Properties of parametrically excited spin waves in ferrites

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An experimental determination has been made of the direction and distribution of the wave vectors of parametrically excited spin waves (PSW) in ferrites. The method of determining these PSW parameters is based on observation of the effect of frequency doubling by means of PSW. This effect is a nonlinear three-step interaction, consisting of a single process of splitting and two processes of merging of quasiparticles, whose probabilities, as determined experimentally, depend strongly on the PSW properties. It was found that the range of excitation of PSW in wave-vector space depends on the polarization of the pumping with respect to the constant magnetic field. It is least for parallel pumping and reaches a maximum for perpendicular pumping. For parallel pumping an effect of primary PSW on the excitation of secondary waves was detected; for perpendicular pumping, this effect is absent.

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This paper describes an experimental study of several characteristics of parametrically excited spin waves (PSW) in ferrites. In the experimental procedure, the method of frequency doubling by means of PSW was used to determine the polar angle θ_k of the spin wave, between the direction of the PSW wave vector **k** and the direction of the external magnetic field **H** in which the ferrite is placed; thus the distribution of the spin waves in wave-vector space was studied. In addition, an attempt was made to investigate the influence of different spin waves on each other in the process of parametric excitation. The object investigated was a sphere of yttrium-iron garnet (YIG) of diameter ~2 mm. The pumping frequency was 9370 MHz, the pulse duration 200 μsec, the repetition frequency 1 to 50 Hz.

FREQUENCY DOUBLING BY MEANS OF PSW

1. For investigation of PSW characteristics, use was made of the phenomenon discovered by us of frequency doubling by means of parametrically excited spin waves. The essence of this phenomenon consists of the following. As a result of parametric excitation, there appear in the specimen PSW with frequency $\omega_k = \omega/2$, where ω is the pumping frequency. PSW, by the process of three-magnon merging, can form new spin waves with frequency and wave vector equal to the sum of the frequencies and wave vectors of the two merging PSW[1]. Since the PSW frequency is $\omega_k = \omega/2$, the frequencies of the new spin waves will be equal to the pumping frequency. Further, each of the new spin waves can merge with a wave of the same kind, but having an opposite wave vector, with the consequence that electromagnetic waves originate with frequency twice the pumping frequency: frequency doubling by means of PSW occurs in the ferrites. We note that the last process—the process of merging of two spin waves with oppositely directed vectors into a single electromagnetic wave—is the process inverse to parametric excitation of spin waves in ferrites by longitudinal pumping[2].

The frequency-doubling process described here is certainly of low probability, and it will have an appreciable magnitude only when the effect of the usual frequency doubling[3] is small; this is greatest under conditions of ferromagnetic resonance at the pumping frequency. On the other hand, frequency doubling by means of PSW will occur at a value of H significantly smaller than the resonance value, where the effect of the usual doubling is extremely small. In fact, it was pointed out earlier that the process of three-magnon merging of PSW with each other plays a part in the mechanism of frequency doubling by means of PSW. The laws of conservation of energy and of momentum impose bounds on the range of constant magnetic fields in which the merging process can occur. The upper bound of this range, which depends on the parameters of the material from which the ferrite is manufactured and on the polar angle θ_k of the PSW, is determined by the following formula:

$$\frac{H_{3m}(\theta_k)}{4\pi M_0} = {}^2/_3(\alpha^2+\sin^4\theta_k)^{1/2} - {}^1/_3(\alpha^2+{}^1/_4\sin^4\theta_k)^{1/2} - {}^1/_2\sin^2\theta_k + {}^1/_3, \quad (1)$$

where M_0 is the saturation magnetization of the ferrite, and where $\alpha = \omega/4\pi\gamma M_0$.

The probability of three-magnon merging of PSW has a sharp maximum at field $H = H_{3m}(\theta_k)$[4], decreases rapidly at fields less than $H_{3m}(\theta_k)$, and is zero for $H > H_{3m}(\theta_k)$. Thus there is a possibility in principle of determining the field $H_{3m}(\theta_k)$ by study of the dependence of the effectiveness of frequency doubling by means of PSW on the constant magnetic field. Then by means of formula (1), with known values of α and M_0, it is possible to obtain information about the polar angle θ_k of the parametric spin waves.

