de Haas–van Alphen effect and helicons in metals

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The helicon electrodynamics features related to the de Haas–van Alphen effect are considered for an uncompensated metal with a closed Fermi surface of arbitrary shape. The analysis is carried out in the local limit in which the metal can be characterized by a static magnetoresistance tensor and a static differential magnetic permeability tensor describing the anisotropy of the de Haas–van Alphen effect. The amplitude of the de Haas–van Alphen effect is assumed to be arbitrary but within the limits defined by the thermodynamic stability of the homogeneously magnetized state. It is shown that the de Haas–van Alphen effect can strongly influence not only the phase velocity but also the damping and polarization of the helicon in the general case. The pronounced effect of the off-diagonal components of the differential magnetic permeability tensor is noted; these components sometimes appear even for very small deviations of the magnetic field from the symmetric direction. Resonance excitation of waves in a plate is discussed. The possible relation between periodic magnetic structures in metals and helicons is considered.

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INTRODUCTION

The propagation of electromagnetic waves in pure metals in a magnetic field at liquid helium temperatures is essentially determined in many cases by the effects of temporal and spatial dispersion.[1] However, there always exists a region of rather low frequencies where these effects do not have to be taken into account and one can use the local relations between the field and current vectors just as in the static case. This frequency range is determined by the conditions:

$$\omega\tau\ll 1,\quad \omega\ll\omega_e,\quad kl\ll 1,\quad kr_e\ll 1, \tag{1}$$

where ω and k are the frequency and wave number of the wave, ω_e and r_e are the frequency and radius of the motion of electrons in the magnetic field, τ and l are the time and the mean free path. The quantities ω_e, k, and r_e depend on the magnetic field in such fashion that the range of frequencies where the conditions (1) are satisfied can be broadened materially by increasing the magnetic field. However, in this case, it may be necessary to take into account the quantization of the electron orbits in the magnetic field.

If the quantum effects are small, then the "local electrodynamics" of the metal are completely determined by the static magnetoresistance tensor.[2] The action of quantum effects on the wave propagation in the metal in the local limit considered here can manifest themselves either in oscillation of the components of the magnetoresistance tensor (the Shubnikov-de Haas effect), or in oscillations of the magnetization (the de Haas—van Alphen effect).

In a number of experimental researches,[3,4] it has been shown that the de Haas—van Alphen effect plays the essential role in most cases; this effect leads to oscillations of the phase velocity of the waves. However, in some cases, for example, in the situation with a magnetic breakdown, oscillations of the magnetoresistance turn out to have the decisive effect on the wave propagation.[5]

In this paper we consider the influence of only the de Haas—van Alphen effect on the propagation of helicons in metals under the local-limit conditions described by the inequalities (1). It was shown in[6] that for sufficiently small wave amplitude, this effect can be taken into account by the introduction of the differential magnetic permeability tensor. The solution of the wave equation and the dispersion relation were obtained in[6] with the diagonal components of the magnetoresistance tensor neglected. At the same time, the wave-propagation problem considered here can be solved completely without adding any limitations on the magnetoresistance tensor, as is indeed done below. The relations thus obtained describe the influence of the de Haas—van Alphen effect not only on the phase velocity of the helicons, but also on the ratio of the real and imaginary parts of the wave vector, and also on its polarization.

THE DIFFERENTIAL MAGNETIC PERMEABILITY TENSOR

It was already noted in the Introduction that the local electrodynamics of the metal without account of quantum effects are determined by the magneto-resistance tensor, which describes the linear relation between the current and the electric field of the wave:

$$\mathbf{e}=\hat{\rho}\mathbf{j}. \tag{2}$$

If the amplitude of the magnetic induction field of the wave **b** is small in comparison with the periods of the de Haas—van Alphen effect, then the connection between the fields **h** and **b** can also be represented by a linear relation[3]:

$$\mathbf{h}=\hat{\mu}\mathbf{b},\qquad \mu_{ik}=\delta_{ik}-4\pi\frac{\partial M_i}{\partial b_k}, \tag{3}$$

where δ_{ik} is the Kronecker symbol and M_i are the components of the oscillating magnetization of the metal.

