Effect of linear transformation on the absolute parametric instability in an inhomogeneous plasma

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The development of absolute parametric instability in an inhomogeneous plasma is investigated in the vicinity of plasma resonance on the basis of a fourth-order equation. It is shown that along with "violet" Langmuir satellites it should also be possible to excite "red" satellites as a result of the plasma becoming more "transparent" in the strong pumping field. The effect of the linear transformation of the pumping electromagnetic wave on the development of the absolute instability is investigated in the geometric-optics approximation. It is shown that the presence of a transformed wave considerably enlarges the instability region without appreciably altering the logarithmic increment.

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INTRODUCTION

The investigation of parametric instabilities in an inhomogeneous plasma has gained greatly in importance in connection with the problem of plasma heating by powerful electromagnetic radiation. The anomalous absorption of large-amplitude electromagnetic waves in a plasma is attributed to these instabilities.[1-2] The inhomogeneity of the plasma greatly influences the parametric interaction of the waves. Allowance for the inhomogeneity can lead to stabilization of the decay instabilities, since the condition for spatial synchronism for the indicated waves is satisfied only in a finite region of the plasma. A stationary theory of decay processes in a weakly inhomogeneous plasma was developed in[3-5] under the condition that the interacting waves are described by the geometrical-optics approximation. An examination of the temporal evolution of the perturbations in the same approximation has shown that in the course of time the amplitudes of the waves increase to a certain maximum value determined by the stationary gain.[6,7] These exist, however, situations wherein absolute instabilities develop in the plasma,[7-10] and the time-growing perturbations are localized in a certain region of space. Then, as a rule, at least for one of the interacting waves in this region, the geometrical-optics approximation is violated, and this leads to a blocking of the oscillations as a result of reflection from the turning point[1] or transformation points.[12] In[13] there was demonstrated the possibility of development of absolute instability in the region of existence of a space-limited pump wave of constant amplitude.

Considerable interest attaches to the decay of a pump wave into a Langmuir wave and an ion-sound wave, since this decay has the largest growth rate in a uniform plasma. This decay was investigated in[8] for the case of oblique incidence of a p-polarized pump wave on an inhomogeneous plasma. It was shown that owing to the particular behavior of the field amplitude in the vicinity at the plasma-resonance point $\omega_0 = \omega_{pe}(z)$ (ω_0 is the frequency of the pump wave, $\omega_{pe}^2(z) = 4\pi e^2 n_0(z)/m$), absolute instability can set in if the intensity of the pump wave is large enough:

$$a^2 = \frac{E_i^2}{4\pi n_0 T_e}\frac{1}{2\pi k_0 L}\left(\frac{L}{\Delta}\right)^3 \gg 1,$$

where E_i is the amplitude of the incident wave, $k_0 = \omega_0/c$, $\Delta = (r_{De}^2 L)^{1/2}$, L is the characteristic scale of the plasma inhomogeneity, and r_{De} is the Debye radius of the electrons. The results of[8] are based on the use of geometric optics for the description of parametrically excited waves in the vicinity of plasma resonance. This method can be used to investigate the excitation of only "violet" Langmuir satellites with frequencies $\omega_1 > \omega_0$. In strong fields ($a^2 \gg 1$), however, the dispersion properties of the medium are strongly altered near plasma resonance. This change leads to "transparentization" of the plasma to the resonantly excited Langmuir perturbation. Not only the perturbations with frequencies $\omega_1 > \omega_0$ ("violet" satellites) are then unstable, but also perturbations with frequencies $\omega_1 < \omega_0$ ("red" satellites), and the latter do not exist at all in the indicated region in the linear approximation. At the same time, an important role can be assumed by the process of linear transformation of a pump wave into a plasma wave, which was likewise not taken into account in[8].