2. The experimental apparatus for observation of frequency doubling by means of PSW was the usual apparatus for observation of the threshold of first-order spin-wave instability in the three-centimeter range of wavelengths of the pumping signal[5], except that to the end of the resonator containing the ferrite, via a circular diaphragm, was soldered a circular one- and-one-half centimeter waveguide. Through this waveguide, the power $P_{2\omega}$ at frequency 2ω reached a measuring receiver of the one-and-one-half centimeter range having sensitivity 10^{-11} W. The pumping angle θ_h between the directions of the constant and alternating magnetic fields could be varied continuously from 0 to 90° by rotation of the electromagnet about an axis coincident with the direction of propagation of the waves in the three-centimeter and one-and-one-half centimeter waveguides.

The experimentally observed value of $P_{2\omega}$ with change of the constant magnetic field had two maxima:

one in the resonance-field region $H \sim 3.4$ kOe; the second, which was significantly smaller in amplitude, shifted, depending on the pumping angle θ_h, from field $H = 1030$ Oe (parallel pumping, $\theta_h = 0$) to field $H = 1340$ Oe ($\theta_h = 90°$, perpendicular pumping). The second maximum, in our view, is caused by frequency doubling in the ferrites by means of excitation of PSW. In favor of this assertion are the following circumstances:

a) Resonance frequency doubling decreases with departure from resonance, and already at fields $H < 2.5$ KOe the power $P_{2\omega}$ becomes less than the sensitivity of the apparatus.

b) Frequency doubling at small fields occurs only after the pumping power exceeds the threshold for spin-wave instability, $P > P_{th}$. Simultaneous observation of the power reflected from the resonator and of the power in the one-and-one-half centimeter waveguide shows that the instants of appearance of instability and of the double-frequency signal coincide.

c) The $P_{2\omega}(P)$ dependence for small fields can be expressed approximately by the law $P_{2\omega} = CP^n$, where $3 < n \lesssim 4$, whereas in the case of resonance frequency doubling, the usual equation for second-order effects, $n = 2$, holds. The difference of n from 2 is caused by the complicated dependence of the spin-wave amplitude on the value of P/P_{th}.

d) The location of the field for three-magnon merging, for longitudinal pumping of spin-wave instability ($\theta_h = 0$), can be determined experimentally by observation of the minimum of the imaginary part of the nonlinear susceptibility[5]. It was found that the maximum of the effectiveness of the frequency doubling corresponds to the minimum of the nonlinear susceptibility; that is, this maximum occurs, as was expected, at field $H_{max} = H_{3m}$.

It seems to us that all these facts convincingly speak in favor of the hypothesis that the frequency doubling at small fields occurs by virtue of parametric excitation of spin waves.

3. Figure 1 shows the dependence upon the pumping angle of the value of the constant magnetic field, H_{max}, at which a maximum of $P_{2\omega}$ is observed. If $P_{2\omega}$ is due to frequency doubling by means of PSW, then H_{max} coincides with the field $H_{3m}(\theta_k)$ for three-magnon merging of spin waves. Consequently, by experimental determination of H_{max} it is possible, by Eq. (1), to determine the value of the angle θ_k of PSW excited at a given pumping angle. The calculation of θ_k on the basis of $H_{max} = H_{3m}(\theta_k)$ is also shown in Fig. 1. For convenience, there are plotted on the H_{max} axis values of the fields for three-magnon merging for PSW having various polar angles θ_k.

4. The shape of the double-frequency emission line near the maximum is shown in Fig. 2. Special interest attaches to the part of the emission line at fields $H > H_{max}$. As was pointed out above, the effect of frequency doubling by means of PSW that have angle θ_k^0, having reached a maximum at field $H_{max} = H_{3m}(\theta_k^0)$, should be equal to zero at all fields $H > H_{3m}(\theta_k^0)$, since here the probability of three-magnon merging vanishes for spin waves with polar angle θ_k^0. The experimental occurrence of a continuous decrease of the value of $P_{2\omega}$ at fields $H > H_{max} = H_{3m}(\theta_k^0)$ indicates that in the PSW system there are spin waves with $\theta_k < \theta_k^0$, for which the value of the field $H_{3m}(\theta_k) > H_{3m}(\theta_k^0)$. In other words, there is parametric excitation of a packet of spin waves, having a non-delta-function distribution about a central polar angle θ_k^0 whose value is determined by the pumping angle θ_h. The width of the PSW packet in polar-angle space, $\Delta\theta$, will be larger, the slower the drop of $P_{2\omega}$ for $H > H_{3m}(\theta_k^0)$.

