Electron kinetics in a low-temperature molecular plasma (the ionosphere)

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Solutions to the kinetic equation for electrons in a low-temperature molecular plasma are obtained. The expressions for the electron distribution function in an alternating electric field are analyzed for stationary conditions, as well as for nonstationary perturbation and relaxation. The interaction of a high-power radio pulse with continuously emitted radio waves in the lower ionosphere is investigated experimentally. It is found that the characteristic time of heating of the plasma by a high-power pulse is an order of magnitude smaller than the relaxation time of the perturbation. The experimental results are shown to be in accord with theory. The kinetic parameters of the ionospheric plasma are determined.

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High-power radio waves give rise to strong perturbations in the ionosphere[1]. Thus, in the experiments[2,3] the mean electron energy in the lower ionosphere increased under the action of the electric field of the radio waves by a factor of 20–30. The distribution function of the electrons over velocity is then severely distorted. The kinetics of the electrons in the alternating electric field of the radio waves determines, to a considerable degree, the nature of the nonlinear phenomena in the ionosphere.

The neutral constituents of the lower ionosphere are molecular nitrogen N_2 and molecular oxygen O_2. In a molecular plasma an important influence on the electron distribution function is exerted by the inelastic collisions. The dominant role in the region of low electron energies $\epsilon \lesssim 5$ eV is then played by the excitation of rotational and vibrational levels of the molecules. This leads to a number of distinctive features in the behavior of the electrons in the low-temperature molecular plasma. The present paper is devoted to the investigation of these features.

1. DESCRIPTION OF THE EXPERIMENT

In the experiment we investigated the effect in the lower ionosphere of a high-power radio pulse on continuously emitted radio waves.

A powerful pulse transmitter, located at a point where the magnetic dip is 69.8° and the magnetic declination is 6.6°, emitted radio waves E_1 vertically upward at a frequency of $\omega_1 = 8.48 \times 10^6$ rad/sec close to the local gyromagnetic electron frequency ω_H. The pulse was a rectangular pulse of duration $\Delta t \approx 0.5 \times 10^{-3}$ sec. At the maximum normalized transmitter's emission power P = 0 dB the amplitude of the electric field of the waves at the lower-ionosphere heights $z \sim 70$–90 km was 5–10 times greater than the characteristic plasma field[1]

$$E_p = (3Tm\delta_{\text{eff}})^{1/2}\frac{\omega_{\text{eff}}}{e}, \quad \omega_{\text{eff}} = \omega_1 + \omega_H \cos\theta. \qquad (1)$$

Here e and m are the electron charge and mass, T is the gas temperature, θ is the angle of inclination of the magnetic field with the vertical (in our case $\theta = 20.2°$), and δ_{eff} is the effective fraction of the energy that is losable. The polarization of the wave was "ordinary." Under the conditions of the lower ionosphere $E_p \approx (400$–$600)$ mV/m. The apparatus used is described in greater detail in[3].

A second transmitter 2, located 42 km to the southeast of the powerful transmitter 1, emitted a continuous wave E_2 at frequencies $\omega_2 = 1.71 \times 10^6$ and $\omega_2 = 2.35 \times 10^6$ rad/sec. This radiation was picked up at the point 3 located 50 km to the northwest of the transmitter 1. The trajectory of the received beam passed through the perturbed region of the ionosphere. The scheme of the experiment is shown in Fig. 1.

Figure 2 shows typical examples of the shape of the E_2 signals received. The black stripes in the diagrams are time markings spaced 100 μsec (Fig. 2a) and 1 msec (Fig. 2, b, c, d) apart. The high-power pulse E_1 was switched on at the moment marked in the diagram by the arrow. It can be seen that the wave E_2 undergoes a considerable perturbation under the action of the wave E_1. The E_2-wave amplitude both decreases—the suppression of the E_2 wave by the E_1 wave—and, conversely, increases (Fig. 2d).

A distinctive feature of the curves shown is the very rapid growth of the perturbation and its rather slow decrease: the relaxation time exceeds the perturbation time by more than an order of magnitude. As will be shown below, this effect is a consequence of the distinctive features of the kinetics of the electrons of the ionosphere in the field of the strong perturbing wave.

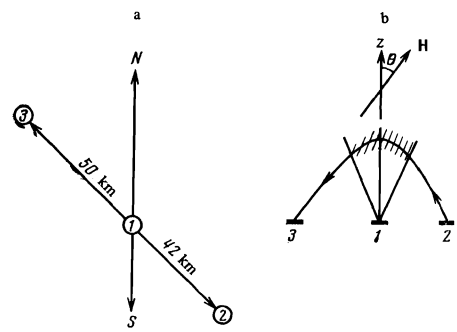

FIG. 1. Scheme of experiment ((a) view from above, (b) side view): 1) high-power pulse transmitter; 2) radio station emitting a continuous wave; 3) signal reception station. The region of interaction of the waves in the ionosphere is hatched.