In Sec. 1 of this paper we investigate parametric interaction of perturbations of the Langmuir and ion-sound type with a spatially-inhomogeneous pump field near plasma resonance in the case when the perturbation localization region is $\delta z \ll \Delta$ (Δ is the half-width of the resonance). In contrast to[8], the analysis is based on an equation of fourth order. This makes it possible not only to justify the geometrical-optics approximation used in[8], but also extend greatly the limits of applicability of the results, and in particular advance into the region of larger pump-field intensities and consider, besides the usual case, also the case of "modified" decay. It is shown at the same time that the case of unmodified decay ($\gamma \ll \omega_2$, where ω_2 is the frequency of the ion sound) is well described by the geometrical-optics approximation.

The latter circumstance allows us to use this approximation to analyze perturbations localized in the region $\delta z \gg \Delta$, when it becomes important to take into account the linear transformation of the pump wave into a plasma wave. The influence of the linear transformation on the development of absolute instability in the case of unmodified decay is the subject of Sec. 2 of this paper, it is shown that in the presence of a transformed wave there appear unstable perturbations, the localization region of which greatly exceeds Δ, whereas the increment does not change substantially.

In strong fields ($a^2 \gg 1$), an important role is played by the self-action of the pump wave. The nonlinear solutions obtained in[14] describe the "transparentization" of the opacity barrier for the pump wave itself beyond the point of plasma resonance z = 0 at $z > a^{2/3}\Delta$. At the same time, the character of the behavior of the field at $z < a^{2/3}\Delta$ is determined qualitatively by the linear theory, and, in particular, a peak of the longitudinal-field intensity exists in the vicinity of z = 0. No account is taken in this paper of the self-action of the pump wave. A correct allowance for this self-action will lead apparently only to a redefinition of the quantity a, on which the increments obtained below depend. This conclusion can be drawn on the basis of the fact that the results of Sec. 1 of the present paper are valid for the parametric interaction in an arbitrary inhomogeneous pump field near the maximum of its intensity.

1. ABSOLUTE PARAMETRIC INSTABILITY NEAR PLASMA RESONANCE

We consider an inhomogeneous plasma with a density that increases monotonically in the positive direction of the z axis. An electromagnetic wave of frequency ω_0 is incident at an angle θ to the density gradient and is polarized in such a way that the electric-field vector lies in the plane of incidence (p-polarization). Near the plasma-resonance point z = 0 there exists a region in which the electric field has an anomalously large value,[15] and the z-component of the electric field at $z \ll -\Delta$ is described by the expression[16]

$$E_{0z}=F_0(z)\exp(i\omega_0 t-ik_x x)+\text{c.c.},$$

$$F_0(z)=\frac{\Phi(\theta)E_i}{(2\pi k_0 L)^{1/2}}\frac{L}{\Delta}\left\{\left(-\frac{z}{\Delta}\right)^{-1}+\left(-\frac{z}{\Delta}\right)^{-1/4}\pi^{1/2}\exp\left[-\frac{i\pi}{4}+i\frac{2}{3}\left(-\frac{z}{\Delta}\right)^{3/2}\right]\right\},\tag{1.1}$$

where $k_x = k_0 \sin\theta$, and $\Phi(\theta)\sim 1$ and differs from zero in a narrow range of incidence angles $\theta \lesssim (k_0L)^{-1/3}$. Expression (1.1) holds in the collisionless limit

$$\frac{\nu}{\omega_0}\frac{L}{\Delta}\ll 1,\tag{1.2}$$

where ν is a certain effective electron collision frequency. The first term in the asymptotic expression (1.1) describes the increase of the electric-field amplitude near the resonance point, and the second corresponds to the appearance of a plasma wave as a result of the linear transformation of the pump wave.

