Manifestation of band structure effects in tunneling in metals

M. A. Belogolovskiĭ, A. A. Galkin, and V. M. Svistunov

Donets Physico-technical Institute, Ukrainian Academy of Sciences
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The possibility of observing in the band structure of metals, with the aid of the tunneling method, critical energies connected with topological changes in the electronic constant-energy surfaces is investigated. The analysis is carried out for finite temperatures in the framework of the WKB approximation. It is shown that in the case of a superconducting injector the corresponding singularities in the tunneling conductivity split up and become sharp. Effects due to the finite thickness of the film in question are studied, and an explicit expression is obtained for the oscillating part of the tunnel-junction conductivity.

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Such experimental methods as the methods of cyclotron resonance, de Haas-van Alphen effect, dimensional and other effects allowing the determination of the dispersion law for electrons in the immediate vicinity of the Fermi surface are widely used at present to study the electronic energy spectrum of metals. At the same time, the method of electron tunneling possesses the important merit that the energy region that can be investigated in it is significantly broader, and is limited only by the height of the tunnel-detector barrier (of the order of several electronvolts). Therefore, the use of the tunneling technique to study the electronic characteristics of a metal can lead to qualitatively new results that are inaccessible to other experimental methods.

It should be emphasized that there are in the literature different opinions about the possibility of investigating the band structure of a metal with the aid of the tunneling technique. For example, the assertion is made in Douglass and Falikov's paper[1] in reference to the well-known paper[2] by Harrison that no specific information about the electronic density of states $N(\epsilon)$ can be obtained from tunneling data. In reality, however, as will be shown below, Harrison's results[2] only indicate that the dependence of the tunnel-junction conductivity on $N(\epsilon)$ for a normal metal has a much more complex character than in the case of a superconductor. This circumstance was noted by Duke in his papers[3], in which he studied the volt-ampere characteristics of tunneling at voltage potentials corresponding to the band edge of one of the metals, the temperature having been assumed to be zero.

In the present paper we discuss the possibility of observing with the aid of the tunneling technique critical energies ϵ_{cr} in the vicinity of which topological changes occur in the electronic constant-energy surfaces of metals. A particular case of such changes can be the appearance, considered by Duke[3], of a new band. We investigate the bias dependence of the tunnel-junction conductivity at finite temperatures in the case of a normal, as well as of a superconducting, injector. The WKB approximation is used to compute the tunneling matrix element.

1. In the framework of the model tunneling-Hamiltonian scheme[4], the dependence of the tunneling current flowing between two normal metals on the voltage potential applied to the barrier is given by the following expression:

$$I_{NN}(U)=\frac{4\pi e}{\hbar}\sum_{\mathbf{p},\mathbf{q}}|T_{\mathbf{pq}}|^2(f_\mathbf{p}-f_\mathbf{q})\delta(\varepsilon_\mathbf{p}+U-\varepsilon_\mathbf{q}), \tag{1}$$

where e is the electron charge; the index $\mathbf{p}$ characterizes the electronic states in the injector; $\mathbf{q}$ characterizes the electronic states in the metal whose band structure is being investigated; $T_{\mathbf{pq}}$ is the matrix element for electron tunneling from the state $\mathbf{p}$ into the state $\mathbf{q}$; $f_\mathbf{p}$ and $f_\mathbf{q}$ are the Fermi distribution functions in the first and second coatings. The voltage potential U is measured in energy units, and all the energies will in this case be measured from the Fermi level of the material under investigation. For simplicity, let us assume that the band structure of the injector does not have singularities in the region of interest to us.

It is natural that, to study the electronic spectrum, we need either single-crystal films or grain-oriented films consisting of sufficiently large crystallites with a defined band structure. Tunneling experiments performed on such films[5,6] clearly demonstrate the specular nature of the penetration of an electron through the tunneling barrier. In this case the components of the momenta $\mathbf{p}$ and $\mathbf{q}$ parallel to the plane of the barrier, $p_\parallel$ and $q_\parallel$, should be conserved. Harrison[2] has computed in the WKB approximation the matrix element $T_{\mathbf{pq}}$, which turned out to be equal to

$$|T_{\mathbf{pq}}|^2=\frac{\partial\varepsilon_\mathbf{p}}{\partial p_\perp}\frac{\partial\varepsilon_\mathbf{q}}{\partial q_\perp}P, \tag{2}$$

where the quantity P exponentially decreases as the angle γ between the direction of the tunneling electron and the normal to the barrier increases; for typical junctions the limiting value of this angle may reach $\sim 5°$[7].