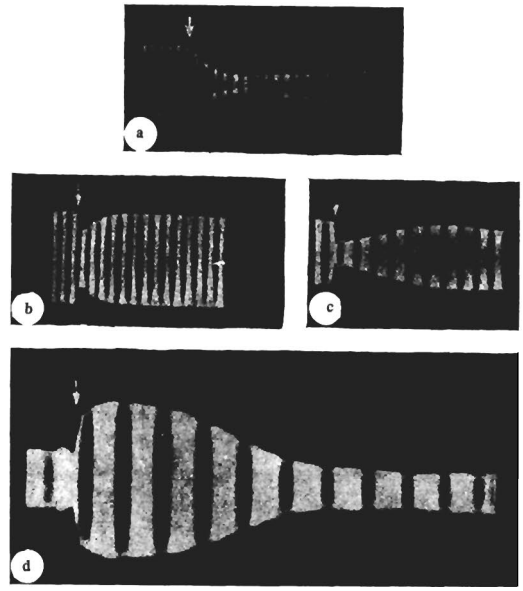

FIG. 2. Shape of the received signals: a) P = 0 dB, $\omega_2 = 2.35 \times 10^6$ rad/sec; April 25, 1967, at 0110 h; b) P = −10 dB, $\omega_2 = 2.35 \times 10^6$ rad/sec; April 25, 1967, at 2130 h; c) P = 0 dB, $\omega_2 = 1.71 \times 10^6$ rad/sec; March 13, 1967, at 2150 h; d) P = 0 dB, $\omega_2 = 2.35 \times 10^6$ rad/sec; March 15, 1967, at 0010 h.

2. THEORY

1. The kinetic equation. The electron distribution function in a molecular plasma located in a variable electric field depends on the modulus of the electron velocity v and the time t, and is described by the kinetic equation[1]

$$\frac{\partial f_0}{\partial t}-\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\delta_{el}\nu\left[\left(\frac{T}{m}+\frac{e^2E_0^2}{3m^2\delta_{el}(\omega^2+\nu^2)}\right)\frac{\partial f_0}{\partial v}+vf_0\right]\right\}+S_{inel}(f_0)=0. \quad (2)$$

Here $\delta_{el} = 2m/M$ is the fraction of the energy lost by an electron in one elastic impact, T is the temperature of the molecules, M is their mass, $\nu = \nu(v) = v\sigma_t(v)N_m$ is the collision rate for an electron moving with a velocity v (N_m is the concentration of the molecules and $\sigma_t(v)$ is the transport collision cross section)[1)]. Further, $S_{inel}(f_0)$ is the inelastic-collision integral:

$$S_{inel}=-\frac{2}{mv}\sum_{k,j}N_k[(\varepsilon+\varepsilon_{k,j})f_0(\varepsilon+\varepsilon_{k,j})\sigma_{k,j}(\varepsilon+\varepsilon_{k,j})-\varepsilon f_0(\varepsilon)\sigma_{k,j}(\varepsilon)$$
$$+(\varepsilon-\varepsilon_{k,-j})f_0(\varepsilon-\varepsilon_{k,-j})\sigma_{k,-j}(\varepsilon-\varepsilon_{k,-j})-\varepsilon f_0(\varepsilon)\sigma_{k,-j}(\varepsilon)]. \quad (3)$$

Here $\epsilon = mv^2/2$, $\epsilon_{k,j}$ is the energy of transition of a molecule from the state k into the state k + j, $\sigma_{k,j} = \int q_{k,j}d\Omega$ is the total cross section for the excitation of the transition $\epsilon_{k,j}$, and N_k is the concentration of the molecules in the state k. The first two terms in (3) describe the excitation of the molecules: the departure and arrival of electrons as a result of the excitation of the quantum $\epsilon_{k,j}$. The last two terms describe impacts of the second kind: the de-excitation of the molecules with a recoil energy $\epsilon_{k,-j}$ to an electron and the transition of the molecule from the state k into the state k − j; $\sigma_{k,-j}$ is the total cross section for this process.

In the case of small energy losses, i.e., for

$$\varepsilon_{k,j};\ \varepsilon_{k,-j}\ll\varepsilon, \quad (4)$$

expanding (3) in a Taylor series, we transform the collision integral into the differential form:

$$S_{inel}=-\frac{2}{mv}\sum_{k,j}N_k\frac{\partial}{\partial\varepsilon}\left\{\left[\varepsilon(\varepsilon_{k,j}\sigma_{k,j}-\varepsilon_{k,-j}\sigma_{k,-j})+\frac{1}{2}(\varepsilon_{k,j}^2\sigma_{k,j}+\varepsilon_{k,-j}^2\sigma_{k,-j})\right]f\right.$$
$$\left.+\frac{\varepsilon}{2}\frac{\partial}{\partial\varepsilon}[(\varepsilon_{k,j}^2\sigma_{k,j}+\varepsilon_{k,-j}^2\sigma_{k,-j})f]\right\}. \quad (5)$$

Let us first consider the excitation of the rotational levels. Diatomic molecules consisting of identical atoms (e.g., H_2, N_2, O_2) do not possess dipole moments. The excitation of rotational levels in the quadrupole interaction of molecules with slow electrons has been investigated by Gerjuoy and Stein[4]. Transitions with angular momentum changes $\Delta J = \pm 2$ are allowed, i.e.,

$$\sigma_{k,j}=\sigma_J\delta(J,J+2),\quad \sigma_{k,-j}=\sigma_{-J}\delta(J,J-2). \quad (6)$$

The total cross sections are then given by

$$\sigma_J=\sigma_0\frac{(J+2)(J+1)}{(2J+3)(2J+1)}\left[1-\frac{2(2J+3)B_0}{\varepsilon}\right]^{1/2},$$
$$\sigma_{-J}=\sigma_0\frac{J(J-1)}{(2J-1)(2J+1)}\left[1+\frac{2(2J-1)B_0}{\varepsilon}\right]^{1/2},$$
$$\sigma_0=8\pi Q^2a_0^2/15. \quad (7)$$

Here Q is the quadrupole constant ($Q \sim 1$), $a_0 = \hbar^2/me^2$ is the Bohr radius. The energy of the rotational quanta is[5]

$$\varepsilon_J=B_0J(J+1), \quad (8)$$

where B_0 is the rotational constant. It follows from (6) and (8) that

$$\varepsilon_{k,j}=\varepsilon_{J+2}-\varepsilon_J=(4J+6)B_0,\quad \varepsilon_{k,-j}=\varepsilon_J-\varepsilon_{J-2}=(4J-2)B_0. \quad (9)$$