We call the tensor $\hat{\mu}$ the differential magnetic permeability tensor of the metal. We note several of its properties:

1) $\mu_{ik} = \mu_{ki}$.

2) The conditions of thermodynamic stability which we shall assume to be satisfied, require that the determinant $|\mu_{ik}|$ and all its principal minors be positive. In particular, this means that all the diagonal components of the matrix μ_{ik} must be positive. Violation of the condition of thermodynamic stability leads to the appearance of diamagnetic domains[7] or periodic structures.[8]

3) The components of μ_{ik} are oscillating functions not only of the magnitude but also of the direction of the constant magnetic field B_0 in which the metal is placed. The periods of the corresponding dependences can be very small, so that even an insignificant deviation of the direction of B_0 from symmetry by an angle of the order of $1/N$, where N is the number quantum magnetic level nearest to the Fermi surface, can lead to off-diagonal components of μ_{ik} that are comparable in value with the diagonal terms.

As to the amplitudes of the oscillations of μ_{ik}, it is assumed that it can take on any value in the limits determined by the conditions of thermodynamic stability of a homogeneously magnetized state.

WAVES IN UNBOUNDED SPACE

We consider plane waves of the form $\exp\{i(\omega t - \mathbf{k}\cdot\mathbf{r})\}$, assuming that the constant magnetic field $\mathbf{B}_0$ makes an arbitrary angle θ with the wave vector $\mathbf{k}$. Substitution in Maxwell's equation, with allowance for the relations (2) and (3), leads to the wave equation

$$[\mathbf{k}\times\hat{\rho}[\mathbf{k}\times\hat{\mu}\mathbf{b}]]=\frac{4\pi i\omega}{c^2}\mathbf{b}. \qquad (4)$$

We introduce a Cartesian set of coordinates with unit vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, where $\mathbf{u}_3 \parallel \mathbf{k}$. Since the vector $\mathbf{b}$ is perpendicular to $\mathbf{k}$, it can always be expressed in a form of linear combination of the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ only. Furthermore, it is easy to see that it suffices to consider in Eq. (4) only the two-dimensional tensors $\hat{\rho}$ and $\hat{\mu}$, which act on the $(\mathbf{u}_1\mathbf{u}_2)$ plane.

The directions of $\mathbf{u}_1$ and $\mathbf{u}_2$ in the plane perpendicular to $\mathbf{k}$ are chosen along the principal axes of the symmetric part of the two-dimensional tensor $\hat{\rho}$, so that this tensor takes the form

$$\hat{\rho}=\begin{Vmatrix}\rho_1 & \rho_{12}\\ -\rho_{12} & \rho_2\end{Vmatrix}=\rho_{12}\begin{Vmatrix}\alpha_1 & 1\\ -1 & \alpha_2\end{Vmatrix}, \quad \alpha_1,\alpha_2<1. \qquad (5)$$

The latter inequality determines the region of existence of the helicons and is usually satisfied in uncompensated metals with closed Fermi surfaces at low temperatures in a sufficiently large magnetic field, which makes an angle $\theta < \frac{1}{2}\pi - (\omega_e\tau)^{-1}$ with $\mathbf{k}$. [2]

The wave equation (4) can be represented in the form

$$-k^2\rho_{12}\hat{\varkappa}\mathbf{b}=4\pi i\omega\mathbf{b}/c^2, \qquad (6)$$

where $\hat{\kappa}$ is a two-dimensional tensor with the components:

$$\varkappa_1=\mu_{12}+\alpha_2\mu_1,\quad \varkappa_2=-\mu_{12}+\alpha_1\mu_2,$$
$$\varkappa_{12}=\mu_2+\alpha_2\mu_{12},\quad \varkappa_{21}=-\mu_1+\alpha_1\mu_{12}. \qquad (7)$$