Quantitatively, the value of $\Delta\theta$ can be described by the value of δH (see Fig. 2), the departure of the $P_{2\omega}(H)$ curve from the vertical drawn through its maximum, at the half-power level. The analytic relation between $\Delta\theta$ and δH can be easily obtained from (1):

$$\frac{\delta H}{4\pi M_0} = \Delta\theta\left(\sin 2\theta_k^0 - \Delta\theta\cos 2\theta_k^0\right)\left(\frac{1}{2} - \frac{2}{3\alpha}\sin^2\theta_k^0\right). \tag{2}$$

In the derivation of (2) the assumption was made that $\alpha^2 \gg \frac{1}{4}\sin^4\theta_k^0$; for the case of YIG under the influence of three-centimeter pumping, this is close to actuality.

Knowledge of the distribution law of spin waves in polar-angle space $\Delta\theta$ enables us, by means of the dispersion relation, to find the width of the PSW packet in wave-number space $\Delta k/k_0$, where k_0 is the wave number corresponding to polar angle θ_k^0.

The experimental dependence of δH on the pumping angle θ_k is shown in Fig. 3. There is evident a rapid increase of the value of δH, and consequently of the related values of $\Delta\theta$ and $\Delta k/k_0$, at pumpings close to perpendicular pumping. An estimate of the value of $\Delta k/k_0$ by means of the data of Fig. 3, formula (2), and the dispersion relation gives for parallel pumping $\Delta k/k_0 \approx 5 \times 10^{-3}$, for perpendicular $\Delta k/k_0 \approx 10^{-1}$.

5. The theory of nonlinear ferromagnetic resonance[4] indicates that in the case of parallel pumping of spin-wave instability ($\theta_h = 0$), in addition to the primary PSW with $\theta_k^0 = 90°$, after the instability threshold has been exceeded by an amount of the order of 10 dB, there should arise in the ferrite secondary PSW with $\theta_k \approx 50°$ and of an amplitude comparable with the amplitude of the primary PSW. To detect secondary, PSW, we made an attempt to record the emission at the double frequency at fields H about 200 Oe larger than the field at which maximum emission of primary waves

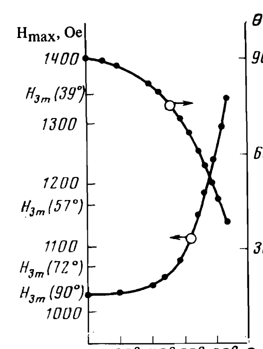

FIG. 1

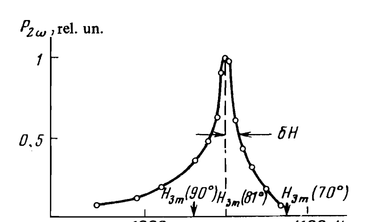

FIG. 2

FIG. 1. Dependence upon pumping angle θ_h of the constant magnetic field H_{max} at which the effectiveness of frequency doubling is greatest, and of the polar angle θ_k of the PSW; H || [111].

FIG. 2. Shape of the double-frequency emission line as a function of constant magnetic field. Pumping angle $\theta_h = 50°$, H || [111]; P/P_{th} = 4 dB.

occurs, since it is in this range that the three-magnon-merging field is located for waves with $\theta_k \approx 50°$. Despite the large reserve of sensitivity of the apparatus (more than 30 dB) and appreciable exceeding of the threshold level of pumping (no less than 15 dB), we did not succeed in detecting for parallel pumping a $P_{2\omega}$ corresponding to a wave with $\theta_k \approx 50°$. On increase of the pumping power, there was observed only a change of shape of the emission line at the double frequency; this change began at excesses of the order of 6–8 dB and consisted in an increase of the value of δH. Thus for $P/P_{th} \lesssim 6$ dB, $\delta H \approx 3$ Oe; but for $P/P_{th} = 10$ dB, the value of δH more than doubled and reached 7 Oe, which corresponds to an increase of the value of $\Delta k/k_0$ from 5×10^{-3} to 10^{-2} and excitation of PSW with polar angles $\theta_k \approx 81°$.