Under the conditions (4), using (6)–(9) in the expression (5), we have

$$S_{inel}^r=-\frac{2}{mv}\sum_J N_J\frac{\partial}{\partial\varepsilon}\left\{\varepsilon\left[4B_0\sigma_0f+4B_0^2\sigma_0(J^2+J+3)\frac{\partial f}{\partial\varepsilon}\right]\right\}. \quad (10)$$

The first term in this formula coincides with the term obtained by Frost and Phelps[6]. In summing over the levels, we should take into account the fact that

$$N_J=\frac{N_m}{p}e^{-\varepsilon_J/T}(2J+1)\varkappa_J,\quad p=\sum\varkappa_J(2J+1)e^{-\varepsilon_J/T},$$

where N_m and T are the total concentration and the temperature of the molecules; p is a normalization factor; κ_J is a constant that is different for even and odd values of J[5]. Then, as is easy to see,

$$\sum N_J=N_m,\quad \sum N_J(J^2+J+3)\approx\frac{T}{B_0}N_m.$$

In the last expression we have assumed that $T \gg B_0$.

Taking into account the fact that $\epsilon = mv^2/2$, we reduce (10) to the standard form[7]

$$S_{inel}^r=-\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2R_r(v)\left[\frac{T}{m}\frac{\partial f_0}{\partial v}+vf_0\right]\right\},$$
$$R_r(v)=8B_0\sigma_0N_m/mv. \quad (11)$$

In molecular nitrogen $B_0 = 2.48 \times 10^{-4}$ eV, Q = 1.04; in oxygen $B_0 = 1.79 \times 10^{-4}$ eV, Q = 1.8[8].

The excitation of vibrational levels is described by the general formula (3). It can be simplified if we assume that the temperature of the molecules is much lower than the energy of the vibrational quanta, i.e., if we assume that $T \ll \hbar\omega_l$. In the lower ionosphere $T \sim 0.02$ eV, while $\hbar\omega_l = 0.292$ eV for nitrogen and $\hbar\omega_l = 0.195$ eV for oxygen. Therefore, the condition $T \ll \hbar\omega_l$ is sufficiently well fulfilled. In this case the number of excited molecules is small and the collisions

involving them are, to a first approximation, unimportant. Then the collision integral (3) assumes the form

$$S_{\text{inel}}^{v^*}=\frac{2N_m}{mv}\left[\varepsilon f(\varepsilon)\sum_{v^*}\sigma_{v^*}(\varepsilon)-\sum_{v^*}(\varepsilon+\varepsilon_{v^*})f(\varepsilon+\varepsilon_{v^*})\sigma_{v^*}(\varepsilon+\varepsilon_{v^*})\right]. \quad (12)$$

Here N_m is the molecule concentration, ϵ_{V^*} is the vibrational-quantum energy[5]

$$\varepsilon_{v^*}=\hbar\omega_i(v^*+{}^1/_2),$$

while σ_{V^*} is the total cross section for the excitation. If the vibrational losses are small in comparison with the electron energy (4), then the integral (12) assumes the simple form (5):

$$S_{\text{inel}}^{v^*}=-\frac{1}{2v^2}\frac{\partial}{\partial v}\{v^3R_vf_0\},$$

$$R_v=\frac{2N_m}{mv}\sum_{v^*}\varepsilon_{v^*}\sigma_{v^*}=\frac{2N_m\hbar\omega_i}{mv}\sum_{v^*}\left(v^*+\frac{1}{2}\right)\sigma_{v^*}. \quad (13)$$

In molecular nitrogen the vibrational levels are efficiently excited by electrons of energy 2 eV $\lesssim \epsilon \lesssim$ 3 eV. The process has a strongly expressed resonance character: The excitation is connected with a temporary capture of an electron[9]. The maximum cross sections $\sigma_{V^*} \sim 5\times10^{-16}$ cm^2. In the region of low energies ϵ < 1.0 eV, the cross sections are small, i.e., $\sigma_{V^*} < 10^{-18}$ cm^2[10]. On the whole, a similar picture obtains in the case of molecular oxygen. The region of efficient excitation here is 0.4 eV $\lesssim \epsilon \lesssim$ 2 eV, and the maximum cross sections are $\sigma_{V^*} \sim 10^{-17}$ cm^2. The vibrational losses in the general case are not small, i.e., the condition (4) is not fulfilled for them, and the general expression (12) should be used for the collision integral. However, in the case of the excitation of the first nitrogen levels, i.e., in the case when v^* = 1, 2, in the principal region of vibrational losses, the ratio $\epsilon_{V^*}/\epsilon \sim 0.2-0.3$. In air and in the ionosphere the contribution of these levels is the dominant contribution when $\epsilon \lesssim$ 2 eV. Therefore, at low energies $\epsilon \lesssim$ 2 eV, we can use here, in some approximation, the expression (13) for $S_{\text{inel}}^{V^*}$. The form of the function R_V for the lower ionosphere (air), constructed from the data of[6,9-11], is shown in Fig. 3.

Data on the cross sections for optical-level excitation, capture, and ionization are given in[10,11]. For the conditions of interest to us, it is sufficient to restrict ourselves to the consideration of the electron distribution function in the low-energy region $\epsilon \lesssim$ 5 eV, where these collisions are unimportant. In this region the kinetic equation (2) assumes the form

$$\frac{\partial f_0}{\partial t}-\frac{1}{2v^2}\frac{\partial}{\partial v}\left\{v^2\left[(\delta_{\text{el}}\nu+R_r)\left[\frac{T}{m}\frac{\partial f_0}{\partial v}+vf_0\right]+\frac{e^2E_0^2\nu}{3m(\omega_{\text{eff}}^2+\nu^2)}\frac{\partial f_0}{\partial v}\right]\right\}+S_{\text{inel}}^{v^*}=0. \quad (14)$$

In our case for an ordinary wave undergoing quasi-longitudinal propagation, $\omega_{\text{eff}} = \omega + \omega_H \cos\theta$[12].