The problem thus reduces to finding the eigenvectors and eigenvalues of the tensor $\hat{\kappa}$. The eigenvectors can be represented as

$$\mathbf{b}_\pm=\mathbf{u}_1+\beta_\pm\mathbf{u}_2,\quad \beta_\pm=\beta_0 e^{\pm i\varphi_\beta},$$
$$\beta_0=\left|\frac{\varkappa_{21}}{\varkappa_{12}}\right|^{1/2},\quad \cos\varphi_\beta=\frac{\varkappa_2-\varkappa_1}{2|\varkappa_{12}\varkappa_{21}|^{1/2}}. \qquad (8)$$

The eigenvalues are of the form

$$\gamma_\pm=\gamma_0 e^{\pm i\varphi_\gamma},$$
$$\gamma_0=(\varkappa_1\varkappa_2-\varkappa_{12}\varkappa_{21})^{1/2},\quad \cos\varphi_\gamma=\frac{\varkappa_1+\varkappa_2}{2(\varkappa_1\varkappa_2-\varkappa_{12}\varkappa_{21})^{1/2}}. \qquad (9)$$

With the aid of (6), we obtain the dispersion relation

$$k_\pm^2=\frac{4\pi\omega}{c^2\rho_{12}\gamma_0}\exp\left\{-i\left(\frac{\pi}{2}\pm\varphi_\gamma\right)\right\}. \qquad (10)$$

Since φ_γ is usually near $\pi/2$, Eq. (10) then determines one weakly and one strongly damped wave.

The magnetic field $\mathbf{h}$ of the wave can be obtained by substituting (8) in (3). The electric field and the current density are calculated with the help of Maxwell's equations. In particular, the transverse electric field is

$$\mathbf{e}_\pm=\left(\frac{\omega\rho_{12}\gamma_0}{4\pi}\right)^{1/2}\exp\left\{-i\left(\frac{\pi}{4}\pm\frac{\varphi_\gamma}{2}\right)\right\}\beta_\pm(u_1-\beta_\pm^{-1}u_2). \qquad (11)$$

Thus all the characteristics of the helicon are determined by the complex numbers $\gamma_\pm$ and $\beta_\pm$ and by giving the directions of $\mathbf{u}_1$ and $\mathbf{u}_2$ on the plane perpendicular to $\mathbf{k}$.

There is greatest interest in the situation in which (see (5)) $\alpha_1 \ll 1$ and $\alpha_2 \ll 1$. In this case, it is convenient to use the approximate expression for the components $\gamma_\pm$ and $\beta_\pm$ which contain terms of no higher order than first in α_1 and α_2. Then, in place of (8) and (9), we have

$$\beta_0=\left(\frac{\mu_1}{\mu_2}\right)^{1/2}\left(1-\frac{1}{2}\frac{\mu_{12}}{\mu_1}\alpha_1-\frac{1}{2}\frac{\mu_{12}}{\mu_2}\alpha_2\right), \qquad (12)$$

$$\cos\varphi_\beta=-\frac{\mu_{12}}{(\mu_1\mu_2)^{1/2}}\left[1-\frac{1}{2\mu_{12}}\left(\frac{\Delta}{\mu_1}\alpha_1-\frac{\Delta}{\mu_2}\alpha_2\right)\right],\quad \mu_{12}\gg\alpha_1,\alpha_2,$$
$$\cos\varphi_\beta=\frac{\alpha_1\mu_2-\alpha_2\mu_1}{2(\mu_1\mu_2)^{1/2}},\quad \mu_{12}\ll\alpha_1,\alpha_2, \qquad (13)$$

$$\Delta=\mu_1\mu_2-\mu_{12}^2, \qquad (14)$$

$$\gamma_0=\Delta^{1/2},\quad \cos\varphi_\gamma=\frac{\alpha_1\mu_2+\alpha_2\mu_1}{2\Delta^{1/2}}. \qquad (15)$$

Substitution of (15) in (10) shows that the quantity γ_0 determines the oscillations of the real part of the wave vector, i.e., the oscillations of the phase velocity of the helicon, [3,4] and Δ is identical with q from [6]. The ratio of the imaginary and real parts of the wave vector and, as will be seen below, the quality factors of the resonances of the standing waves in a plate are determined by the quantity $\cos\varphi_\gamma$.

