Attenuation of longitudinal sound in superconductors

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The attenuation of longitudinal sound in superconducting alloys is investigated in the free electron model on the basis of a gauge-invariant set of equations. It is found that at low frequencies the expression for the coefficient of the sound attenuation due to the electric fields differs from the BCS formula.

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INTRODUCTION

In the study of the attenuation of sound in metals, we distinguish between direct deformation absorption and the Joule losses connected with the presence of deformation macroscopic electromagnetic fields. The appearance of these components is determined in the final analysis by a single cause: the deformation of the crystal lattice. However, direct deformation losses are not due to the macroscopic currents, while the Joule losses are determined by just these currents. In a superconductor, the current is the sum of a current of excitations and the superfluid current and, for a given value of the total current, the relative contributions of these components and perhaps the value of the dissipation also can change as a function of the scales of the spatial and temporal dispersions. In this connection, the Joule losses when transverse sound propagates in a superconductor have a complicated frequency dependence (see, for example,[1,2]). It seems natural to expect something similar in the propagation of longitudinal sound in superconducting alloys, inasmuch as the scales of the dispersion in them change, in particular, with change in the electron mean free path.

An investigation of Joule losses in superconducting alloys was carried by Tsuneto,[3] who assumed, however, the contribution of the superfluid flow to be purely adiabatic. The calculation of the Joule losses of a longitudinal sound wave is carried out below on the basis of a gauge-invariant system of equations for the superconductor. Inasmuch as we are interested in Joule losses, we limit ourselves to the free electron model, in which the losses of the sound wave are connected only with macroscopic electric fields.[4]

1. THE INTERACTION HAMILTONIAN OF ELECTRONS WITH SOUND

We consider the propagation of a longitudinal sound wave in an isotropic metal. In the laboratory system of coordinates, the electrons interact with the sound wave through the electromagnetic fields $\mathbf{A}$ and φ, and through collisions with moving impurities.

The motion of the impurities is taken into account by expansion of the potential of the impurities in terms of the lattice displacements. By assuming the deformation of the crystal lattice to be small and concerning ourselves with the linear response of the system, we restrict our consideration to the first terms of the expansion

$$V_{imp}(\mathbf{r},t)=\sum_a V(\mathbf{r}-\mathbf{r}_a)-\sum_a \frac{\partial V(\mathbf{r}-\mathbf{r}_a)}{\partial \mathbf{r}}\mathbf{u}(\mathbf{r}_a,t), \qquad (1)$$

Here $\mathbf{r}_a$ is the location of the a-th impurity in the undeformed lattice, and $\mathbf{u}(\mathbf{r}_a, t)$ is the displacement of the impurity. Further, we consider only the Born approximation for electron scattering by the impurities.

In the diagrams, we denote the static part of the potential of the impurity by a cross,[5] and the scattering by the dynamic part (the second term in the expansion (1)) by a cross with a circle. The crosses which refer to the same impurity are joined by a dashed line. Then, in the calculation of the Green's functions, the ordinary crosses, which are not connected with any other crosses, can be ascribed to the renormalization of the chemical potential.[5] It is not difficult to understand that, in complete analogy, circled crosses not connected with other crosses by a dashed line can be ascribed to the scalar electrochemical potential φ. Thus, disregarding as is customary,[5] diagrams with crossing of dashed lines, we shall in what follows consider only diagrams of the type shown in Fig. 1a with all possible directions of the arrows on the electron lines between the dynamic and static crosses.

As an example, we give the analytic expressions corresponding to the diagrams shown in Figs. 1b and 1c, which, in correspondence with (1), are of the form

$$-in_{imp}|V|^2\int(\mathbf{p}'-\mathbf{p}+\mathbf{k},\mathbf{u})G_\varepsilon^+(\mathbf{p}')\frac{d^3p'}{(2\pi)^3},$$

$$-in_{imp}|V|^2\int(\mathbf{p}-\mathbf{p}',\mathbf{u})G_{\varepsilon-\omega}^+(\mathbf{p}')\frac{d^3p'}{(2\pi)^3},$$

where n_{imp} is the number of impurity atoms per unit volume.