2. The stationary distribution function. The steady-state solutions to the kinetic equation (14) were determined numerically. The form of the distribution function $f_0(v)$ in a high-frequency ($\omega_{\text{eff}}^2 \gg \nu^2$) electric field for the plasma in the lower ionosphere (air) is shown in Fig. 4. In the figure can be seen the characteristic feature of the distribution function for $E_0/E_p \lesssim 1$, namely, its slow decrease in the velocity region $v \lesssim 4\times10^7$ cm/sec (i.e., for $\epsilon \lesssim$ 0.4 eV), which leads to the formation of an extended "tail" for the function $f_0(v)$. The reason for this is easy to understand. The main electron-energy losses when $\epsilon \lesssim$ 0.4 eV are connected with the excitation of the rotational levels: They decrease with increasing electron velocity, i.e., $R_r \sim 1/v$. At the same time, the energy acquired by an electron under the action of the high-frequency field increases with v in proportion to the collision rate $\nu(v)$ (e.g., in air $\nu(v) \sim v^{2\alpha}$, where $\alpha \approx {}^5/_6$[13]). Therefore, the fast electrons rapidly acquire energy, i.e., they are accelerated, which leads to the formation of the "tail." Neglecting in (14) the losses due to vibrational-level excitation and the elastic losses $\delta_{\text{el}}\nu$, we have in the steady-state case ($\partial f_0/\partial t$ = 0) the expression

$$f_0(v)=C\exp\left\{\frac{3}{2^{3/4}a^{3/4}}\left[\frac{1}{2}\ln\frac{1+2^{1/2}Z+Z^2}{1-2^{1/2}Z+Z^2}-\operatorname{arctg}(2^{1/2}Z-1)-\operatorname{arctg}(2^{1/2}Z+1)\right]\right\},$$

$$Z=a^{1/4}\left(\frac{mv^2}{2T}\right)^{1/8},\quad a=\frac{e^2E_0^2\nu(v_T)}{3mT\omega_{\text{eff}}^2R_r(v_T)},\quad v_T=\left(\frac{2T}{m}\right)^{1/2}. \quad (15)$$

The constant C is determined by the normalization conditions

$$4\pi\int f_0(v)v^2\,dv=1. \quad (16)$$

It can be seen from (15) that for Z $\gg$ 1 the function $f_0(v) \to \text{const} = C\exp\{-3\pi/2^{3/2}a^{3/4}\}$. Actually this asymptotic form is not reached: At $\epsilon \approx$ 0.5–2 eV the distribution function is truncated, owing to the excitation of the vibrational levels.

In Fig. 5 we show the mean energy $\bar{\epsilon}$ and the effective collision rate $\bar{\nu}$ for an electron as functions of E_0/E_p for air ($\nu(v) = \nu_0 v^{2\alpha}$); here E_p is the characteristic plasma field (1):

$$\bar{\varepsilon}=2\pi m\int_0^\infty v^4f_0\,dv;\quad \bar{\nu}=\frac{4\pi(3+2\alpha)}{3}\int_0^\infty v^2\nu f_0\,dv. \quad (17)$$

It can be seen that the mean electron energy increases rapidly to $\bar{\epsilon}/T \sim 30$ when $E_0 \sim E_p$. Subsequently, the growth of $\bar{\epsilon}$ slows down. The collision rate $\bar{\nu}$ varies in entirely similar fashion. The existence of the region of rapid growth is due to the decrease of $R_r(v)$. It is natural to call this phenomenon a rotational jump. It is close to the analogous cutoff of T_e in a highly ionized plasma—the so-called superheat instability[1].

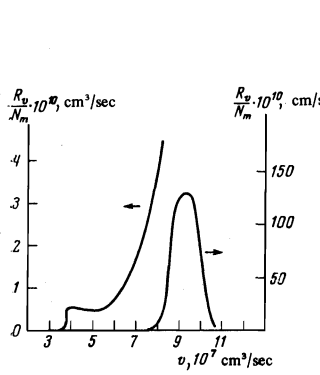

FIG. 3

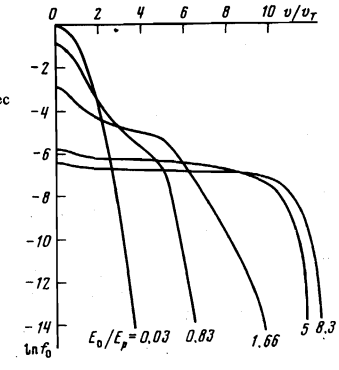

FIG. 4

FIG. 3. The vibrational-loss function.

FIG. 4. Electron distribution function for $\omega_{\text{eff}} \gg \nu$, T = 185 K, and $vT = 0.76\times10^7$ cm/sec ($\delta_0 = 5\times10^{-3}$).

Notice that in the approximate "elementary" theory[12,13] the mean energy and the electron collision rate are determined by the equations

$$\frac{dT_e}{dt}+\delta_{\text{eff}}(T_e)\bar{\nu}(T_e-T)=\frac{e^2E_0^2\bar{\nu}}{3m\omega_{\text{eff}}^2},$$

$$\bar{\nu}(T_e)=\bar{\nu}_0(T_e/T)^{\alpha},\quad \bar{\varepsilon}=3/2T_e. \tag{18}$$

Here δ_{eff} is the effective parameter characterizing the electron energy losses. It is shown in Fig. 6, where δ_{eff} is defined in such a way that the steady-state value of $\bar{\epsilon}$ computed in the elementary theory from (18) coincides with the result of the exact computation (17). It can be seen that when $T_e \lesssim 1000$ K the parameter δ_{eff} energetically decreases with T_e.