Generally speaking, as is evident from (15), this quantity should be an oscillating function of the magnetic field. However, if $\alpha_1 = \alpha_2 = \alpha$ and $\mu_{12} = 0$, then

$$\cos\varphi_\gamma=\frac{\mu_1+\mu_2}{2(\mu_1\mu_2)^{1/2}}\alpha. \qquad (16)$$

The coefficient for α in Eq. (16) is equal to unity for $\mu_1 = \mu_2$ and differs little from it when $\mu_1 \neq \mu_2$, with the exception of the immediate vicinity of the boundary of thermodynamic stability.

If $\mu_{12} = 0$, but $\alpha_2 = \alpha_1 + \epsilon$, then the oscillating part of $\cos\varphi_\gamma$ has the form

$$\frac{\varepsilon}{2}\left(\frac{\mu_1}{\mu_2}\right)^{1/2},\quad \mu_1\neq\mu_2. \qquad (17)$$

In those cases in which μ_{12} has a value that is appreciable in comparison with unity, oscillations of the damping of the helicon always take place.

In the consideration of the polarization of the helicon, we can omit components of the order of α in (12). Substitution in (8) shows that the field $\mathbf{b}_\pm$ is polarized along an ellipse with semiaxes

$$\mathbf{a}_1=\mathbf{u}_1\cos\frac{\varphi_\beta\mp\delta}{2}+\left(\frac{\mu_1}{\mu_2}\right)^{1/2}\mathbf{u}_2\cos\frac{\varphi_\beta\pm\delta}{2},$$
$$\mathbf{a}_2=\mathbf{u}_1\sin\frac{\delta\mp\varphi_\beta}{2}+\left(\frac{\mu_1}{\mu_2}\right)^{1/2}\mathbf{u}_2\sin\frac{\delta\pm\varphi_\beta}{2}, \qquad (18)$$
$$\operatorname{tg}\delta=\frac{\mu_2-\mu_1}{\mu_1+\mu_2}\operatorname{tg}\varphi_\beta.$$

For an isotropic metal with a spherical Fermi surface, if we choose the $\mathbf{u}_2$ axis perpendicular to $\mathbf{k}$ and $\mathbf{B}_0$, we get

$$\mu_{12}=0,\quad \mu_2=1,\quad 0<\mu_1<1,\quad \alpha_1=\alpha_2=\alpha\approx(\omega_e\tau)^{-1},$$
$$\cos\varphi_\beta=\frac{2\pi(\partial M/\partial B)\sin^2\theta}{[1-4\pi(\partial M/\partial B)\sin^2\theta]^{1/2}}\alpha. \tag{19}$$

With the exception of the immediate vicinity of the boundary of the stability region, we have $\varphi_\beta \sim \delta \sim \pi/2$. It is seen from Eqs. (18) in this case that the axes of the polarization ellipse are parallel to the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$, and the ratio of their lengths is equal to $\mu_1^{1/2}$. A similar situation occurs in an arbitrary metal if the magnetic field lies in the plane of symmetry, and α_1, $\alpha_2 \ll 1$.

If the magnetic field deviates from the symmetry plane, even by a small angle, then another situation is possible, for example, in indium or aluminum (see below), in which

$$\alpha_1,\ \alpha_2\ll\mu_{12}<1,\quad \mu_1=\mu_2=1, \tag{20}$$

here (13) yields

$$\cos\varphi_\beta\approx-\mu_{12}, \tag{21}$$

and the semiaxes of the polarization ellipse are equal to

$$a_1=a_2=(1\pm\mu_{12})^{1/2} \tag{22}$$

and make 45° angles with the vectors $\mathbf{u}_1$ and $\mathbf{u}_2$.