At $z > -\Delta$, in the approximation (1.2), the field amplitude is approximated by the expression[15]

$$F_0(z)=\frac{\Phi(\theta)E_i}{(2\pi k_0 L)^{1/2}}\frac{L}{\Delta}\left(\frac{z}{\Delta}+i\right)^{-1}.\tag{1.3}$$

We consider the parametric interaction of perturbations of the Langmuir and ion-sound type with frequencies ω_1 and ω_2, on the one hand, and the high-frequency field E_{0z}, on the other. We assume the frequency-resonance condition

$$\omega_0+\delta=\omega_1+\omega_2,\tag{1.4}$$

where δ is the frequency detuning, to be satisfied. Then, in the given pump-field approximation, this decay is described by the system of coupled equations

$$\left[\frac{\partial^2}{\partial t^2}+2\nu_1\frac{\partial}{\partial t}+\omega_{pe}^2-v_{Te}^2\frac{\partial^2}{\partial z^2}\right]E_z=-\frac{4\pi e^2}{m}n_S E_{0z},$$
$$\left[\frac{\partial^2}{\partial t^2}+2\nu_2\frac{\partial}{\partial t}-v_S^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\right]n_S=\frac{e^2 n_0}{mM\omega_0\omega_1}\frac{\partial^2}{\partial z^2}(E_{0z}E_z),\tag{1.5}$$

where E_z and n_S are the perturbations of the longitudinal electric field and the ion density; ν_1 and ν_2 are the effective field-damping decrements; $v_{Te}^2 = 3T_e/m$, $v_S^2 = 3T_e/M$. It is assumed for simplicity that the Langmuir wave propagates along the density gradient. In the derivation of (1.5) we discarded terms of order Δ/L in comparison with unity. The system (1.5) was obtained in standard fashion from the equations of hydrodynamics and from Maxwell's equations (see, e.g.,[17]).

Let us investigate the stability of the perturbations in such an interaction; to this end, we represent them in the form

$$E_z=f(z)F_0(z)\exp\{i\omega_1 t+pt\}+\text{c.c.},\quad n_S=g^*(z)\exp\{i(\omega_2-\delta)t-ik_x x+pt\}+\text{c.c.}$$

The system (1.5) then reduces to the form

$$f''+2\frac{F_0'}{F_0}f'+\left(\frac{F_0''}{F_0}+Q_1^2\right)f=\frac{1}{n_0 r_{De}^2}g,$$
$$g''+Q_2^2 g=-\frac{\omega_{pe}^2}{4\pi M\omega_0\omega_1 v_S^2}(|F_0|^2 f)'',\tag{1.6}$$

where

$$Q_1^2=k_1^2\left[1-\frac{2i(p+\nu_1)}{k_1 v_1}\right],\quad Q_2^2=k_2^2\left[1+\frac{2iB}{k_2v_2}\right],\quad k_1^2=\frac{\omega_1^2-\omega_0^2}{v_{Te}^2},$$
$$k_2^2=\frac{\omega_2^2-k_x^2v_S^2}{v_S^2},\quad v_1=\frac{k_1v_{Te}^2}{\omega_1},\quad v_2=v_S,\quad B=\frac{-(p+\nu_2+i\delta)^2}{2i\omega_2}+(p+\nu_2+i\delta),$$

and the prime denotes differentiation with respect to z.

In this section we investigate those eigenfunctions of the system (1.6) which are localized near z = 0 in a region $\delta z \ll \Delta$, where the approximation (1.3) for the pump field is valid. Neglecting in (1.6) the terms of order $(|f'/f|\Delta)^{-1}$ for such functions and eliminating g, we obtain for the amplitude of the Langmuir perturbation an equation of fourth order:

$$f^{IV}+(Q_1^2+Q_2^2)f''+Q_1^2Q_2^2 f=-a^2\left(\frac{f}{z^2+\Delta^2}\right)''.\tag{1.7}$$

The quantities Q_1^2 and Q_2^2 can be reduced to a form that lends itself more readily to further analysis:

$$Q_1^2=k_1^2b^{-1}(\Omega-i\Gamma_1),\quad Q_2^2=k_2^2(\Omega-i\Gamma_2)^2,$$
$$\Omega=b+\frac{\Delta\omega}{k_2v_2},\quad \Gamma_{1,2}=\frac{\gamma+\nu_{1,2}}{k_2v_2},\quad b=\frac{k_1v_1}{2k_2v_2}.$$