Thus in parallel pumping of spin-wave instability, the primary PSW packet retains its width until excesses above the threshold of 6–9 dB; at larger excesses, new spin waves are excited, with polar angles close to the primary PSW.

For perpendicular pumping of PSW ($\theta_h \approx 90°$), we did not succeed in observing a change of $\Delta k/k$ on increase of the pumping power; this may be due to the large error of measurement for $\theta_k = 90°$ (see Fig. 3). Therefore to investigate the influence of primary PSW on the excitation of secondary spin waves in this case, a measurement was made of the instability threshold for perpendicular pumping in constant magnetic fields near the field H_c, the minimum of the parallel-pumping threshold. Here the frequency $\omega/2$ goes into a spectrum of spin waves with $k = 0$, and, because of the dependence of the spin-wave damping parameter ΔH_k on k[6], the threshold for excitation of PSW, as a function of k, has two minima. The first minimum occurs at $k = 2.5 \times 10^5$ cm^{-1}, which corresponds to $\theta_k = 45°$; the second, at $k = 0$, $\theta_k \approx 60$–$70°$. The relative depth of the minima depends on the difference $H - H_c$. For $H \leq H_c$, short spin waves ($\theta_k = 45°$) have the lowest threshold; for $H \gtrsim H_c + 50$ Oe, long PSW have a lowest threshold. The threshold for short and for long waves differs experimentally, since for short spin waves there is hard excitation and hysteresis of the threshold, for long waves such phenomena are not observed.

Figure 4 shows the variation of the instability threshold for $\theta_h = 90°$ near the field H_c. For $H > H'$ long PSW are the first to be excited, but excitation of short waves also occurs; from the form of the graph it is clear that

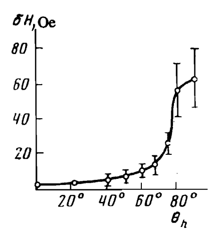

FIG. 3

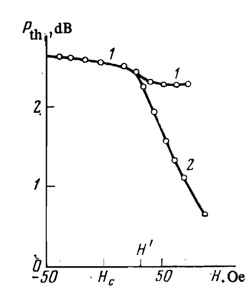

FIG. 4

FIG. 3. Dependence of δH (see Fig. 2) on pumping angle θ_h; H || [111], $P/P_{th} = 4$ dB.

FIG. 4. Dependence upon constant magnetic field of threshold for excitation of PSW in perpendicular pumping ($\theta_h = 90°$). Curve 1, threshold for excitation of short spin waves with $k \approx 2.5 \cdot 10^5$ cm^{-1}; curve 2, threshold for excitation of PSW with $k \to 0$. $H_c = 1530$ Oe; H || [111].

secondary spin waves can arise for very different excesses over the threshold. Furthermore it is evident that the presence in the ferrite of long PSW has almost no influence on the field dependence of the threshold for short spin waves; there is only a slight bending of the experimental curve near the field H'.

6. Self-modulation of the magnetization exerts a very strong influence on frequency doubling by means of PSW. Immediately after the onset of strong self-modulations[7], the growth of $P_{2\omega}$ with increase of pumping power practically ceases. In this connection, the shape of the $P_{2\omega}$ emission pulse differs significantly for different orientations of the crystal with respect to the external field. For $\theta_h = 0$, H || [100], when strong self-modulation is absent, the beginning of the pulse coincides with the beginning of the spin-wave excitation; the end, with the end of the pumping pulse. For $\theta_h = 0$, H || [111], the beginning of the $P_{2\omega}$ pulse has the same location, but its end coincides with the beginning of self-modulation; consequently the double-frequency pulse in this case has a duration of order 1 μsec. For pumpings close to perpendicular ($\theta_h = 90°$), for arbitrary specimen orientations the duration of the $P_{2\omega}$ pulse does not exceed 1 μsec.