Thus the de Haas—Van Alphen effect can have a significant influence not only on the phase velocity but also on the damping and the polarization of the helicon.

Very strong effects should be expected, as was shown above, from the off-diagonal components of μ_{12}. We shall demonstrate by a single example, that the amplitude of the oscillations of these components can be appreciable.

As a medium in which the helicon is propagated, let us consider indium. Under certain experimental conditions, which include the magnitude and direction of the magnetic field and the temperature and quality of the crystal, the de Haas—van Alphen effect in this metal [9] is determined by the portions of the Fermi surface which have the form of tubes which lie in a plane perpendicular to a fourfold axis of fourth order, and which form a toroid in the shape of a square. The oscillating magnetic moment corresponding to each tube is directed along it, independently of the direction of the magnetic field, so that the total moment can be written in the form

$$\mathbf{M}=m_1\mathbf{n}_1\sin\frac{2\pi F_0}{\mathbf{n}_1\mathbf{B}}+m_2\mathbf{n}_2\sin\frac{2\pi F_0}{\mathbf{n}_2\mathbf{B}}, \tag{23}$$

$\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors directed along the tubes, while m_1 and m_2 are amplitude factors that depend on the direction of the magnetic field. Therefore generally speaking $m_1 \neq m_2$. The dependence of the amplitude of the magnetic moment on the magnetic field will not be taken into consideration.

Let the wave propagate perpendicular to the plane in which the tubes lie. We choose the coordinate system so that the basis vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ are directed along the diagonals of the square toroid, and $\mathbf{u}_3 \parallel \mathbf{k}$. The magnetic field $\mathbf{B}_0$ makes an angle θ with the wave vector and is almost parallel to the $(\mathbf{u}_1\mathbf{u}_3)$ plane, so that it makes only a small angle δ with this plane. The order of magnitude of this angle will be made clear later on. Thus, the coordinates of all the vectors in the problem are as follows:

$$\mathbf{k}=\{0,0,k\},\quad \mathbf{B}_0=\{B_0\sin\theta,\ \delta B_0\sin\theta,\ B_0\cos\theta\},$$
$$\mathbf{n}_1=\left\{\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right\},\quad \mathbf{n}_2=\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\right\},\quad \mathbf{b}=\{b_1,b_2,0\}; \tag{24}$$

furthermore, it is clear that in the given geometry the amplitudes of both components in (23) are equal: $m_1 = m_2 = m$.

With the help of (23), we obtain the components of the tensor $\hat{\mu}$:

$$\mu_1=\mu_2=1-4\pi A\cos\frac{2\pi F_0}{B_0\sin\theta}\cos\left[\frac{2\pi F_0}{B_0\sin\theta}\delta\right],$$
$$\mu_{12}=4\pi A\sin\frac{2\pi F_0}{B_0\sin\theta}\sin\left[\frac{2\pi F_0}{B_0\sin\theta}\delta\right]. \tag{25}$$

If $\delta = 0$ in (25), then $\mu_{12} = 0$, but rotation of the field by an angle

$$\delta_c=\frac{1}{4}\frac{B_0\sin\theta}{F_0} \tag{26}$$

is sufficient to make the amplitude of the oscillations of μ_{12} the same as for μ_1 and μ_2 at $\delta = 0$. Using the values $F_0 = 4.6\times10^6$ G for indium [9], we obtain $\delta_c \approx 0.1°$ in a field of 2×10^4 G. Rotation of the field through such an angle has no effect on the magnetoresistance tensor; however, as shown above, it significantly changes all the characteristics of the helicon. To observe this purely quantum effect, we must naturally prepare a very nearly perfect crystal, so that the disorientation of its individual parts is less than δ_c.