The dashed curve in Fig. 5 shows the collision frequency defined according to (18) for $\alpha = 5/6$. Let us emphasize that the parameter α in (18) is determined by the dependence of the frequency ν on v in the exact kinetic theory: If $\nu \sim v^{2\alpha}$, then $\bar{\nu} \sim (T_e/T)^{\alpha}$. In the case when the electron distribution function is nearly Maxwellian, taking (11) into account, we find (see[13]) that

$$\delta_{\text{eff}}\bar{\nu}=\frac{16\sqrt{2}}{3\pi^{1/2}}\frac{B_0\sigma_0N_m}{(T_em)^{1/2}}. \tag{19}$$

3. The nonstationary processes. Let us now consider the non-stationary processes of perturbation and relaxation of the electron distribution function that occur when the plasma is acted upon by a high-power electromagnetic pulse. In the case of heating in a strong electric field for which

$$E_0^2/E_p^2 \gg 1, \tag{20}$$

the relaxation terms in Eq. (14) at the initial stage are unimportant, and the equation assumes the form[2)]

$$\frac{\partial f_0}{\partial t}-\frac{e^2E_0^2}{6m^2\omega_{\text{eff}}^2}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\nu(v)\frac{\partial f_0}{\partial v}\right)=0. \tag{21}$$

Let us take into account the fact that in the perturbation of the distribution function by the strong electric field (20) the initial thermal spread of the electrons over velocity does not play any role, i.e., $f_0(v, 0) \sim \delta(v)$. In this case for $\nu(v) = \nu_0 v^{2\alpha}$, no characteristic parameters having the dimensionality of the velocity v are contained in either the initial conditions of the problem, or in the Eq. (21) itself. This means that the sought-for nonstationary solution to Eq. (21) should be self-similar in nature, i.e., it should depend on only the ratio v^{β}/t. Let us, therefore, seek the distribution function in the form

$$f_0(v,t)=t^{\gamma}F(\xi),\quad \xi=v^{\beta}/t. \tag{22}$$

Substituting (22) into Eq. (21), we find for $\nu(v) = \nu_0 v^{2\alpha}$ that $\beta = 2(1-\alpha)$, $\gamma = -3/(2-2\alpha)$, and

$$F(\xi)=\exp(-\xi/D),\quad D=\frac{2(1-\alpha)^2e^2E_0^2\nu_0}{3m^2\omega_{\text{eff}}^2}.$$

Finally, taking the normalization condition (16) into account, we have

$$f_0(v,t)=\frac{1-\alpha}{2\pi(Dt)^{3/(2-2\alpha)}}\Gamma^{-1}\left(\frac{3}{2-2\alpha}\right)\exp\left(-\frac{v^{2(1-\alpha)}}{Dt}\right). \tag{23}$$

In the lower ionosphere (air) $\alpha \approx 5/6$. It then follows from (23) that upon the action on the ionospheric plasma of a high-power electromagnetic pulse there is produced an electron distribution function $f_0 \sim \exp(-\epsilon^{1/6}t^{-1})$ that decreases very slowly with increasing electron energy ϵ. The cause of this phenomenon consists in the fact that the fast electrons efficiently acquire energy from the alternating field, which leads to their rapid acceleration. Of course, Eq. (21) and, consequently, the expression (23) for f_0 are valid only when the relaxation processes are negligible, i.e., as long as

$$f_0(v,t)\ll f_{0s}(v),$$

where $f_{0s}(v)$ is the stationary distribution function determined above. As $f_0(v, t)$ approaches $f_{0s}(v)$, the relaxation processes become important, and the function $f_0(v, t)$ gradually reaches the steady-state value. It is clear from (23) that when the plasma is heated by a short pulse the stationary distribution is established first of all in the high-velocity region.

The process of relaxation of the electron distribution function after the perturbing field has been switched off is completely determined by the collisions. In the region of considerable electron energies ($\epsilon \gg T$), the term proportional to T in Eq. (14) can be neglected. The equation then assumes the form

$$\frac{\partial f_0}{\partial t}-\frac{1}{2v^2}\frac{\partial}{\partial v}\{v^3(\delta_{\text{el}}\nu+R_r)f_0\}+S_{\text{inel}}^{v^*}=0. \tag{24}$$

In the vibrational region $0.4\ \text{eV} \le \epsilon \le 4$ eV the term in the curly brackets in Eq. (24) is unimportant. Furthermore, in the expression (12) for $S_{\text{inel}}^{v^*}$ we can approximately neglect the second term. Then

$$f_0(v,t)=f_0(v,0)\exp\{-\nu_{\text{inel}}t\},\ \nu_{\text{inel}}=vN_m\sum_{v^*}\sigma_{v^*}, \tag{25}$$

where $f_0(v, 0)$ is the initial distribution function. Relaxation here proceeds rapidly, especially in the region of effective excitation of nitrogen ($\epsilon \sim 2-4$ eV).

