Magnetoacoustic effect in the ultraquantum limit

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The absorption of ultrasound (of frequency $\omega = sk$) by small groups of charge carriers in conductors placed in a strong magnetic field is studied theoretically for distances between the Landau levels of the order of the Fermi energy $\mu(H)$. For electron spectrum models corresponding to singly connected Fermi surfaces, the creation of a new cavity and overlapping of electron and hole bands, we have determined the absorption peak that arises at $\mu(H) = (ms - \hbar k/2)^2/2m$. No absorption occurs at smaller band depths.

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The contribution of small groups of conduction electrons to the thermodynamic and kinetic properties of conducting crystals has been studied intensively in recent years. The possibility of easy variation of the characteristics of the small groups by changes in the pressure, concentration of impurities, etc., has stimulated the search for effects that are sensitive to such changes. Davydov and Kaganov[1] investigated the features of ultrasonic absorption in metals near an electron transition of order $2\frac{1}{2}$.[2] On the other hand, the ultraquantum limit can be achieved in an external magnetic field for small groups of electrons. In this case, the distance between the Landau levels becomes of the order of the Fermi energy. This leads to an interesting dependence of the sound absorption coefficient on the magnetic field and the concentration of charge carriers; the purpose of the current work is to obtain this dependence.

1) In a quantizing magnetic field, the sound absorption coefficient can be calculated from perturbation theory, in accord with[3]:

$$\Gamma = \frac{\pi}{\rho u_0^2 \omega V} \sum_{\alpha,\alpha'} \left\{ f\left(\frac{\varepsilon_\alpha - \mu}{T}\right) - f\left(\frac{\varepsilon_{\alpha'} - \mu}{T}\right) \right\} |M_{\alpha\alpha'}|^2 \delta(\varepsilon_{\alpha'} - \varepsilon_\alpha - \hbar\omega), \quad (1)$$

where ρ and V are the density and volume, respectively, of the sample, $f(x) = (1 + e^x)^{-1}$ is the Fermi function, and α is the set of quantum numbers of the electron n, p_H, σ. If the sound wave $u = u_0 \exp(ikz - i\omega t)$ is propagated along the magnetic field H, then the square of the matrix element of deformation[1)] interaction of the electrons with the sound does not depend on H:

$$|M_{\alpha\alpha'}|^2 = |\langle\alpha|\Lambda_{ik} u_{ik}^0 \exp(ikz)|\alpha'\rangle|^2 = |\Lambda_{ik} u_{ik}^0|^2 \delta_{n,n'} \delta_{\sigma,\sigma'} \delta_{p_z + \hbar k,\, p_{z'}}. \quad (2)$$

In what follows, we shall be interested in the case of such strong magnetic fields that not more than one Landau level will be located below the Fermi level for the given group of electrons: $\hbar\Omega \equiv e\hbar/mc \gtrsim \Delta\epsilon_F$ ($\Delta\epsilon_F$ is the depth of the band for H = 0). Then the absorption coefficient for low temperatures, $T \ll \hbar\Omega$, is simply

$$\Gamma = \Gamma_0 \frac{\Omega}{2\omega} \left\{ f\left[\frac{\Delta - \mu(H)}{T}\right] - f\left[\frac{\Delta + \hbar\omega - \mu(H)}{T}\right] \right\}, \quad (3)$$

and is completely determined by the value of the chemical potential $\mu(H)$ measured from the bottom of the lowest magnetic subband. The quantity Γ_0 is the absorption in the absence of a magnetic field (for $\Delta\epsilon_F > \Delta$, see[1]); for a quadratic dispersion,

$$\Gamma_0 = m^2 |\Lambda_{ik} u_{ik}^0|^2 / 2\pi\hbar^3 \rho u_0^2 k. \quad (4)$$

It is important that the electrons that are closer to the bottom of the magnetic subband $\epsilon_{n\sigma}(0)$ than

$$\Delta \equiv \varepsilon_{n\sigma}(p_{z0}) - \varepsilon_{n\sigma}(0)$$
$$(\varepsilon_{n\sigma}(p_{z0} + \hbar k) - \varepsilon_{n\sigma}(p_{z0}) = \hbar\omega), \quad (5)$$

cannot absorb a sound quantum of frequency $\omega = sk$ by virtue of the conservation laws.

2. In the simplest case which, however, is achieved in some semiconductors, there is just one band of carriers:

$$\varepsilon_{n,\pm}(p_z) = p_z^2/2m + (n + \tfrac{1}{2})\hbar\Omega \pm \mu_B H \quad (6)$$

($\mu_B = e\hbar/2m_0c$ is the Bohr magneton). Since only the region near the bottom ($p_z \sim p_{z0}$ is important for sound absorption, as follows from (5), the quadratic dispersion is a sufficient approximation; generalization to the anisotropic case is obvious.