Let us go over in (1) to integration over $q_\perp$ and $q_\parallel$, and then go over from the variable $q_\perp$ to the variable $\epsilon_\mathbf{q}$; the singularity arising in the last transformation in $(\partial\epsilon_\mathbf{q}/\partial q_\perp)^{-1}$ is canceled out by a corresponding term in $|T_{\mathbf{pq}}|^2$. It is precisely this circumstance that is sometimes interpreted as an indication that the tunneling current does not depend on the band structure. In actual fact, this means that the integral over $\epsilon_\mathbf{q}$ is regular near the singular points, but that the behavior of the double integral over $q_\parallel$ remains singular.

Let us assume that there occurs in the metal under consideration at a point q_{cr} lying in the direction n_{cr} a change in the topology of the constant-energy surfaces, and let us consider in the integral over $\mathbf{q}$ in (1) a small region in the vicinity of the point q_{cr} where the quantity P in (2) can be considered to be a constant. It is precisely this region that gives the nonlinear part of the change δI in the current. For definiteness, let us assume that the corresponding critical energies $\epsilon_{cr} = \epsilon(q_{cr})$ are located above the Fermi level and that $\epsilon_{cr} \gg T$.

Assuming that the temperature is sufficiently low in comparison to the region where the quantity P changes appreciably (tens of degrees), let us extend the domain of integration over the energies in δI to infinity and differentiate with respect to U. Then for the nonlinear part of the change, $\delta\sigma_{NN}$, in the conductivity of the tunneling current, we obtain

$$\delta\sigma_{NN}(U)=CD_{cr}\int_{-\infty}^{\infty} d\varepsilon\left[-\frac{\partial f}{\partial\varepsilon}(\varepsilon-U)\right]\delta S(\varepsilon), \quad (3)$$

where $D_{cr} \equiv D(n_{cr})$ is a quantity characterizing the transparency of the junction in the direction n_{cr} for n_0 perpendicular to the barrier plane ($D(n_0) = 1$), $\delta S(\epsilon)$ is the projection of the constant-energy surface $\epsilon_q = \text{const}$ in the vicinity of q_{cr} on the barrier plane, and C is some constant.

Since D_{cr} decreases rapidly as the angle γ between n_{cr} and the normal to the barrier increases, grain-oriented films should be oriented in such a way that the direction of n_{cr} is perpendicular to the barrier plane; on the other hand, by changing the angle between n_{cr} and the normal we can study the dependence of D_{cr} on n_{cr}, i.e., the character of the penetration of the electrons through the barrier. We shall not need the specific expression for the numerical coefficient C in (3), since we are interested only in finding the ratio of the correction $\delta\sigma$ to the total quantity σ_0, which is equal (at T = 0) to: $\sigma_0 \approx 2\pi C m^* \epsilon_F$ (m^* is the effective mass of the electron).

2. There exist two principal types of topological transitions: the appearance (disappearance) of a new cavity of the constant-energy surface and the change from open to closed constant-energy surfaces, i.e., the rupture of some connecting neck (or, conversely, its appearance)[8]. In the first case the dispersion law near $q = q_{cr}$ is determined in the properly chosen coordinate system by the formula

$$\varepsilon_q=\varepsilon_{cr}+\frac{q_1^2}{2m_1}+\frac{q_2^2}{2m_2}+\frac{q_3^2}{2m_3}, \quad (4)$$

where the vector q is measured from the point q_{cr}. Let us denote the direction cosines of the normal to the barrier surface, computed with respect to the principal axes of the effective-mass tensor by α_i. Then the area of the projection of the constant-energy surface on the barrier plane is equal to

$$\delta S(\varepsilon)=2\pi m_\perp(\varepsilon-\varepsilon_{cr})\theta(\varepsilon-\varepsilon_{cr}) \quad (5)$$

and, consequently,

$$\delta\sigma_{NN}(U)=\eta T\ln\left[1+\exp\left(\frac{U-\varepsilon_{cr}}{T}\right)\right], \quad (6)$$

where we have introduced the notation: $\eta = 2\pi C D_{cr} m_\perp$ and

$$m_\perp=\left(\frac{\alpha_1^2}{m_2m_3}+\frac{\alpha_2^2}{m_1m_3}+\frac{\alpha_3^2}{m_1m_2}\right)^{-1/2}. \quad (7)$$