In the rotational region $\epsilon \le 0.4$ eV the relaxation slows down sharply. Here, solving Eq. (24) for $S_{\text{inel}}^{v^*} = 0$, we find

$$f(v,t)=\frac{v_0^2A(v_0)}{v^2A(v)}f_0(v_0,0),\quad A=v(\delta_{\text{el}}\nu+R_r), \tag{26}$$

where the velocity $v = v(v_0, t)$ is determined by the equation

$$dv/dt=-1/2A(v),\quad v_0=v(0).$$

Taking (11) and the fact that $R_r \gg \delta_{\text{el}}\nu$ into account, we have from (26) that

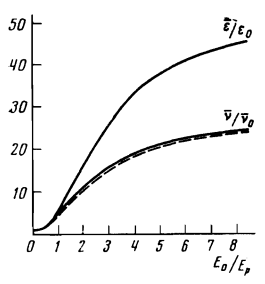

FIG. 5

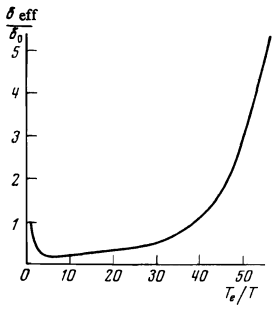

FIG. 6

FIG. 5. Dependences of the mean energy $\bar{\epsilon}$ and the effective electron collision rate $\bar{\nu}$ on E_0/E_p for $\omega_{\text{eff}} \gg \nu$, T = 185 K. The dashed curve is the plot, according to (18), of $\bar{\nu}$.

FIG. 6. The mean fraction of energy lost for $\omega_{\text{eff}} \gg \nu$, T = 185 K, and $\delta_0 = 5 \times 10^{-3}$.

$$f(v,t)=(1+A_0t/2v)^2 f_0(v+A_0t/2,0),$$
$$A_0=8\sigma_0 B_0 N_m/m,\quad v_0=v+{}^1/_2 A_0 t. \tag{27}$$

It can be seen from this that the characteristic time of relaxation of the distribution function in the rotational region

$$t_r\approx\frac{2v}{A_0}\approx\frac{\varepsilon_0^{1/2}m^{1/2}}{2^{3/2}B_0\sigma_0 N_m}.$$

Here ϵ_0 is the energy from which the rotational relaxation starts (in air $\epsilon_0\approx 0.4$ eV).

3. INTERACTION OF RADIO WAVES IN THE IONOSPHERE. DISCUSSION OF THE EXPERIMENTAL RESULTS

The interaction of radio waves in the lower ionosphere is due primarily to a change in their absorption. Indeed, the amplitude of the wave E_2 after traversing a distance s in the ionosphere is equal to[12]

$$E_2=E_{20}\exp\left\{-\frac{\omega}{c}\int\varkappa\,ds\right\}, \tag{28}$$

where κ is the coefficient of absorption of the wave:

$$\varkappa\sim N\bar\nu/(\omega_2^2+\bar\nu^2). \tag{29}$$

As the electrons in the plasma are heated up under the action of the electric field of the high-power wave, the collision rate $\bar\nu$ changes and, with it, the absorption coefficient κ and the amplitude of the wave E_2[3]. It can be seen from (28) and (29) that if $\omega_2^2\gg\bar\nu^2$, then the absorption of the E_2 wave rises under the action of the high-power E_1 wave (the case of suppression, Figs. 2a–2c); if, on the other hand, $\omega_2^2<\bar\nu^2$, then κ, on the contrary, decreases (brightening, Fig. 2d). And, indeed, the cases of suppression are observed usually in the evening and night time, when the condition $\omega_2^2\gg\bar\nu^2$ is fulfilled in the interaction region $z\sim 80$–90 km. The cases of brightening are, however, observed when the E_2 wave propagates in the D-region ($z\lesssim 60$–70 km), where $\omega_2\lesssim\bar\nu$.

To compute the amplitude of the field E_2, we should, as can be seen from (28) and (29), know the distribution of the concentration and collision rate of the electrons along the entire trajectory of the beam in the ionosphere. We can, however, assume that the interaction is localized at some altitude[4]. Then, for example, under the conditions of suppression, $\omega_2^2\gg\bar\nu^2$, we have from (28) that

$$E_2=E_{20}\exp\{-K_0(\bar\nu/\bar\nu_0-1)\}. \tag{30}$$

Here E_{20} is the unperturbed, while E_2 is the perturbed (by the E_1 wave), amplitude of the field, $\bar\nu_0$ and $\bar\nu$ are respectively the unperturbed and perturbed collision rates of the electrons in the interaction region, and K_0 is a quantity characterizing the total absorption of the unperturbed wave E_{20} in the interaction region. The variation in time of the quantity K_0 is determined by the variability of the ionosphere, i.e., it is characterized by a time $t\sim 10$ sec. Therefore, during the time $\Delta t\le 10^{-2}$ of interaction with the high-power pulse and of relaxation of the perturbation, the quantity K_0 can be assumed to be constant. Consequently, the changes in the field amplitude E_2 in our experiments are determined by only the change in the electron collision rate $\bar\nu$ in the interaction region.

Let us first consider how the collision rate $\bar\nu$ increases with time during the heating of the plasma. In the approximation of the elementary theory, it follows from (18) that

$$\frac{T_e}{T}=(1+Gt)^{1/(1-\alpha)},\quad G=\frac{e^2E_0^2\bar\nu_0}{3m\omega_{\text{eff}}^2},\quad \bar\nu=\bar\nu_0(1+Gt)^{\alpha/(1-\alpha)}. \tag{31}$$

It can be seen from these relations that the nature of the growth of the collision rate $\bar\nu$ essentially depends on the kinetic constant α. The growth of $\bar\nu$ with t is especially rapid in the case when α is close to unity. The formula (31) has been obtained under the neglect of the relaxation term $\delta_{\text{eff}}\bar\nu(T_e-T)$: It is valid as long as the electron temperature is less than its steady-state value T_{es}. As T_e approaches T_{es}, the growth of T_e sharply slows down. Such a dependence of $\bar\nu$ on the time t is also obtained from the formulas (17) and (23) of the exact kinetic theory.