We shall denote the vertex with the electromagnetic fields $\mathbf{A}$ and φ by a point with a wavy line. Then the graphical representation of the first-order correction to the Green's function G_0^+ will take the form shown in Fig. 2. Similar graphical representations exist also for the functions G_1^-, F_1^+, and F_1.

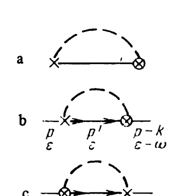

FIG. 1

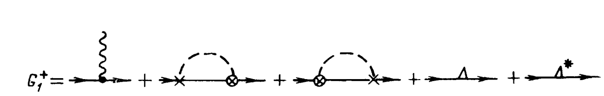

FIG. 2

2. DERIVATION OF THE BASIC EQUATIONS

To calculate the sound attenuation caused by the electromagnetic fields, it is necessary to find the electric field $\mathbf{E}$ produced in the superconductor by the sound wave. Here, since the calculations will be carried out in an arbitrary gauge with the use of two-gauge invariant potentials, the scalar Φ and the vector $\mathbf{Q}$, we need a set of two equations for the determination of these quantities. The first of these equations expresses the condition of electrical neutrality, which can be written, for a harmonic wave in the laboratory set of coordinates, as the equality of the electron and ion components of the current:

$$\mathbf{j}_e = en\dot{\mathbf{u}}, \tag{2}$$

where n is the number of electrons per unit volume.

As the second equation, we can employ the continuity equation. However, since it is identical in a superconductor with the self-consistent equation for $\Delta - \Delta^*$, we use this equation, and this reduces the volume of the calculations. Thus the necessary equations in the Matsubara technique are written in the form

$$en\dot{\mathbf{u}} = \frac{2e}{m}T\sum_{\varepsilon}\int \mathbf{p}G^{+}_{1\varepsilon,\varepsilon-\omega}\left(\mathbf{p}+\frac{\mathbf{k}}{2},\mathbf{p}-\frac{\mathbf{k}}{2}\right)\frac{d^3p}{(2\pi)^3} - \frac{ne^2}{m}\mathbf{A}(\omega,\mathbf{k}), \tag{3}$$

$$\Delta-\Delta^* = |g|T\sum_{\varepsilon}\int [F_{1\varepsilon,\varepsilon-\omega}(\mathbf{p}_+,\mathbf{p}_-) - \overset{+}{F}_{1\varepsilon,\varepsilon-\omega}(\mathbf{p}_+,\mathbf{p}_-)]\frac{d^3p}{(2\pi)^3}, \tag{4}$$

where ω and $\mathbf{k}$ are the frequency and wave vector of the sound wave.

Equations (3) and (4) must be continued analytically in ϵ and ω to the real axis, and also averaged over the impurities. Here we shall use the technique developed by Gor'kov and Éliashberg,[6,7] without however imposing any limitations to the path length l other than the condition $p_F l \gg 1$, which enables us to take only the ladder diagrams into account in averaging over the impurities. The technique of averaging over the positions of the impurities, suggested by Gor'kov and Éliashberg,[7] allows us to obtain the answer much more economically than in the use of the well-known procedure of[5], and consists in the fact that the averaging over the impurities of the superconducting Green's function reduces in definite fashion to averaging of the Green's function of the normal metal. In this case four types of vertices to be averaged arise for Eqs. (3) and (4).[7] Omitting the calculations, we write down the result immediately:

$$\left\langle \int G_{\xi_m}{}^R(\mathbf{p}_+)\frac{\mathbf{pu}}{2\tau}G_{\xi_n}{}^A(\mathbf{p}_-)\frac{d^3p}{(2\pi)^3}\right\rangle_{imp} + \text{c.c.} = \frac{p_F{}^3ku}{6\pi}\left[\frac{1}{i+\Omega_{mn}\tau}\frac{\alpha_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\right], \tag{5}$$

