shall assume that apart from (1.1) the following inequality holds:

$$\frac{\lambda^2 q_z^2}{2}\frac{\Omega}{\omega - s\Omega} \ll 1, \tag{1.2}$$

i.e., just as before[1,2], we shall study quantum effects connected with only the quantization of the orbital motion of the charged particles.

Under the above-mentioned conditions and if we use the fact that in a strong magnetic field in the cases of practical interest we have always $\Omega^2 \gg \omega_p^2$ (ω_p is here the electron plasma frequency), the components of the dielectric tensor of a quantum plasma with a Maxwellian distribution function take the following form:

$$\begin{gathered}
\varepsilon_{11} = 1 - \frac{|s|}{\alpha_{qu}} F\left(1 - i\left(\frac{\pi}{2}\right)^{1/2} \beta_s \frac{s\Omega}{\omega} \exp\left(-\frac{\beta_s^2}{2}\right)\right),\\
\varepsilon_{22} = 1 - \frac{\alpha_{qu}}{|s|} F\left(1 - i\left(\frac{\pi}{2}\right)^{1/2} \beta_s \exp\left(-\frac{\beta_s^2}{2}\right)\right),\\
\varepsilon_{33} = 1 - \frac{2T}{|s|h\Omega} F\left(1 - i\left(\frac{\pi}{2}\right)^{1/2} \beta_s^3 \exp\left(-\frac{\beta_s^2}{2}\right)\right),\\
\varepsilon_{12} = -\varepsilon_{21} = iF\left(1 - i\left(\frac{\pi}{2}\right)^{1/2} \beta_s \exp\left(-\frac{\beta_s^2}{2}\right)\right),\\
\varepsilon_{13} = \varepsilon_{23} = 0,
\end{gathered} \tag{1.3}$$

where for $1 < \alpha_{qu} < e^{\varphi}/2$

$$F = \frac{\omega_p^2}{2\omega(\omega - s\Omega)} \frac{\alpha_{qu}^s}{s!} e^{-\alpha_{qu}}, \tag{1.4}$$

while for $\alpha_{qu} > e^{\varphi}/2$

$$F = \frac{\omega_p^2}{\omega(\omega - s\Omega)} \frac{e^{-\alpha_{qu}}}{2(4\pi\alpha_{qu})^{1/2}}. \tag{1.5}$$

Here

$$\beta_s = \omega - s\Omega/|q_z|v_T, \quad \varphi = \hbar\Omega/2T.$$

2. QUANTUM CYCLOTRON WAVES IN AN ELECTRON PLASMA

To find the spectra of the quantum waves we substitute Eq. (1.3) into the dispersion equation of arbitrary non-potential waves (see, e.g.,[6]):

$$\left|\varepsilon_{ij} + \frac{q_i q_j}{\omega^2}c^2 - \frac{q^2c^2}{\omega^2}\delta_{ij}\right| = 0. \tag{2.1}$$

Bearing in mind that $n^2 \gg 1$ under the conditions considered by us, we get two equations from (2.1):

$$\varepsilon_{33} - \frac{q_\perp^2 c^2}{\omega^2} = 0, \quad \varepsilon_{11}\left(\varepsilon_{22} - \frac{q^2c^2}{\omega^2}\right) + \varepsilon_{12}^2 = 0, \tag{2.2}$$

which determine the spectra of the ordinary and the extraordinary electromagnetic waves.

We find from the first Eq. (2.2) the following spectrum of the ordinary quantum wave:

$$\frac{\omega}{s\Omega} = \begin{cases} 1 - \dfrac{\omega_p^2}{\Omega^2}\dfrac{\alpha_{qu}^s}{|s|\xi_{qu} s!} e^{-\alpha_{qu}} & \text{if} \quad 1 < \alpha_{qu} < e^{\varphi}/2, \\ 1 - \dfrac{\omega_p^2}{\Omega^2}\dfrac{e^{-\alpha_{qu}}}{s\xi_{qu}(4\pi\alpha_{qu})^{1/2}} & \text{if} \quad \alpha_{qu} > e^{\varphi}/2, \end{cases}$$

$$\gamma = -\left(\frac{\pi}{2}\right)^{1/2} \frac{(\omega - s\Omega)^4}{|q_z|^3 v_T^3} \exp\left(-\frac{\beta_s^2}{2}\right), \tag{2.3}$$