Taking the logarithm of the ratio E_{20}/E_2 twice, we have from (30) and (31) that

$$\ln\ln(E_{20}/E_2)=\ln K_0+\ln[(1+Gt)^{\alpha/(1-\alpha)}-1]. \tag{32}$$

It can be seen from this relation that at large values of Gt (before the steady-state value is reached) $\ln\ln(E_{20}/E_2)\sim\ln t$.

A typical, experimentally observable dependence of $\ln\ln(E_{20}/E_2)$ on $\ln t$ is shown in Fig. 7. Here I denotes the region of small values of $\ln t\to 0$ (it is extended because of the logarithmic conversion of the scale), II denotes the primary region, i.e., the region of rapid growth of $\bar\nu$, and III indicates the region where the steady-state value is reached. The maximum of the derivative

$$q=\left|\frac{d\ln\ln E_{20}/E_2}{d\ln t}\right| \tag{33}$$

is attained at the point, $t=t_m$, of inflection of the curve. From the formulas (31) and (32) we have

$$q_m=\frac{\alpha}{1-\alpha}\lambda_m,\quad q_m=q(t_m),$$
$$\lambda_m=\lambda\left(\frac{T_{em}}{T}\right),$$
$$\lambda\left(\frac{T_e}{T}\right)=\frac{(T_e/T)^{2\alpha-1}[(T_e/T)^{1-\alpha}-1]}{(T_e/T)^{\alpha}-1}. \tag{34}$$

The quantity λ_m can be assumed to be a constant and equal to[5] $\bar\lambda_m=0.44$. It then follows from (34) that

$$\alpha=(1+\bar\lambda_m/q_m)^{-1}. \tag{35}$$

Thus, by determining the quantity q_m from the experimental data, we can find from the formula (35) the kinetic parameter α characterizing the dependence of the collision rate ν, or of the transport cross section

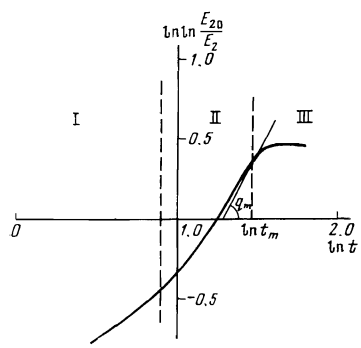

FIG. 7. The growth of the perturbation of the E_2 wave; P = 0 dB, $\omega_2=1.71\times10^6$ rad/sec, March 13, 1967, at 2215 h.

σ_t for electron-molecule collisions, on the electron velocity v: $\nu = \nu_0 v^{2\alpha}$, $\sigma_t = \sigma_0 v^{2\alpha-1}$. The value of α thus measured in the lower ionosphere for different conditions turns out to be stable and equal to

$$\alpha = 0.83 \pm 0.02. \quad (36)$$

In molecular composition, the lower ionosphere is similar to air. By interpolating the results of the laboratory measurements carried out in[13] in nitrogen and oxygen, we obtain the value $\alpha = 5/6 \approx 0.83$ for air. It is in agreement with (36).

It can be seen from the expressions (32) and (34) that the dimensionless quantity q is determined by the characteristics of the plasma kinetics, and does not depend on the specific values of the parameters K_0 and $\bar{\nu}_0$ characterizing the dimensions and location of the region in the ionosphere where the radio waves interact. In Fig. 8 we have plotted according to (34) and (36) the dependence of q/q_m on T_e/T_{em}. On the other hand, taking into account the fact that, according to (31),

$$\frac{T_e}{T_{em}} = \frac{T}{T_{em}}\left\{1 + \left[\left(\frac{T_{em}}{T}\right)^{1-\alpha} - 1\right]\frac{t}{t_m}\right\}^{1/1-\alpha},$$

we can construct the same dependence just on the basis of the experimental data. It is shown in Fig. 8 for several specific examples. Satisfactory agreement between the theory and experiment can be seen[6)].

Let us now consider the relaxation of the electron collision rate $\bar{\nu}$ after the perturbing field E_1 has been switched off. In the vibrational region $\epsilon > 0.4$ eV the relaxation proceeds rapidly, (25). Only the rotational region $\epsilon < 0.4$ eV is therefore important. The relaxation of the distribution function in this region is described by the formulas (27).

Let us take into account the fact that after the intense perturbation of the plasma by the alternating electric field (20), the distribution function $f_0(v)$ falls off slowly to some cutoff velocity v_c, after which, to the contrary, its very rapid decrease-truncation-begins (see Fig. 4). During the relaxation in the rotational region, the distribution function, according to (27), preserves the same form—only the location of the cutoff boundary v_c changes:

$$v_c = v_0 - A_0 t/2, \quad A_0 = 8B_0\sigma_0 N_m/m, \quad (37)$$

Here v_0 is the boundary of the rotational region. According to (17), the effective collision rate for $v_c \gg v_T$ is equal to

$$\bar{\nu} \approx \frac{3+2\alpha}{3}\nu(v_c) = \frac{3+2\alpha}{3}\nu_0 v_c^{2\alpha}. \quad (38)$$

The formulas (37) and (38) describe the variation in time of the electron collision rate for $v_c \gg v_T$ and, consequently, the process of relaxation of the perturbation of the wave E_2, (30). Taking (30), (37), (38), and (11) into account, we have in the notation of (33) that

$$q \simeq \alpha \bar{R} t \simeq \alpha A_0 t/v_c = \alpha R_r(v_c)\, t, \quad \bar{R} \simeq (v_c^2 - v_r^2)^{-1} dv_c^2/dt. \quad (39)$$