$$\left\langle \int \mathbf{p}G_{\xi_m}{}^R(\mathbf{p}_+)\frac{\mathbf{pu}}{2\tau}G_{\xi_n}{}^A(\mathbf{p}_-)\frac{d^3p}{(2\pi)^3}\right\rangle_{imp} + \text{c.c.} = \frac{mp_F{}^3\mathbf{u}}{6\pi}\left[\frac{\Omega_{mn}}{i+\Omega_{mn}\tau}\frac{\alpha_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\right], \tag{6}$$

$$\left\langle \int G_{\xi_m}{}^R(\mathbf{p}_+)G_{\xi_n}{}^A(\mathbf{p}_-)\frac{d^3p}{(2\pi)^3}\right\rangle_{imp} + \text{c.c.} = \frac{mp_F}{\pi}\left[\frac{\beta_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\right], \tag{7}$$

$$\left\langle \int \mathbf{p}G_{\xi_m}{}^R(\mathbf{p}_+)G_{\xi_n}{}^A(\mathbf{p}_-)\frac{d^3p}{(2\pi)^3}\right\rangle_{imp} + \text{c.c.} = \frac{mp_F\mathbf{k}l}{3\pi}\left[\frac{1}{i+\Omega_{mn}\tau}\frac{\alpha_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\right]. \tag{8}$$

Here, in correspondence with[7],

$$G_{\xi}^{R(A)}(\mathbf{p}) = -\frac{1}{\xi-\xi_p \underset{(-)}{+} i/2\tau}$$

is the retarded (advanced) Green's function of the normal metal, and the following notation is used:

$$\alpha_{mn} = \frac{3(a-\operatorname{arctg} a)}{a^3}, \quad \beta_{mn} = \frac{\operatorname{arctg} a}{a}, \quad a = \frac{kl}{1-i\Omega_{mn}\tau},$$

$$\Omega_{mn} = \xi_m - \xi_n, \quad D = \frac{lv_F}{3}\frac{\alpha_{mn}}{(1-i\Omega_{mn}\tau)^2}, \quad \tau = \frac{l}{v_F},$$

and by the arctan z we mean the principal branch of the arctangent, defined in the complex plane with cuts along the imaginary axis $(-i\infty, -i)$ and $(i, +i\infty)$.

The expression $\mathbf{p}\cdot\mathbf{u}/2\tau$ results from the block with the dynamic and static crosses, joined by the dashed line (see Fig. 1). For terms with the vector potential $\mathbf{A}$ in (5) and (6), we must replace $\mathbf{u}$ by $2e\tau\mathbf{A}/m$. After averaging over the impurities and analytic continuation over ϵ and ω with account of (5)—(8), Eqs. (3) and (4) take the form