In the absence of a right-hand side, the solution of (1.7) is obviously a superposition of plane waves with a linear dispersion law. The presence of the right-hand side, however, leads to the appearance of spatially localized particular solutions $f\to 0$ as $z\to\pm\infty$. The determination of these particular solutions reduces to an investigation of the eigenvalue problem for Eq. (1.7). If it turns out here that there are eigenvalues with Re p > 0, then the corresponding solutions are absolutely unstable with increment γ = Re p. The quantity $\Delta\omega$ = Im p determines the nonlinear shift of the perturbation frequency. In contrast to[8], where the geometrical-optics approximation was used for the perturbation, we can now consider not only the case $k_1^2 > 0$ $(\omega_1 > \omega_0)$, which corresponds to excitation of a "violet" satellite, but also the case $\omega_1 < \omega_0$ ("red" satellites). The excitation of "red satellites will be treated by making the substitutions $b \to -b$ and $k_1 \to |k_1|$ in all the formulas encountered below.

The use of the Fourier transformation reduces the investigation of the eigenvalues of (1.7) to the eigenvalue problem for an equation of second order in k-space. The regions of localization of the function f(z) and of its Fourier transform f(k) are then connected by the relation[1)] $\delta z\delta k \approx n$.

Taking the Fourier transform of (1.7) with the variable k and putting, by virtue of the assumed smallness of the localization region of the eigenfunctions, $(z^2+\Delta^2$

$+ \Delta^2)^{-1} \approx \Delta^2(1 - z^2/\Delta^2)$, we obtain after making the change of variable $\xi = k\Delta\sqrt{a}$,

$$f''(\xi)-(\xi^2-\alpha+\beta/\xi^2)f(\xi)=0, \qquad (1.8)$$

where

$$\alpha=a+(Q_1^2+Q_2^2)\Delta^2/a, \quad \beta=Q_1^2Q_2^2\Delta^4/a^2.$$

We note that at $\beta = 0$ Eq. (1.8) coincides with the equation for the quantum oscillator.

Substitution, in (1.8), of the solution in the form

$$f(\xi)=e^{-\xi^2/2}\xi^s u(\xi), \qquad (1.9)$$

where $s(s - 1) = \beta$, leads to an equation for the function $u(\xi)$:

$$u''-2(\xi-s/\xi)u'+[\alpha-(2s+1)]u=0, \qquad (1.10)$$

the solutions of which at $s = 0$ are Hermite functions. If we seek the solution of (1.10) in the form of the series

$$u=\xi^r\sum_{n=0}^{\infty} a_n\xi^n, \qquad (1.11)$$

then we obtain an equation for r:

$$r(r-1+2s)=0,$$

and the recurrence relations

$$a_{2n+2}=\frac{2r+4n-\alpha+2s+1}{(r+2n+2)(r+2n+2s+1)}a_{2n}, \quad a_{2n+1}=0 \quad (n=0,1,\ldots).$$

Since $(a_{2n+2}/a_{2n}) \sim n^{-1}$ as $n \to \infty$, the solution (1.11) behaves as $\xi \to \pm\infty$ like $u \sim \xi^r \exp(\xi^2)$, so that the function f diverges at infinity. However, under the condition $2r + 4n - \alpha + 2s + 1 = 0$ the infinite series in (1.11) degenerates into a polynomial of degree $2n$, and the function f in (1.9) tends to zero as $\xi \to \pm\infty$, i.e., it is localized. The condition for the termination of the series (1.11) yields for the eigenvalues an equation that can be rewritten in the form

$$4n-\alpha+2s_l+1=0, \quad l=1,\ 2, \qquad (1.12)$$

where $s_{1,2} = \frac{1}{2} \pm (\frac{1}{4} + \beta)^{1/2}$.