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where $\xi_{qu} = 2\varphi c^2 q_\perp^2/\Omega^2 = 2\varphi\xi_{cl}$. On the other hand, the second Eq. (2.2) gives the spectrum of the plasma quantum wave (under the conditions considered the extra-ordinary wave degenerates into the plasma wave):

$$\omega^2 = \begin{cases} s^2\Omega^2 + \dfrac{\omega_p{}^2 \alpha_{qu}^{s-1}}{(s-1)!} e^{-\alpha_{qu}} & \text{if} \quad 1 < \alpha_{qu} < e^\varphi/2, \\ s^2\Omega^2 + \dfrac{|s|\omega_p{}^2}{(4\pi\alpha_{qu}{}^3)^{1/2}} e^{-\alpha_{qu}} & \text{if} \quad \alpha_{qu} > e^\varphi/2, \end{cases} \tag{2.4}$$

$$\gamma = -\left(\frac{\pi}{2}\right)^{1/2} \frac{|s|\Omega}{\omega} \frac{(\omega - s\Omega)^2(\omega + s\Omega)}{2\omega|q_z|v_T} \exp\left(-\frac{\beta_s{}^2}{2}\right).$$

The frequency spectra obtained are clearly also valid when $q_z = 0$, i.e., for waves propagating strictly at right angles to the magnetic field. In that case there exist quantum waves also in the region of small refractive indices of the medium ($\alpha_{qu} < 1$). We get in that limit from the dispersion relations (2.2) the following spectra for the ordinary and the extra-ordinary waves:

$$\omega = s\Omega + \frac{\omega_p{}^2}{\Omega^2} \frac{\alpha_{qu}{}^s}{s^2 \cdot s!} \frac{T}{\hbar},$$

$$\omega = s\Omega + \frac{\omega_p{}^2}{2\Omega^2} \frac{\alpha_{qu}{}^{s-1}}{s!}, \tag{2.5}$$

which under the condition $\Omega^2 \gg \omega_p^2$ apart from a coefficient $1/2$ are the same as Korneev and Starostin's results.[4] We note that in their paper[4] the spectra of the quantum waves were obtained with the electron spin taken into account.

We now give for a comparison the classical limit of the waves considered when the thermal energy of the electrons is appreciably larger than the energy of the Larmor quanta ($\hbar\Omega_e \ll T_e$). In that limit the dispersion relation (2.1) splits into three equations for electromagnetic waves with a frequency close to the electron cyclotron frequency $\omega \approx s\Omega$ and with a large refractive index. These equations determine the spectra of the ordinary, the extra-ordinary, and the plasma waves:[7]

$$\omega_1 = s\Omega\left[1 - \frac{\omega_p{}^2}{\Omega^2} \frac{1}{\xi_{cl}(2\pi\alpha_{qu})^{1/2}}\right],$$

$$\gamma_1 = -\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega}{|s|\Omega} \frac{(\omega - s\Omega)^4}{|q_z|^3 v_T{}^3} \exp\left(-\frac{\beta_s{}^2}{2}\right);$$

$$\omega_2 = s\Omega\left[1 - \frac{\omega_p{}^2}{\Omega^2} \frac{\sin^2\vartheta}{\xi_{cl}(2\pi\alpha_{cl})^{1/2}}\right],$$

$$\gamma_2 = -\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega}{s\Omega} \frac{(\omega - s\Omega)^2}{|q_z|v_T} \exp\left(-\frac{\beta_s{}^2}{2}\right), \tag{2.6}$$

$$\omega_3{}^2 = s^2\Omega^2 + 2\frac{s^2\omega_p{}^2}{(2\pi\alpha_{cl}{}^3)^{1/2}},$$

$$\gamma_3 = -\left(\frac{\pi}{2}\right)^{1/2} \frac{(\omega - s\Omega)^2(\omega + s\Omega)}{2\omega|q_z|v_T} \exp\left(-\frac{\beta_s{}^2}{2}\right),$$

where $\alpha_{cl} = q_\perp^2 v_T^2/\Omega^2$.

Comparing the spectra in the classical and the quantal limits we can reach the following conclusions.

a) In the classical limit ($\hbar\Omega_e \ll T_e$) there are in the frequency region considered three waves with the spectra (2.6), two of which—the ordinary and the extraordinary ones—merge into one when $q_z = 0$. In the quantum case, however, two waves propagate with the spectrum (2.4); the extraordinary wave degenerates into the plasma wave.

b) The dispersion law of the quantum waves changes with increasing α_{qu}: when $1 < \alpha_{qu} < e^\varphi/2$ the frequencies of the quantum waves are proportional to α_{qu}^n and when $\alpha_{qu} > e^\varphi/2$ they are inversely proportional to $\alpha_{qu}^{m/2}$. This is connected with the fact that the argument of the Bessel function $I_s^{qu}(\alpha_{qu}/\operatorname{cosech}\varphi)$ for quantum media with a large refractive index can be both larger than and smaller than unity. However, for a classical plasma in the region $\alpha_{cl} \gg 1$ the argument of the function $I_s^{cl}(\alpha_{cl})$ is always larger than unity.