$$\begin{aligned}\frac{6\pi^2}{iep_F{}^3}en\dot{\mathbf{u}} = \int\frac{d\varepsilon\, d\xi_m\, d\xi_n}{(2\pi i)^2}\Big\{\Big(&\frac{\alpha_{mn}}{i+\Omega_{mn}\tau}\frac{\Omega_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\Big)\\ \times\Big[&\frac{pu}{2}\frac{(\varepsilon+\xi_m)(\varepsilon-\omega+\xi_n)+\Delta^2}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\Big)\\ &-\frac{pu}{2}\frac{\Delta(\varepsilon-\omega+\xi_m)+(\varepsilon+\xi_n)\Delta}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big(\frac{\Delta}{\xi_\varepsilon}-\frac{\Delta}{\xi_{\varepsilon-\omega}}\Big)\\ &-\frac{e}{c}vA\tau\frac{(\varepsilon+\xi_m)(\varepsilon-\omega+\xi_n)+\Delta^2}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big]+\Big(\frac{\alpha_{mn}}{i+\Omega_{mn}\tau}\frac{1}{Dk^2-i\Omega_{mn}}+\text{c.c.}\Big)\\ \times\Big[&kle\varphi\frac{(\varepsilon+\xi_m)(\varepsilon-\omega+\xi_n)-\Delta^2}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}-kl\frac{\Delta_1\Delta(\varepsilon+\xi_m)+\Delta_1{}^*\Delta(\varepsilon-\omega+\xi_n)}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big]\Big\},\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{i(\Delta_1-\Delta_1{}^*)}{\lambda} = \int\frac{d\varepsilon\, d\xi_m\, d\xi_n}{(2\pi i)^2}\Big\{\Big(&\frac{\alpha_{mn}}{i+\Omega_{mn}\tau}\frac{1}{Dk^2-i\Omega_{mn}}+\text{c.c.}\Big)\\ \times\Big[&\frac{2}{3}kl\frac{e}{c}vA\frac{(\xi_m-\xi_n)\Delta}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\Big)+\frac{2}{3}kl\frac{e}{c}vA\\ &\times\frac{\varepsilon\xi_n-(\varepsilon-\omega)\xi_m}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big(\frac{\Delta}{\xi_\varepsilon}-\frac{\Delta}{\xi_{\varepsilon-\omega}}\Big)-\frac{1}{12}p_Fv_Fku\frac{\Delta(\xi_m-\xi_n)}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Big]\\ -\Big(&\frac{\beta_{mn}}{Dk^2-i\Omega_{mn}}+\text{c.c.}\Big)\Big[e\varphi\frac{2\Delta\omega}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\\ &-\frac{(\xi_m+\varepsilon)(\xi_n-\varepsilon+\omega)+\Delta^2}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Delta_1+\frac{(\xi_m-\varepsilon)(\xi_n+\varepsilon-\omega)+\Delta^2}{(\xi_m{}^2-\xi_\varepsilon{}^2)(\xi_n{}^2-\xi_{\varepsilon-\omega}^2)}\Delta_1{}^*\Big]\Big\},\end{aligned} \tag{10}$$

$\lambda \equiv |g|\,mp_F/2\pi^2$. Here and below, all the integrals over ϵ must be understood in the analytically continued form, i.e.,

$$\int d\varepsilon\, F(\xi_\varepsilon,\xi_{\varepsilon-\omega}) = \int d\varepsilon\Big\{\operatorname{th}\frac{\varepsilon-\omega}{2T}F(\xi_\varepsilon{}^R,\xi_{\varepsilon-\omega}^R)-\operatorname{th}\frac{\varepsilon}{2T}F(\xi_\varepsilon{}^A,\xi_{\varepsilon-\omega}^A) -\Big(\operatorname{th}\frac{\varepsilon-\omega}{2T}-\operatorname{th}\frac{\varepsilon}{2T}\Big)F(\xi_\varepsilon{}^R,\xi_{\varepsilon-\omega}^A)\Big\}, \tag{11}$$

$$\xi_\varepsilon{}^R = -(\xi_\varepsilon{}^A)^* = \begin{cases}\operatorname{sign}\varepsilon(\varepsilon^2-\Delta^2)^{1/2}, & \varepsilon^2>\Delta^2\\ i(\Delta^2-\varepsilon^2)^{1/2}, & \varepsilon^2<\Delta^2.\end{cases}$$

The expressions (9) and (10) are easily integrated over ξ_m and ξ_n, by calculating the corresponding residues. It is necessary here to consider the fact that at the point $\xi_m - \xi_n = -i/\tau$ the singularity in the integrand is removable and the arctangent is taken in the sense of the principal value. Introducing the phase of the gap by the equation

$$\Delta_1-\Delta_1{}^* = 4ie\Delta\chi/c \tag{12}$$

and using the following relations

$$(\xi_\varepsilon^{R(A)}+\xi_{\varepsilon-\omega}^{R(A)})\Big(\frac{\varepsilon}{\xi_\varepsilon^{R(A)}}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}^{R(A)}}\Big) = \omega\Big(1-\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon^{R(A)}\xi_{\varepsilon-\omega}^{R(A)}}\Big),$$