Equation (1.8) is not valid in the entire range of variation of ξ. Indeed, in the expansion of $(z^2 + \Delta^2)^{-1}$ we have discarded terms of higher order, which would lead to the appearance of higher-order derivatives in (1.8). For example, the term proportional to z^4/Δ^4 leads to the appearance of the fourth derivative $d^4f/ad\xi^4$ in (1.8). It is obvious that the condition for discarding this term takes the form $d^2f/d\xi^2 \ll af$, according to (1.8), or

$$(|\beta|/a)^{1/4}\ll|\xi|\ll a^{1/2}\equiv\xi_{max}.$$

It is precisely in this region that equation (1.8) is valid. On the other hand, according to (1.9), the region of localization of $f(\xi)$ is of the order of $d\xi_f \sim (2n + 1)^{1/2}$. To be able to speak of the existence of solutions that are localized in k-space, it is therefore necessary to satisfy the inequality $d\xi_f \ll \xi_{max}$ or $a \gg 2n + 1$. Then, since the region of localization of $f(z)$ is of the order of $\delta z_f \sim (\delta k_f)^{-1} \sim \Delta[(2n + 1)/a]^{1/2}$, the initial assumption $\delta z_f \ll \Delta$ is automatically satisfied. Thus, the inequality $a \gg 1$ yields the lower limit of the intensities of the pump wave, for which the described results hold true.

We return now to the eigenvalue equation (1.12) and rewrite it, discarding terms small in comparison with a:

$$[Q_1^2+Q_2^2+\varkappa_n^2]^2=4Q_1^2Q_2^2, \qquad (1.13)$$

where we put $\kappa_n^2 = a^2[1 - 2(2n + 1)/a]\Delta^{-2}$. Equation (1.13) is algebraic and of fourth-degree in $(\Omega - i\Gamma)$. It has roots with $\Gamma > 0$ at all values of the parameters. Particularly simple equations for the roots are obtained in two limiting cases, $|Q_1| \gg |Q_2|$ and $|Q_1| \ll |Q_2|$.

In the former case Eq. (1.13) has the solution

$$\gamma+\nu_1=\frac{1}{2}\frac{\varkappa_n^3v_1^2}{k_1^2v_2}, \quad \Delta\omega=-\frac{k_1v_1}{2}\left(1+\frac{\varkappa_n^2}{k_1^2}\right). \qquad (1.14a)$$

This case is realized in sufficiently weak pump fields:

$$\varkappa_n\ll 2k_1v_2/v_1.$$

In strong fields, when the opposite inequality holds, we obtain

$$\gamma+\nu_2=\varkappa_n v_2, \quad \Delta\omega=-k_1v_1/2. \qquad (1.14b)$$

It is seen from these expressions that the degree of dependence of the instability increment on the pump-wave amplitude changes appreciably with change of the latter. In the case of weak pump fields, the instability threshold is determined by the damping of the Langmuir wave, whereas in sufficiently strong fields, the threshold may be determined by the damping of the ion sound. The growth rate decreases with increasing number of the state. Expressions (1.14a) and (1.14b) describe also the "modified" decay ($\gamma \gg \omega_2$), when the stability is aperiodic.

2. INFLUENCE OF LINEAR PUMP-WAVE TRANSFORMATION ON THE ABSOLUTE INSTABILITY

In this section we investigate the eigenfunctions of the system (1.6), which are localized in a region whose width greatly exceeds the half-width of the plasma resonance ($\delta z \gg \Delta$). An important influence on the development of the absolute instability is exerted here by the presence of a transformed plasma wave, described by the asymptotic expression (1.1), in the region $z < -\Delta$. It is impossible as yet to investigate the eigenfunction of the system (1.6) in such a complicated field by the method developed in the preceding section. We shall therefore use for the description of the unstabile perturbations the geometrical-optics approximation, which has, however, a narrower applicability range. We seek the solution of the system (1.6) in the form

$$f=a_1(z)\exp(-ik_1z), \quad g=a_2(z)\exp(-ik_2z), \qquad (2.1)$$

with $\omega_1 > \omega_0$, $k_1 = k_2 = k > 0$. (The results are not changed if k is replaced by $-k$. It is necessary to change here from $v_{1,2}$ to $|v_{1,2}|$.)