7. The experimental determination of the width of the emission spectrum at the double frequency was carried out only at the maximal excesses $P/P_{th} \approx 15$ dB, and the magnitude of the signal was found to be at the limit of sensitivity of the spectral analyzer. According to these measurements, the width of the $P_{2\omega}$ emission spectrum is independent of the orientation of the specimen with respect to the direction of H and has a value not less than 10 MHz.

DISCUSSION OF RESULTS

1. As a result of the experiments performed, a number of properties of PSW in ferrites have been established. In particular, the interval Δk of excitation of PSW in wave-vector space was determined; the value $\Delta k/k_0$ of this interval for parallel pumping of spin-wave instability was found to be significantly smaller than for perpendicular (5×10^{-3} and 10^{-1} respectively). This fact can be explained if one takes into account that a reason for the existence of a finite interval of excitation of PSW, $\Delta k \neq 0$, along with two-magnon scattering on crystal inhomogeneities, is nonlinear damping[4,8], whose probability is proportional to $\sin 2\theta_k$ and reaches a maximum for PSW excited perpendicular to the pumping. The value of $\Delta k/k_0$ can be measured by the method considered in the paper only at a single value of the constant magnetic field: $H = H_{3m}$. In view of the fact that the probability of nonlinear damping is a maximum at this field[8], it can be expected that in an arbitrary field $H_{3p} < H < H_c$ the interval $\Delta k/k_0$ of excitation of PSW does not exceed the values measured in this research. Here H_{3p} is the value of the constant magnetic field below which there becomes possible a process of three-magnon splitting of PSW, accompanied by heating up of the reservoir of thermal spin waves at frequencies different from $\omega/2$[4].

The effect of two-magnon scattering on the interval of excitation of PSW was established in measurement of the value of δH for polycrystalline YIG; for parallel pumping of spin-wave instability, it was found in this case that $\delta H \geq 20$ Oe (we recall that for a perfect monocrystal, $\delta H \approx 3$ Oe). The values obtained in this work for the interval of excitation of spin waves indicate comparatively low spatial coherence of PSW even in the optimal case of parallel pumping of spin-wave instability. It is quite possible that this circumstance explains the absence of a positive result in experiments on scattering of light on PSW.

2. The results of the investigation of the $\delta H(P)$ dependence enable us to suggest a model for the development of instability of PSW in ferrites. In particular, in parallel pumping there is generated a packet of primary spin waves with polar angles close to $\theta_k = 90°$. The width of this packet in polar-angle space, $\Delta\theta$, is determined by two-magnon scattering and nonlinear damping. The primary PSW impede excitation of secondary spin waves by the pump up to excesses $P/P_{th} \lesssim 8$ dB, since the PSW and the pump act on spin waves in phase opposition[4]. For $P/P_{th} \geq 8$ dB, the PSW can no longer compensate the effect of the pump, and there begin to be excited a new group of spin waves, possessing a minimal threshold; this leads to an increase of the width $\Delta\theta$ of the PSW packet with increase of power.

The absence in the specimen of secondary spin waves with $\theta_k \approx 50°$ may be due to the fact that the theory of their excitation[4], for simplicity of the calculation, neglects processes of nonlinear damping and two-magnon scattering of spin waves; this leads to the result $\Delta k = 0$. As was shown above, these processes have a significant influence on the properties of the PSW, producing in a real situation a finite interval of excitation of PSW, $\Delta k \neq 0$. It is quite possible that under real conditions the generation of secondary spin waves occurs at significantly larger excesses over the threshold, as was pointed out in the theory[4]; this fact will promote a dependence of spin-wave damping on θ_k, which was not taken into account in[4] in obtaining the threshold for secondary PSW. In the case of perpendicular pumping, where there are small spatial coherence and a complicated dependence of the value of the threshold for excitation of spin waves upon their wave vector, the effect of PSW on the excitation of secondary spin waves is absent.

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153