$$(\xi_\varepsilon^{R(A)}+\xi_{\varepsilon-\omega}^{R(A)})\left(1-\frac{\varepsilon(\varepsilon-\omega)-\Delta^2}{\xi_\varepsilon^{R(A)}\xi_{\varepsilon-\omega}^{R(A)}}\right)=\omega\left(\frac{\varepsilon}{\xi_\varepsilon^{R(A)}}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}^{R(A)}}\right),$$

$$\int d\varepsilon\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right)=4\omega,\quad \int d\varepsilon\left(\frac{1}{\xi_\varepsilon}+\frac{1}{\xi_{\varepsilon-\omega}}\right)=\frac{4}{\lambda}, \tag{13}$$

we obtain, after straightforward but cumbersome calculations,

$$Q\int d\varepsilon\frac{\alpha}{\Omega+i/\tau}\frac{\Omega}{\Omega+iDk^2}\left(1-\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\right)-\mathbf{k}\Phi\int d\varepsilon\frac{\alpha}{\Omega+i/\tau}\frac{1}{\Omega+iDk^2}$$
$$\times\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right)=\frac{m\dot{u}}{e}\int d\varepsilon\frac{\alpha-\beta}{\Omega+iDk^2}\frac{\Omega}{\omega}\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right), \tag{14}$$

$$\frac{kv_F^2}{3}Q\int d\varepsilon\frac{\Delta}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\frac{\alpha}{\Omega+i/\tau}\frac{\Omega}{\Omega+iDk^2}-\Phi\int d\varepsilon\frac{\omega\Delta}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\frac{\beta}{\Omega+iDk^2}$$
$$=\frac{\delta\mu_0}{e}\int d\varepsilon\frac{\omega\Delta}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\frac{\alpha-\beta}{\Omega+iDk^2}, \tag{15}$$

in which α, β, a and D are given by the formulas for α_{mm}, β_{mm}, etc. (see above), and

$$\delta\mu_0=-{}^2/{}_3\varepsilon_F\,\mathrm{div}\,\mathbf{u},\quad \Omega=\xi_\varepsilon+\xi_{\varepsilon-\omega}; \tag{16}$$

$$\mathbf{Q}=m\dot{\mathbf{u}}/e+\mathbf{A}-\nabla\chi,\quad \Phi=\varphi+\delta\mu_0/e+\dot{\chi} \tag{17}$$

are the generalized gauge invariant vectors and scalar potentials. The choice of the potentials in the form (17) is dictated by the requirement of Galilean and gauge invariance.

3. SOUND ATTENUATION

We first consider the sound attenuation in a normal metal. In this case the equation for the phase (15) is satisfied automatically, and from the equation of electrical neutrality (14), setting $\Delta = 0$, for the electric field[8]

$$E=i\omega Q-ik\Phi \tag{18}$$

we immediately find

$$E=(\beta-\alpha)en\dot{u}/\sigma, \tag{19}$$

$$\sigma=\frac{ne^2\tau}{m}\frac{\alpha}{1-i\omega\tau} \tag{20}$$

is the conductivity of the metal. The power of the Joule losses of the sound wave is determined by the expression $P = {}^1\!/_2\mathrm{Re}\,(\mathbf{E}^*\cdot\dot{\mathbf{u}}en)$ and thus, in accord with (19), for the sound attenuation coefficient $\gamma = P|{}^1\!/_2\rho|\dot{u}|^2 s$, where ρ is the density of the material and s the speed of sound, in the limit $\omega\tau \ll 1$, we obtain the Pippard formula[9]

$$\gamma_n=\frac{nm}{\rho s\tau}\left(\frac{a^2\,\mathrm{arctg}\,a}{3(a-\mathrm{arctg}\,a)}-1\right). \tag{21}$$

here $a = kl$. The formula (19) is valid also in the case $\omega\tau \gg 1$. In this limit we obtain

$$\gamma_n=\frac{2}{9\pi}\frac{m^2\varepsilon_F^2}{\rho s^2}\omega, \tag{22}$$

which coincides with (21) in the limit $kl \gg 1$.