The function $\mu(H)$ is, as usual, found from the condition of conservation of the concentration of the electrons in the magnetic field, $N(\mu, T, H) = N(\mu, T, 0)$. If the electron gas is degenerate at $H = 0$ ($\mu_0 \gg T$), then at high fields, $\hbar\Omega > \mu_0$, and we get

$$\mu(H) \approx 16\mu_0^3/9(\hbar\Omega)^2, \quad \mu_0^3/(\hbar\Omega)^2 \gg T, \quad (7a)$$

$$\mu(H) \approx T \ln\left(\frac{8}{3\sqrt{\pi}} \frac{\mu_0^{3/2}}{\hbar\Omega\sqrt{T}}\right), \quad \mu_0^3/(\hbar\Omega)^2 \ll T. \quad (7b)$$

Substitution of (7) in (3) gives the sound absorption coefficient as a function of the magnetic field, the temperature and the electron concentration; the result turns out to be different, depending on the ratio of the quantities $\hbar\omega$ and T. In the case of high frequencies ($\hbar\omega \gg T$),

$$\frac{\Gamma(H)}{\Gamma_0} \approx \begin{cases} \dfrac{\Omega}{2\omega}\left\{ f\left(\dfrac{\Delta}{T} - \dfrac{16\mu_0^3}{9(\hbar\Omega)^2 T}\right) - f\left(\dfrac{\Delta + \hbar\omega}{T} - \dfrac{16\mu_0^3}{9(\hbar\Omega)^2 T}\right)\right\} & (8a) \\ (4/3\sqrt{\pi})(\mu_0^{3/2}/\hbar\omega\sqrt{T}) \exp(-\Delta/T). & (8b) \end{cases}$$

The absorption is exponentially small when $\mu(H) > \Delta + \hbar\omega$; thereafter, it increases and depends linearly on H for $\Delta + \hbar\omega > \mu(H) > \Delta$; to the right of this region, if $\Delta \gg T$, it again falls off to the constant value (8b).

In the opposite case, when $\hbar\omega \ll T$, if $\Delta \gg T$, the dependence $\Gamma(H)$ has the form of a sharp peak for $\mu(H) = \Delta$, i.e., this is none other than the last quantum oscillation of the absorption, the shape of which, however, differs from the quasiclassical case:[3]

$$\frac{\Gamma(H)}{\Gamma_0} \approx \begin{cases} \dfrac{\hbar\Omega}{8T} \mathrm{ch}^{-2}\left[\dfrac{\Delta}{2T} - \dfrac{8\mu_0^3}{9(\hbar\Omega)^2 T}\right], & (9a) \\ (4/3\sqrt{\pi})(\mu_0/T)^{3/2} \exp(-\Delta/T). & (9b) \end{cases}$$

We note that the absorption in a strong magnetic field $(\hbar\Omega)^2 \gg \mu_0^3/T$ is not necessarily small, since it can turn out for given T and ω that the "absorption

region'' (5), $p_{z0} = ms - \hbar k/2$ is very close to the bottom of the band:

$$\Delta=(ms-\hbar k/2)^2/2m\ll T. \tag{10}$$

Then, as is seen from Eqs. (8) and (9), the sound absorption coefficient in strong fields comes out from saturation and becomes much larger than in the absence of the magnetic field (see Fig. 1).

In the frequency dependence, Γ has a sharp maximum for $\omega = 2ms^2/\hbar$, if $\hbar\omega \gg T$.

3. We now consider the situation that is characteristic for metals near an electronic transition of order $2\frac{1}{2}$ under pressure,[2] when, a new cavity of the Fermi surface is formed at $\epsilon_F = \epsilon_k$. The result of Davydov and Kaganov[1] can easily be generalized to the case of an external magnetic field.

In a sufficiently strong field, for the new band,

$$\varepsilon=\varepsilon_k+p_z^2/2m+(n+{}^1/_2)\hbar\Omega\pm\mu_B H,$$

the ultraquantum limit is reached, while the remaining electrons can be described quasiclassically:

$$\varepsilon_F\gg\hbar\Omega>Z\equiv\varepsilon_F-\varepsilon_k\gg T. \tag{11}$$