If the geometrical-optics approximation is to be valid in the entire region where the field exist, the following inequality must be satisfied:[2)]

$$|da_{1,2}/dz|\ll|k_{1,2}a_{1,2}|. \qquad (2.2)$$

Substituting (2.1) in (1.6), abbreviating the equations (i.e., discarding $a''_{1,2}$), and eliminating a_2, we obtain an equation for the amplitude of the Langmuir wave:

$$(1-\varepsilon)a_1''+\left(\frac{p+\nu_1}{v_1}-\frac{B}{v_2}+\frac{F_0'}{F_0}+2ik\varepsilon\right)a_1' + \left[-\left(\frac{p+\nu_1}{v_1}+\frac{F_0'}{F_0}\right)\frac{B}{v_2}+k^2\varepsilon\right]a_1=0, \qquad (2.3)$$

where

$$\varepsilon=\frac{a^2}{4(k\Delta)^2}\varphi\left(\frac{z}{\Delta}\right), \quad \varphi\left(\frac{z}{\Delta}\right)=\begin{cases}(1+z^2/\Delta^2)^{-1}, & z>-\Delta,\\ \pi(-z/\Delta)^{-1/2} & z<-\Delta.\end{cases}$$

Equation (2.3) is valid if $z > -\Delta(k\Delta)^2$, and terms of order $(k\Delta)^{-1}$ and Δ/L have been discarded from it. We consider pump-wave intensities in the range $1 \ll a \ll k\Delta$. The first inequality coincides with that used in Sec. 1, and the second is the necessary condition for neglecting the higher-order derivatives in (2.3), for in this case we have $\epsilon \ll 1$ in the entire region, so that the coefficient of the second derivative is not small anywhere and can be set equal to unity. The transformation

$$a_1=\psi(z,p)\exp\left\{-\frac{1}{2}\left(\frac{p+\nu_1}{v_1}-\frac{B}{v_2}\right)z-\int^z\left(\frac{F_0'}{2F_0}+ik\varepsilon\right)dz\right\} \quad (2.4)$$

reduces (2.3) to the equation

$$\psi''+U(z,p)\psi=0, \quad (2.5)$$

where $U(z, p)$ is generally-speaking a complex function of elaborate form. We confine ourselves to parameter values for which $U(z, p)$ takes a form that admits of a simple investigation of the eigenvalue problem (2.5).

In the region $z > -\Delta$, where the representation (1.3) is valid for the pump field, we can approximate $U(z, p)$ by the expression

$$U(z,p)=\frac{a^2}{z^2+\Delta^2}-\frac{1}{4}\left(\frac{p+\nu_1}{v_1}+\frac{B}{v_2}\right)^2, \quad (2.6)$$

if the inequality $1 \ll \Delta\,|(p+\nu_1)/v_1+B/v_2| \ll k\Delta$ is satisfied.

Under the same assumptions in the region $-\Delta(k\Delta)^2 \ll z \ll -\Delta$, using (1.1), we obtain

$$U(z,p)=\frac{\pi a^2}{\Delta^2}\left(-\frac{z}{\Delta}\right)^{-1/2}-\frac{1}{4}\left[\frac{p+\nu_1}{v_1}+\frac{B}{v_2}-\frac{2i}{\Delta}\left(-\frac{z}{\Delta}\right)^{1/2}\right]^2. \quad (2.7)$$