c) The spectra of the classical and the quantum waves (for $\alpha_{qu} > e^\varphi/2$) have the same dispersion law and differ only in the regions in which the particles are localized: $\alpha_{qu} = \lambda^2 q_\perp^2/2$ is the ratio of the quantum region of particle localization λ to the transverse wavelength, while $\alpha_{cl} = \rho^2 q_\perp^2$ is the ratio of the electron Larmor radius (classical localization region) to the transverse wavelength. For waves with spectra determined by the particle localization region, the quantum effects thus manifest themselves in a change in the size of the particle localization region. The frequency of the quantum plasma wave is smaller in order of magnitude than the frequency of the corresponding classical wave, while the frequency of the ordinary quantum wave is larger than the frequency of the classical wave.

d) In a collisionless electron plasma the dissipation of the wave is determined by the longitudinal motion (along the magnetic field) of the particles. As we took into account the quantum nature of the electron motion only in the transverse direction it is natural that the damping rates of the quantum and classical waves are the same.

In conclusion we discuss the problem of possibilities for an experimental observation of quantum waves in a solid-state plasma in the limit ($\alpha_{qu} \gtrsim 1$). It is well known[8] that a strong magnetic field assists the lifting of the degeneracy in the particle energy distribution. Following well-known discussions[9,10] one can reach the conclusion that in a semiconductor plasma at a temperature $T \gtrsim 10$ K in a quantum magnetic field $B_0 \gtrsim 100$ kOe the particle distribution function for concentrations $n_0 \lesssim 10^{16}\ \text{cm}^{-3}$ as a Maxwellian form. The cyclotron frequency for electrons with an effective mass $m_* \approx 10^{-2} m_0$ is equal to $\Omega = |e|B_0/mc \approx 10^{14}$ Hz. Therefore for cyclotron waves $\omega - \Omega \sim 10^{11}$ Hz the condition $\omega - \Omega$ $\nu_e \approx 10^9$ to 10^{10} Hz is satisfied which justifies the consideration of a collisionless plasma. When the inequality $\alpha_{qu} \gtrsim 1$ is satisfied and if we restrict ourselves then only to intraband transitions, it is necessary that the wavelength $q^{-1} < 10^{-6}$ cm.

Determining the refractive indexes for quantum ordinary and plasma (extraordinary) waves from (2.3) and (2.4)

$$n^2 = \begin{cases} \dfrac{c^2}{\lambda^2\omega^2\sin^2\vartheta}\left[2\hat{P}_1\left(\dfrac{\omega_p{}^2}{\omega(s\Omega - \omega)} \dfrac{\gamma^2}{2 \cdot s!}\right)\right] & \text{if} \quad 1 < \alpha_{qu} < e^\varphi/2, \\ \dfrac{c^2}{\lambda^2\omega^2\sin^2\vartheta}\left[2\hat{P}_2\left(\dfrac{\omega_p{}^2}{\omega(s\Omega - \omega)} \dfrac{\gamma^2}{4\pi^{1/2}}\right)\right] & \text{if} \quad \alpha_{qu} > e^\varphi/2; \end{cases}$$

$$n^2 = \begin{cases} \dfrac{c^2}{\lambda^2\omega^2\sin^2\vartheta}\left[2\hat{P}_1\left(\dfrac{\omega_p{}^2}{(\omega^2 - s^2\Omega^2)(s-1)!}\right)\right] & \text{if} \quad 1 < \alpha_{qu} < e^\varphi/2, \\ \dfrac{c^2}{\lambda^2\omega_p{}^2\sin^2\vartheta}\left[2\hat{P}_2\left(\dfrac{\omega_p{}^2}{(\omega^2 - s^2\Omega^2)(4\pi)^{1/2}}\right)\right] & \text{if} \quad \alpha_{qu} > e^\varphi/2; \end{cases}$$

$$\hat{P}_1 = [\ln + (s-1)\ln\ln], \quad \hat{P}_2 = [\ln - {}^3/_2\ln\ln], \text{ where } = v_T/c,$$

we find that $n \sim 10^2$ to 10^3, i.e., the inequality $q_\perp^{-1} < 10^{-6}$ cm is easily satisfied. The restriction to solely intraband transitions is therefore not in contradiction to the condition $\alpha_{qu} > 1$. Finally, if we take into account that for carriers with $m_* = 10^{-2} m_0$ we can neglect the electron spin[8] we are led to the conclusion that the quantum waves considered above are completely observable under experimental conditions.

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