Then, solving the equation $(N(H) = N(0)$, we can easily show that the chemical potential ''does not feel'' the small group of electrons, with accuracy to $n/N = 8\pi(2mZ)^{3/2}/3N\hbar^3$; since $\epsilon_F \gg \hbar\Omega$, the chemical potential does not depend on the magnetic field:

$$|\mu-\varepsilon_F|\sim\hbar\Omega(\hbar\Omega/\varepsilon_F)^{1/2}.$$

Now, substituting the value of the chemical potential (calculated from the bottom of the new band) in (3),

$$\begin{gathered}\mu(H)=Z-\alpha H,\\ \alpha\equiv e\hbar/2mc-\mu_B=(m^{-1}-m_s^{-1})e\hbar/2c,\end{gathered} \tag{12}$$

we obtain the contribution to the absorption coefficient due to this band:

$$\Delta\Gamma\approx\begin{cases}\Gamma_0(\Omega/2\omega)\exp((Z-\Delta-\alpha H)/T), & Z-\alpha H<\Delta,\\ \Gamma_0(\Omega/2\omega), & \Delta<Z-\alpha H<\Delta+\hbar\omega,\\ \Gamma_0(\Omega/2\omega)\exp((\Delta+\hbar\omega+\alpha H-Z)/T), & \Delta+\hbar\omega<Z-\alpha H,\end{cases}$$
$$\Delta=(ms-\hbar k/2)^2/2m,\quad \hbar\omega\gg T. \tag{13}$$

The dependence of $\Delta\Gamma$ on Z has the form of a rectangular pulse of height of the order of the maximal sound absorption by the remaining electrons. If the bottom of the new band is shifted in the magnetic field, the absorption edge shifts by αH in comparison with the case $H = 0$.

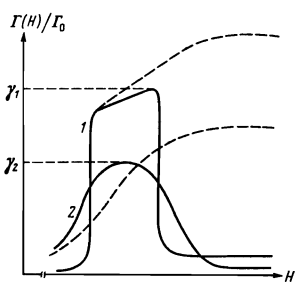

FIG. 1

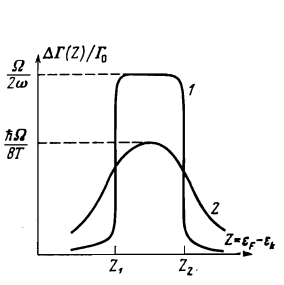

FIG. 2

FIG. 1. Dependence of the absorption on the magnetic field for $\hbar\omega \gg T$ (curve 1) and $\hbar\omega \ll T$ (curve 2). The dashed lines indicate the case $\Delta \ll T$; $\gamma_1 = 2\mu_0^{3/2}/3\hbar\omega\sqrt{\Delta}$, $\gamma_2 = \mu_0^{3/2}/6T\sqrt{\Delta}$.

FIG. 2. Sound absorption by electrons of the new cavity as a function of distance from the Fermi level in the cases $\hbar\omega \gg T$ (curve 1) and $\hbar\omega \ll T$ (curve 2), $Z_1 = \Delta + \alpha H$, $Z_2 d = \Delta + \hbar\omega + \alpha H$.

If $\hbar\omega \ll T$, then the new cavity gives an aborption spike of the following form for $Z - \alpha H = \Delta$ (Fig. 2):

$$\Delta\Gamma\approx\Gamma_0\frac{\hbar\Omega}{8T}\operatorname{ch}^{-2}\left(\frac{Z-\alpha H-\Delta}{2T}\right). \tag{14}$$

The contribution of the small group of electrons to the sound absorption is easily separated experimentally if, as usual, their effective mass is small ($m \ll M$) since the absorption by the remaining electrons depends monotonically on H and Z[2)]

If $e\hbar H/Mc > T$ nevertheless, the dependence (13) or (14) will modulate the more rapid oscillations of $\Gamma(Z, H)$ due to the quantization of the energy of the electrons of the large bands.

4. Another type of conductor, for which the ultraquantum limit can be achieved in comparatively small magnetic fields, is the case of a semimetal of the bismuth type, with overlap of electron and hole bands:

$$\begin{gathered}\varepsilon_i=E_i+(-1)^i[p_{zi}^2/2m_i+(n_i+{}^1/_2\pm m_i/2m_{si})\hbar\Omega_i],\\ i=1.\,2,\quad \Delta E=E_1-E_2>0.\end{gathered} \tag{15}$$

Since the sound ''absorption region'' ($p_{zi} \sim p_{z0i}$) is close to the bottom of the magnetic subband, we limit ourselves here to the quadratic dispersion law.