The function (2.6) has a turning point $z_{1,2}=\pm\Delta(\mu^2-1)^{1/2}$, where

$$\mu=2a\left[\left(\frac{p+\nu_1}{v_1}+\frac{B}{v_2}\right)\Delta\right]^{-1}.$$

Let $\mu \gg 1$. Then in the region where the approximation (2.6) is valid the potential $U(z, p)$ does not vanish at $z < 0$), and to find the left-hand turning point it is necessary to use expression (2.7). If at the same time[3] $\pi\mu^3 \ll 2a$ and $\pi^2\mu^4 \ll (k\Delta)^2$, then the function (2.7) has in the considered region the turning point $z=-z_0=-\pi^2\mu^4\Delta$.

It follows from (2.6) and (2.7) that in the region $|z| \ll \Delta(k\Delta)^2$ the function ψ has asymptotic forms

$$\psi\sim\exp\left[\pm\frac{1}{2}\left(\frac{p+\nu_1}{v_1}+\frac{B}{v_2}\right)z\right],$$

whereas according to (2.4) the asymptotic forms of a_1 are

$$a_1\sim\exp\left(-\frac{p+\nu_1}{v_1}z\right),\quad \exp\left(\frac{B}{v_2}z\right). \quad (2.8)$$

If at the same time the equation

$$\int_{-z_0}^{z_1}\sqrt{U(z,p)}\,dz=\pi(n+1/2) \quad (n-\text{is an integer}) \quad (2.9)$$

has roots p with $\operatorname{Re} p > 0$ and $\operatorname{Re} B > 0$, then Eq. (2.5), and together with (2.4), has localized solutions that increase exponentially with time.

The integral (2.9) can be calculated with good accuracy in the following manner: The function U is approximated by the expression (2.7) in the region $-z_0 < z < -\Delta$ and by the expression (2.6) in the region $-\Delta < z < z_1$. Simple calculations show that the principal terms of the asymptotic expansion of the integrals at $\mu \gg 1$ take the form

$$\int_{-z_0}^{-\Delta}\sqrt{U}\,dz\approx\frac{\pi^3}{4}a\mu^3,\quad \int_{-\Delta}^{z_1}\sqrt{U}\,dz\approx a\ln 4\mu. \quad (2.10)$$

It follows therefore that the main contribution to the integral (2.9) is given by the region in which there exist a transformed plasma wave. From (2.9) and (2.10) we obtain an equation for the determination of the eigenvalues p:

$$\frac{p+\nu_1}{v_1}+\frac{B}{v_2}=\lambda_n,\quad \lambda_n=\frac{a}{2\Delta}\left(\frac{4\pi^2a}{2n+1}\right)^{1/4}, \quad (2.11)$$

which can also be rewritten in the form

$$(Q_1^2-Q_2^2)^2=-4k^2\lambda_n^2.$$

In the case $|Q_1| \ll |Q_2|$ we obtain

$$\gamma+\nu_2=(\lambda_n k)^{1/2}v_2,\quad \Delta\omega=-kv_1/2, \quad (2.12a)$$

with $\lambda_n \gg k(v_2/v_1)^2$. If the inequality is reversed, have

$$\gamma+\nu_1=\lambda_n v_1,\quad \Delta\omega=-kv_1/2. \quad (2.12b)$$

The previously employed conditions $\pi\mu^3 \ll 2a$, $\pi^2\mu^4 \ll (k\Delta)^2$, and $\lambda_n\Delta \gg 1$ impose an upper bound on the number n of modes for which the foregoing analysis is valid:

$$2n+1\ll\pi a^2, \quad (2.13)$$

and the conditions $\mu \gg \sqrt{2}$ and $\lambda_n \ll k$ yield the lower bound