We proceed to the consideration of the attenuation of a longitudinal sound of frequency $\omega \ll \min\{T, \Delta\}$ in superconductors. In view of the fact that Eqs. (14) and (15) are rather involved, we limit ourselves to the consideration of a few limiting cases. We begin with the case of a high concentration of impurities ($kl \ll 1$, $\Delta\tau \ll 1$). Solving the set of Eqs. (14) and (15), we obtain the following for the ratio of the attenuation coefficients in the superconducting γ_s and normal γ_n states:

$$\frac{\gamma_s}{\gamma_n}=\mathrm{Re}\left\{\frac{2\omega}{\Delta}\left[\frac{\omega}{2\Delta}I_3+\frac{i\omega}{Dk^2}I_1I_2\Big/\left(I_1-\frac{i\omega}{Dk^2}I_2\right)\right]^{-1}\right\},$$

$$I_1=\int d\varepsilon\frac{\Delta}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\frac{\Omega}{\Omega+iDk^2},\quad I_2=\int d\varepsilon\frac{\Delta}{\xi_\varepsilon\xi_{\varepsilon-\omega}}\frac{\omega}{\Omega+iDk^2},$$

$$I_3=\int d\varepsilon\frac{1}{\Omega+iDk^2}\left(\frac{\varepsilon}{\xi_\varepsilon}-\frac{\varepsilon-\omega}{\xi_{\varepsilon-\omega}}\right). \tag{23}$$

We first consider the case of weak spatial dispersion, when

$$Dk^2\ll\omega, \tag{24}$$

or, what amounts to the same thing,

$$\omega\tau\ll(s/v_F)^2.$$

We can then carry out an expansion in the integrand in powers of Dk^2/Ω. There are two possibilities here:

$$\Delta\tau\gg(s/v_F)^2, \tag{25a}$$

$$\Delta\tau\ll(s/v_F)^2. \tag{25b}$$

In the limit (25a), Eq. (23) becomes

$$\frac{\gamma_s}{\gamma_n}=\mathrm{Re}\frac{2\omega/\Delta}{\omega I_3/2\Delta+i\omega I_2/Dk^2}$$
$$=\begin{cases}\left(\dfrac{Dk^2}{\omega}\right)^2\dfrac{2T}{\Delta}2e^{-\Delta/T} & T\ll\Delta\\ \left(1-\dfrac{\Delta}{T}\right)\Big/\left[\left(1-\dfrac{\Delta}{T}\right)^2+\left(\dfrac{Dk^2}{\omega}+\dfrac{\pi}{4}\dfrac{\omega}{Dk^2}\dfrac{\Delta}{T}+\dfrac{\pi}{16}\dfrac{\omega}{\Delta}\right)^2\right], & T\gg\Delta\end{cases} \tag{26}$$

and in the case (25b), we have

$$\frac{\gamma_s}{\gamma_n}=\mathrm{Re}\frac{2\omega/\Delta}{\omega I_3/2\Delta+I_1}$$
$$=\begin{cases}\dfrac{\omega}{\pi\Delta}\dfrac{\omega}{\pi T}\ln\left(\dfrac{4T}{\gamma\omega}\right)2e^{-\Delta/T}, & T\ll\Delta\\ \dfrac{2T\omega}{\pi\Delta^2}\left(1+\dfrac{\Delta}{T}\ln\dfrac{8\Delta}{\omega}\right), & \Delta\ll T\ll\dfrac{\Delta^2}{\omega},\\ \left(1+\dfrac{\Delta}{T}\ln\dfrac{8\Delta}{\omega}\right)^{-1}, & \dfrac{\Delta^2}{\omega}\ll T\end{cases} \tag{27}$$
$$\ln\gamma=0.577.$$

The limit (24) corresponds to a small spatial dispersion. If the spatial dispersion is large ($Dk^2 \gg \Delta$), then we immediately get from (23) the BCS relation:

$$\gamma_s/\gamma_n=2f_F(\Delta), \tag{28}$$

where f_F is the Fermi function and $\Delta = \Delta(T)$ is the gap in the energy spectrum of the superconductor for a given temperature T.