EXCITATION OF A WAVE IN A HALF-SPACE

Since we are dealing with low-frequency waves of the acoustic range, they are usually excited with a coil carrying an alternating current of given amplitude. If the metal has a plane surface, then the coil is so placed that its field is parallel to this surface. We shall therefore assume that, on the surface of a metal that fills the half-space $x_3 > 0$, where x_3 is the coordinate in the direction of the unit vector $\mathbf{u}_3$, the field $\mathbf{h}$, tangent to the surface and parallel to the unit vector $\mathbf{u}_1$ is produced by an external source. The choice of the basis vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ is the same as above.

To make use of the results already obtained, we resolve the field $\mathbf{h}$ in the metal (its part parallel to the surface) into the components $\mathbf{h}_\pm = \mu\mathbf{b}_\pm$, where $\hat{\mu}$ is a two-dimensional tensor. Then the boundary condition at the surface takes the form

$$\mathbf{u}_1=\eta_+\mathbf{h}_++\eta_-\mathbf{h}_-, \tag{27}$$

where $\eta_\pm$ are complex coefficients subject to determination. Simple calculations yield

$$\eta_+=\frac{\beta_--\mu_2+\mu_{12}}{(\beta_--\beta_+)\Delta},\quad \eta_-=-\frac{\beta_+\mu_2+\mu_{12}}{(\beta_--\beta_+)\Delta}. \tag{28}$$

The transverse part of the electric field $\mathbf{e}$, the magnetic induction $\mathbf{b}$, and the current density $\mathbf{j}$ can also be represented as linear combinations of the natural solutions for an unbounded metal with the same coefficients η_+ and η_-. Of course, each component should be multiplied by $\exp[i(\omega t - k_\pm x_3)]$. The time-averaged value of the energy flux through the surface of the metal is

$$\bar{S}=\frac{c}{8\pi}\mathrm{Re}[\eta_+\mathbf{e}_++\eta_-\mathbf{e}_-,\mathbf{u}_1].$$

With the help of (11), we get

$$\bar{S}=\frac{c}{8\pi}\operatorname{Re}\left[\left(\frac{\omega\rho_{12}\gamma_0}{4\pi}\right)^{1/2}\eta_+\exp\left[i\left(\frac{\pi}{4}+\frac{\varphi_T}{2}\right)\right]+\left(\frac{\omega\rho_{12}\gamma_0}{4\pi}\right)^{1/2}\eta_-\exp\left[i\left(\frac{\pi}{4}-\frac{\varphi_T}{2}\right)\right]\right]. \quad (29)$$

In the zeroth approximation in α_1 and α_2, we have

$$\eta_+=\eta_-=\mu_2/2\Delta. \quad (30)$$

Substituting (15) and (30) in (29), we get

$$\bar{S}=\frac{c}{16\pi}\left(\frac{\omega\rho_{12}}{4\pi}\right)^{1/2}\frac{\mu_2}{\Delta}. \quad (31)$$

RESONANCE IN A PLANE PARALLEL PLATE

Let the metal fill a plane-parallel layer $-d < x_3 < d$ and let the external source produce the same field of unit amplitude $\mathbf{h} = \mathbf{u}_1$ on both surfaces. The fields $\mathbf{h}$ and $\mathbf{b}$ evidently depend on the coordinate x_3 in the same fashion. In particular, the field $\mathbf{b}$ inside the layer is of the form

$$\mathbf{b}=\left(\eta_+\mathbf{b}_+\frac{\cos k_+x_3}{\cos k_-d}+\eta_-\mathbf{b}_-\frac{\cos k_-x_3}{\cos k_-d}\right)e^{i\omega t}. \quad (32)$$

For $\mathbf{b}_\pm$, $\mathbf{k}_\pm$, and $\eta_\pm$ we must use (8), (10), and (28).