In the case when $|U - \epsilon_{cr}| \gg T$ we obtain that for $U > \epsilon_{cr}$ the quantity $\delta\sigma_{NN} = \eta(U - \epsilon_{cr})$, while for $U < \epsilon_{cr}$

$$\delta\sigma_{NN}=\eta T\exp(-|U-\varepsilon_{cr}|/T).$$

Let us now proceed to the second case connected with the behavior of the constant-energy surfaces in the vicinity of the conic point where

$$\varepsilon_q=\varepsilon_{cr}+\frac{q_1^2}{2m_1}+\frac{q_2^2}{2m_2}-\frac{q_3^2}{2m_3}. \quad (8)$$

Analyzing the shape of the corresponding projection, $\delta S(\epsilon)$, on the boundary plane between the two metallic films, we find that for

$$\frac{1}{M^2}=\frac{\alpha_1^2}{m_2m_3}+\frac{\alpha_2^2}{m_1m_3}-\frac{\alpha_3^2}{m_1m_2}>0,$$

the nonlinear part of the change $\delta S(\epsilon)$ has the form

$$\delta S(\varepsilon)=M(\varepsilon_{cr}-\varepsilon)\ln|\varepsilon/\varepsilon_{cr}-1|. \quad (9)$$

If $M^2 < 0$, then

$$\delta S(\varepsilon)=-2\pi|M|(\varepsilon-\varepsilon_{cr})\theta(\varepsilon-\varepsilon_{cr}).$$

This result coincides up to a constant factor with (5); therefore, it is nothing new and will not be considered further.

For $M^2 > 0$, when the formula (9) is valid, we obtain, for $|U - \epsilon_{cr}| \gg T$, that

$$\delta\sigma_{NN}(U)=\chi\left[(\varepsilon_{cr}-U)\ln\left|\frac{U}{\varepsilon_{cr}}-1\right|+\frac{\pi^2T^2}{6(\varepsilon_{cr}-U)}\right]. \quad (10)$$

Here $\chi = CD_{cr}M$. In Fig. 1 we show the dependence of $\delta(d^2I/dU^2)_{NN}$ on $(U - \epsilon_{cr})/T$ for two types of topological changes.

In both cases the contribution of $\delta\sigma$ to σ_0 is extremely small; in particular, it follows from (6) that for $U \gg \epsilon_{cr}$ the quantity $\delta\sigma \sim D_{cr}\sigma_0(U - \epsilon_{cr})/\epsilon_F$. To observe such a singularity, we need higher-order derivatives: at least $d\sigma/dU = d^2I/dU^2$. This circumstance has been demonstrated in experiments[5] on $Al\text{-}Al_2O_3\text{-}Au$ junctions. The qualitative agreement of the shape of the d^2I/dU^2 curves near the singular points in the band structure of gold[5] with the theoretical dependences $\delta(d^2I/dU^2)_{NN}$, shown in Fig. 1, confirms the ideas expounded in the present paper that it is possible to observe details of the band structure of metals with the aid of the tunneling technique.