In a sufficiently strong field ($\hbar\Omega_i \sim \Delta E$) only one Landau level remains in each of the bands; here the sound absorption coefficient is the sum (for electrons and holes) of the functions (3), in which the position of the Fermi level relative to the bottom of each of the bands is determined from the condition $N_1(H) = H_2(H)$:

$$\mu_{1,2}(H)=\frac{m_{2,1}}{\sum m_i}\left[\Delta E-\sum\frac{\hbar\Omega_i}{2}\left(1-\frac{m_i}{m_{si}}\right)\right]. \tag{16}$$

If, in accord with[4], $\Sigma(m_i/m_{si}) < 2$ and the overlap of the bands decreases in the magnetic field, the dependence $\Gamma(H)$ is the sum of spikes of the form (13) for $\hbar\omega \gg T$ or (14) for $\hbar\omega \ll T$, falling off sharply when the depth of each of the bands is comparable with

$$\Delta_i=(m_i s-\hbar k/2)^2/2m_i. \tag{17}$$

In large fields (or small depth of the bands), the charge carriers practically do not interact with the sound wave and the total absorption is equal to the lattice absorption, as in a dielectric.

It must be remarked that a single-electron description can turn out to be inadequate near the ''semi-metal-dielectric'' transition,[4,5] But, relative to sound, such a transition, as has been shown, takes place before the overlap of the bands disappears, which allows us to rely at least on the qualitative validity of the result.[3)]

5. The effects considered above should take place upon satisfaction of the inequalities

$$\hbar\Omega>\Delta\varepsilon_F>(ms^2;\hbar^2k^2/m)\gg T, \tag{18}$$

where the parentheses denote the larger of the quantities enclosed in them. Account of scattering by the carriers with the help of the Dingle factor[6] leads to the replacement of T by $T + \hbar/2\tau$, where τ is the time of free flight. Inasmuch as the effective mass of the carriers of the small groups is usually small ($m \sim 0.1$–$0.01\ m_0$), in the case of hypersound, $\omega \gtrsim 10^{10}$ sec^{-1}) the inequality (18) yields $H \gtrsim 10^4$ Oe, $T \lesssim 1°$K, $\tau \lesssim 10^{-11}$ sec. Thus, in the presence of small groups of charge car-

riers, the magnetoacoustic effects turn out to be quite accessible to observation in the ultraquantum limit.

Such experiments can give information on the parameters of the electron energy spectrum, which determines the location of the anomalies in the resultant dependences of Γ on H and ϵ_F. We need only take it into account that the anisotropy of the Fermi surface leads to a replacement of m by m_z in the quantity Δ (see (10)) and by $m_\perp = (m_x m_y)^{1/2}$ in the quantity Ω, if the sound propagates along the symmetry axis of the crystal. The shift in the magnetic field of the jump $\Gamma(\epsilon_F)$ in the formation of a new band can make easier the observations of transitions of order $2\frac{1}{2}$ under pressure, since it is difficult to change the latter smoothly over wide limits.

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[1)]Account of other interaction mechanisms, for example, the piezoelectric one, would lead only to a change in the value of the matrix element.

[2)]We note that in the anisotropic case $\Gamma_0 \propto m_\perp m_z$ and therefore smallness of only the transverse mass is required; the relative value of the jump $\Delta\Gamma$ does not contain the small parameter $m_\perp/M_\perp$.

[3)]The effective masses are usually small in semimetals and the dielectric constant ϵ_0 is large, so that at frequencies $\omega \sim 10^{10}$ sec^{-1}, $\Delta \sim \hbar^2 k^2/m \gg m e^4/\epsilon_0^2 \hbar^2$ and, according to [5] the Coulomb interaction is small. It also follows from [5] that the interband coupling is unimportant in the given case (the extrema of the bands at different points of p-space).

[1] V. N. Davydov and M. I. Kaganov, Zh. Eksp. Teor. Fiz. **67**, 1491 (1974) [Sov. Phys.-JETP **40**, 741 (1974)].
[2] I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **38**, 1569 (1960) [Sov. Phys.-JETP **11**, 1130 (1960)].
[3] V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 786 (1961) [Sov. Phys.-JETP **13**, 552 (1961)].
[4] M. Ya. Azbel', N. B. Brandt, Zh. Eksp. Teor. Fiz. **48**, 1206 (1965) [Sov. Phys.-JETP **21**, 804 (1965)]; Fiz. Tverd. Tela **16**, 1360 (1974) [Sov. Phys. Solid State **16**, 876 (1974)].
[5] A. A. Abrikosov, and S. D. Beneslavskiĭ, Zh. Eksp. Teor. Fiz. **59**, 1280 (1970) [Sov. Phys.-JETP **32**, 699 (1971)]; L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela **6**, 2791 (1964) [Sov. Phys.-Solid State **6**, 2219 (1965)].
[6] R. B. Dingle, Proc. Roy. Soc. (London) **A211**, 517 (1952).

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