Here we have taken into account the fact that under the conditions when $v_c \gg v_T$ the inequality $\bar{\nu} \gg \bar{\nu}_0$ always holds. Now, expressing A_0 in terms of q and t with the aid of (37) and (39), we find

$$A_0/v_0 = 2q/t(2\alpha + q). \quad (40)$$

Consequently,

$$v_c/v_0 = 2\alpha/(2\alpha + q). \quad (41)$$

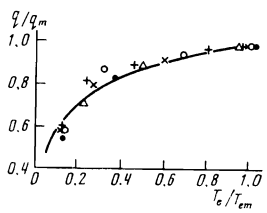

FIG. 8

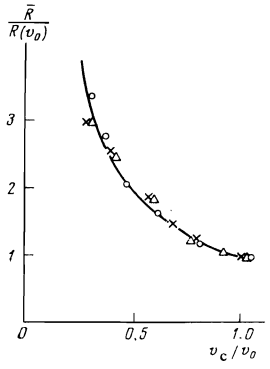

FIG. 9

FIG. 8. Universal heating curve for $T_{em}/T = 20$. The points indicate the results of an experiment performed on March 14, 1967, at: ○) 2206 h, △) 2211 h, +) 2214 h, X) 2217 h, ●) 2223 h.

FIG. 9. Universal relaxation curve. The points indicate experimental results obtained on: X) April 25, 1967, at 2140 h; ○) March 13, 1967, at 2210 h; △) March 13, 1967, 2220 h.

The formulas (39) and (41) express in an implicit form the dependence of the averaged rotational-loss function $\bar{R}$ on the velocity v_c if the function q(t) is known. The latter can be directly determined from experimental data. In Fig. 9 this dependence has been constructed for several concrete examples. It has a highly stable form. The continuous curve in the figure is the theoretical curve; it is valid for $v_c \gg v_T$, when, according to (39), $\bar{R} = R_r(v_c)$ and, consequently, from (11)

$$\bar{R}/R(v_0) = R_r(v_c)/R_r(v_0) = v_0/v_c.$$

It can be seen from Fig. 9 that the theory is in good agreement with experiment[7)]. Consequently, the relaxation of a perturbation of the electrons in the lower ionosphere is indeed determined by the excitation of the rotational levels of the molecules O_2 and N_2.

Let us consider the period, $t \to \infty$, of completion of the relaxation. The distribution function in this case is close to a Maxwellian distribution with $T_e = T$. As follows from (18) and (30), the quantity q/t for $t \to \infty$ is a constant equal to $\bar{R} = \delta_{eff}\bar{\nu}_0$, (19). Further, taking (39) and (11) into account, we have

$$\frac{\bar{R}}{R(v_0)} = \frac{4\alpha}{3\pi^{1/2}}\left(\frac{\epsilon_0}{T}\right)^{1/2}.$$

Under our conditions, $T \approx 180$–200 K, $\epsilon_0 \approx 0.4$ eV, $\alpha = 0.83$, and the magnitude of the constant $\bar{R}/R(v_0) \approx 3.0$–$3.3$. The observed values of this ratio lie in the interval 3–3.5.

Taking into account the fact that for air $\overline{B_0 Q^2} = (B_0Q^2)_{N_2} + (B_0Q^2)_{O_2} \approx 3.3 \times 10^{-4}$ eV, we find from the formula (11) that

$$A_0/v_0 \approx 6 \cdot 10^{-12} N_m. \quad (42)$$

Determining the constant A_0/v_0 from the experimental data in accordance with (40), we can find from (42) the concentration of the molecules, N_m, in the interaction region. It turns out to be equal to $(3$–$10) \times 10^{13}$ cm^{-3}. This means that the interaction of the waves occurs in the ionosphere at heights $z \approx 80$–90 km, which corresponds with the well-known results of the cross-modulation investigation[14].

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[1)]The collisions between the electrons in the lower ionosphere (i.e., at heights $z < 100$ km) can be neglected.

[2)]It can be shown on the basis of the exact kinetic equation that the electron distribution function becomes symmetric within a time interval $\Delta t \sim (1-3)/\nu$, i.e., Eq. (21) is valid after the expiration of such a time period from the moment the strong electric field is switched on.

[3)]The change in the electron concentration in the lower ionosphere during a short period of time of the order of the duration of the perturbing pulse, $\Delta t \approx 5 \times 10^{-4}$ sec, is unimportant.

[4)]The basis for such an assumption consists in the fact that, owing, on the one hand, to the exponential growth with altitude of the electron concentration N and, on the other, to the exponential decrease of the collision rate $\bar{\nu}$, the product $N\bar{\nu}$ has quite a sharp maximum. The numerical computations show that the region of localization of the perturbation is of the order of 1–2 km in height.

[5)]Indeed, the quantity T_e/T at the point of inflection is close to the steady-state value $(T_e/T)_s$. According to the results of the measurements and of the numerical computations of [3], in our experiments $20 \leqslant T_e/T \leqslant 30$. Taking into account the fact that $\alpha \approx 5/6$ (see below), we have from (34) that $0.43 \leqslant \lambda_m \leqslant 0.46$. It is not difficult to verify that allowance for the variation of λ within these limits leads, in the computation of α, only to corrections of less than 1%.

[6)]The parameter T_{em}/T was assumed to be equal to 20. When the value of T_{em}/T is varied within the limits $20 \lesssim T_{em}/T \lesssim 30$, both the theoretical and the experimental curves change little.

[7)]Notice that under the conditions when $T_e \gg T$ the elementary theory leads to the same dependence as (19): $\bar{R}/R(T_{em}) = (T_{em}/T_e)^{1/2}$.

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