$$2n+1\gg\pi^2 a. \quad (2.14)$$

Thus, for the lowest modes $2n+1 \sim \pi^2 a$ the growth rate is determined by the quantity $\lambda_n \sim a/\Delta$ and decreases with increasing mode number. The condition $\lambda_n \ll k$ makes it possible to consider only unmodified decays by this method. The same method can be used to investigate the instability of modes localized in the narrow region $|z| \ll \Delta$. The numbers of these modes are determined by the inequality $2n+1 \ll a$ (see Sec. 1), and the growth rate and the frequency shift are obtained from (2.12) by replacing λ_n with κ_n. Consequently, the growth rate of the modes localized in the narrow region agrees in order of magnitude with the growth rate of the lowest unstable modes localized in a broader region. The dependence of the growth rates on the group velocities $v_{1,2}$ and also the instability thresholds, coincide with those obtained in the preceding section.

The results are only in qualitative agreement with the results of Sec. 1. The reason is that the use of geometrical optics necessitated a number of rough assumptions (see footnote 2), so that Eq. (2.11) is essentially approximate.

Our analysis allows us to draw the following conclusions: The development of absolute instability near plasma resonance is connected with the fact that the action of dynamic pressure produces in this region an effective "potential well." Perturbations that interact resonantly with the pump field are captured in this "well" and can exist in it in various states, which turn out to be absolutely unstable. The perturbations corresponding to "states" with small numbers $(2n+1 \ll a)$ are localized in a narrow region $\delta z \ll \Delta$ near the plasma resonance, where the pump field can be approximated by a simple parabolic dependence. With increasing number of the state, the localization region increases, and the growth rate decreases somewhat. When the localization region becomes of the same order as or larger than Δ, the dependence of the pump field on the coordinate becomes more complicated, and this is due to the appearance of a transformed plasma wave. States with numbers (2.13) and (2.14) correspond to unstable perturbations localized in the region $\delta z \sim a^{4/3}\Delta \gg \Delta$. The growth rate of the lowest modes localized in so broad a region is of the same order as the growth rate of the modes localized in the narrow region. Allowance for the linear transformation of the pump wave leads consequently to an appreciable increase of the instability region without an appreciable decrease of the growth rate.

It is obvious that the results of the present paper can be generalized to include parametric instability in an arbitrary inhomogeneous pump-wave field near the maximum of its intensity, where the representation $|E_{0z}|^2 \approx E_0^2(1 - z^2/z_0^2)$ is valid (z_0 is the characteristic scale of the field inhomogeneity). To this end it suffices to make everywhere in Sec. 1 the substitution

$$a^2 \to \frac{E_0^2}{4\pi n_0 T_e}\frac{z_0^2}{r_{De}^2}.$$

Such an inhomogeneous pump field can be realized, for example, not only in the case considered above, but also in the case of parametric excitation in the focus of a laser, and also in strongly nonlinear formations such as solitons and collapsing caverns. The instabilities of this type can constitute an effective mechanism for the absorption of electromagnetic radiation when plasma is heated and the energy of solitons or collapsing caverns is dissipated.

Let us estimate the pump-wave intensities needed to ensure the condition $a > 1$. For the parameters of a neodymium laser and the plasma used in the droplet variant of the laser thermonuclear reaction ($\omega_0 \approx 10^{15}$ sec^{-1}, $L \sim (10^{-2}$—$10^{-1})$ cm, $T_e \approx 10$ keV, $n_0 \approx 10^{21}$ cm^{-3}), this condition is equivalent to the requirement that the plasma be irradiated by an electromagnetic wave with an energy flux density $Q > (10^{11} - 10^{12})$ W/cm^2. Such laser powers are easily attainable at present (see, e.g.,[2]).

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1)We can use the approximate-equality sign because, as will become evident later on, the eigenfunctions in k-space are close to the wave functions of a quantum oscillator.

2)It will be shown later on that $|da_1/dz| \approx ka_1/2 < ka_1$. Thus, the geometrical-optics approximation is quite crude. Its use leads, nevertheless, to a qualitatively correct result.

3)This condition makes it possible to neglect the last term in the square brackets of (2.7).

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