Equations (14) and (15) allow a transition to the case of a pure superconductor ($\omega\tau \gg 1$). In contrast to the case of high concentration of impurities, the ratio γ_s/γ_n here, for all frequencies of interest to us ($\omega \ll \min\{T, \Delta\}$), is determined by the BCS relation (28) with insignificant corrections of the order of $(s/v_F)^2$.

DISCUSSION OF THE RESULTS

As is seen from the results above, the ratio γ_s/γ_n turns out to be sensitive to the relation of the scales of spatial and temporal dispersions. In the region of weak spatial dispersion (24), which corresponds to the quasi-homogeneous situation, γ_s/γ_n falls off more rapidly with temperature than in the case of strong dispersion ($Dk^2 \gg \Delta$), whereas the BCS formula (28) holds for γ_s/γ_n.[1)] Here, there exist two possibilities for the quantity $\Delta\tau$ in the region $\omega\tau \ll (s/v_F)^2$: (25a) and (25b), which lead to different dependences of the sound attenuation coefficient on the impurity concentration. In this connection, we note that Eq. (15), in the absence of sound and electromagnetic fields, is the wave equation for the phase of the gap and in the considered case $\Delta\tau \ll 1$ the velocity which enters into this equation is given by the expression

$$w = v_F(\pi\Delta\tau/3)^{1/2}, \quad T \ll \Delta,$$
$$w = v_F(2\Delta\tau/3)^{1/2}, \quad T \gg \Delta \qquad (29)$$

and the inequalities (25a) and (25b) correspond to the cases $w \gg s$ and $w \ll s$.

Thus w is the velocity of the Bogolyubov collective excitations in the neutral Fermi gas with a finite path length of the particles of the gas, and differs from $v_F/3^{1/2}$—the velocity of the collective excitations in the case of an infinite path length—by an amount $\sim(\Delta\tau)^{1/2}$. Thus, in the case (25b) the sound wavelength is large in comparison with the wavelength of the collective excitations of the same frequency and an entirely homogeneous situation is realized here, which is seen from (27), where there is no dependence on Dk^2. In this case γ_S/γ_n is given by the ratio of the conductivity of the metal in the normal state to the effective conductivity of the superconductor at the frequency ω. The latter circumstance very clearly reveals the effect of screening of the electromagnetic fields by the superfluid currents similar to the case for propagation of transverse sound in a superconductor.

A quantitative comparison of the results obtained in the present research with the experimental data is difficult, since the regions in which the predicted phenomena are important have not been specially investigated. However, we note that a number of experiments[10] on measurements of the attenuation of sound in superconducting alloys point to a large decrease in the attenuation coefficient with temperature at frequencies below 10^2 MHz, which can evidently be connected with the results obtained here.

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Noted added in proof (August 22, 1975) in a recent work by Schmeil and Schön (Phys. Rev. Lett. 34, 941 (1975)), it was shown that near T_C, collective excitations can exist with the dispersion law $\omega = -(i\pi\Delta^2/4T) \pm [\omega^2k^2 - (\pi\Delta^2/4T)^2]^{1/2}$, where the velocity w is given by Eq. (29). This dispersion law is easily obtained by setting the determinant of the set of equations for the potentials Q and Φ ((14) and (15) of our present paper) equal to zero.

[1)] In a pure superconductor ($\omega\tau \gg 1$) such a quasihomogeneous situation is not realized since we always have $kv_F \gg \omega$.

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