If we record the helicon signal with a receiving coil whose turns are perpendicular to the turns of the exciting coil, then the emf at the open ends of the receiving coil can be calculated in the following way:

$$v_2=-\frac{1}{c}\frac{d}{dt}\int_{-d}^{d}(\mathbf{b}\mathbf{u}_2)\,dx_3,$$

where $\mathbf{b}$ must be substituted from (32). The result can be represented in the form of a sum of resonant components, as was done by Penz:[10]

$$v_2=-\frac{8i\omega de^{i\omega t}}{\pi^2c(\alpha_1\mu_2+\alpha_2\mu_1)}\left\{\left[1-\frac{\mu_{12}(\mu_{12}+2\alpha_2\mu_1+\alpha_1\mu_2)}{\Delta}\right]\times\sum_0^\infty\frac{1}{(2n+1)^2}\frac{1}{1+iQ(\omega/\omega_n-\omega_n/\omega)}+i\mu_{12}\left(\frac{1+\alpha_1\alpha_2}{\Delta}\right)^{1/2}\times\sum_0^\infty\frac{1}{(2n+1)^2}\frac{\omega_n/\omega}{1+iQ(\omega/\omega_n-\omega_n/\omega)}\right\}, \quad (33)$$

where

$$Q=\frac{\Delta^{1/2}(1+\alpha_1\alpha_2)^{1/2}}{\alpha_1\mu_2+\alpha_2\mu_1},\qquad \omega_n=(2n+1)^2\frac{\pi^2}{d^2}\frac{c^2}{16\pi}\rho_{12}\Delta^{1/2}(1+\alpha_1\alpha_2)^{1/2}. \quad (34)$$

The relations (33), (34) are exact and in the absence of the de Haas—van Alphen effect they go over into the corresponding formula given by Penz.[10]

A synchronous detector, constructed in such fashion that it responds either to the real or to the imaginary part of the expression (33), is employed in the usual experimental scheme for detecting the signal v_2 of (33). In the absence of the de Haas—van Alphen effect the resonance curve has either a symmetric or an antisymmetric shape as a function of the frequency. As is seen from Eq. (33), the shape of the resonance curve in the experiment described above can be strongly distorted if the component μ_{12} has a value that is not too small in comparison with unity. But if $\mu_{12} = 0$, then the shape of the resonance curve is not distorted; however, all the remaining characteristics—the resonance frequency, the height and width—can oscillate upon variation of the magnitude or direction of the magnetic field.

CONCLUSION

The formulas obtained in the paper are a general character in the limits of validity of the conditions (1), and certain particular variants discussed in the text, by no means exhaust the abundance of possibilities which are determined principally by the shape of the Fermi surface of the investigated metal.

We turn our attention to still another circumstance. The fact is that the helicons exist in the metal not only in the form of waves, excited from without, but also as elementary thermal excitations. We now trace how the frequency of such an excitation with a certain wave vector $\mathbf{k}$ depends on the temperature. If the experimental conditions are chosen in such fashion that, upon lowering the temperature, we reach the boundary of stability ($\Delta = 0$) of a homogeneously magnetized state at finite temperature, then, as is seen from the dispersion relation (10), the frequency of the helicon vanishes on this boundary. In the local theory, the vanishing of the frequency takes place simultaneously for all branches of the spectrum that correspond to the given direction $\mathbf{k}$. However, in the nonlocal case, when the components of the tensors $\hat{\rho}$ and $\hat{\mu}$ depend on $\mathbf{k}$, one can imagine that any sufficiently narrow region of $\mathbf{k}$ space turns out to be separated out in the sense that the stability boundary for it will be achieved earlier for the highest temperature. Then the transition of the metal into an inhomogeneously magnetized state, corresponding to the given helicon mode, can take place. Thus, the considerations given above indicate the possibility of consideration of periodic diamagnetic structures[8] as a consequence of the "freezing" of certain helicon modes.

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