3. If the singularities in the band structure of the material under investigation are only slightly smeared and the corresponding tunneling curves are sufficiently sharp, then the enhancement of these singularities requires the use of a superconductor as an injector. In considering the behavior of the conductivity of such a junction, we shall, for simplicity, neglect the electron-phonon interaction in the superconductor and assume the energy gap Δ to be isotropic. Using the expression for the current in the S-I-N system (see, for example,[9]), we find that the nonlinear correction to the current in

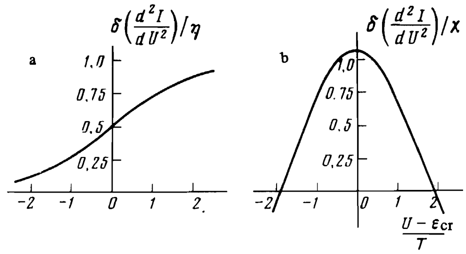

FIG. 1. The nonlinear part of the dependence of $\delta(d^2I/dU^2)$ on $(U-\epsilon_{cr})/T$ for N–I–N junctions: a) in the case of the formation of a new constant-energy recess; b) in the case of a topological transition from closed to open constant-energy surfaces.

the presence of topological changes in the N metal is equal to

$$\delta I_{SN}(U)=CD_{cr}\int_{-\infty}^{\infty}d\varepsilon\,\mathrm{Re}\left\{\frac{|U-\varepsilon|}{[(U-\varepsilon)^2-\Delta^2]^{1/2}}\right\}[f(\varepsilon-U)-f(\varepsilon)]\delta S(\varepsilon). \quad (11)$$

It can be seen from (11) that S-I-N junctions possess two advantages over N-I-N junctions: First, the singularity in the density of states of a superconductor at $T = 0$ is more pronounced than in a normal metal; second, this singularity does not get washed out as the temperature is increased right up to $T = T_{cr}$, but only approaches ϵ_F as the gap decreases in magnitude.

In contrast to the N-I-N junction, the conductivity of the S-I-N system will possess singularities at voltage potential $U_{cr} = \epsilon_{cr} + \Delta$ and, in the case of a finite temperature, also at $U_c = \epsilon_{cr} - \Delta$. The analytic behavior of $\delta\sigma_{SN}$ near these points can be found by differentiating the expression (11); the corresponding calculations are similar to those already carried out; therefore, below we shall write out only the final results. The asymptotic forms of the $\delta\sigma_{SN}$ curve for biases on the barrier close to U_{cr} and U_c are given by:

a) in the case of the appearance of a new constant—energy recess

$$\delta\sigma_{SN}=\begin{cases}\text{const}, & U<U_{cr},\\ \text{const}+2\pi CD_{cr}m_\perp[2\Delta(U-U_{cr})]^{1/2}[1+e^{-\Delta/T}]^{-1}, & U>U_{cr};\end{cases} \quad (12)$$

$$\delta\sigma_{SN}=\begin{cases}\text{const}-2\pi CD_{cr}m_\perp[2\Delta(U_c-U)]^{1/2}[1+e^{\Delta/T}]^{-1}, & U<U_c,\\ \text{const}, & U>U_c;\end{cases} \quad (13)$$

b) in the case of a topological transition from closed to open constant-energy surfaces

$$\delta\sigma_{SN}=\begin{cases}\text{const}+2\pi CD_{cr}M[2\Delta(U_{cr}-U)]^{1/2}[1+e^{-\Delta/T}]^{-1}, & U<U_{cr},\\ \text{const}, & U>U_{cr};\end{cases} \quad (14)$$

$$\delta\sigma_{SN}=\begin{cases}\text{const}, & U<U_c,\\ \text{const}-2\pi CD_{cr}M[2\Delta(U-U_c)]^{1/2}[1+e^{\Delta/T}]^{-1}, & U>U_c.\end{cases} \quad (15)$$

Notice that as the temperature is decreased, the amplitude of the singularities in the vicinity of $U = U_c$ decreases according to the law $e^{-\Delta/T}$ for $\Delta \gg T$ and $U < U_{cr}$. In Fig. 2 we schematically show the corresponding singularities in the second derivatives of the tunneling current with respect to the bias.

4. The traditional materials for investigation in the tunneling technique are metal films. The effects connected with the finite tunnel-junction film thickness have been studied in Gogadze and Kulik's paper[10], where the authors consider the case of equidistantly spaced energy levels, i.e., they assume the electronic band to be substantially filled. Below we find the oscillatory correction to the junction conductivity for voltage potentials somewhat above the critical bias, when the band is just beginning to fill up. We investigate the topological transition connected with the formation of a

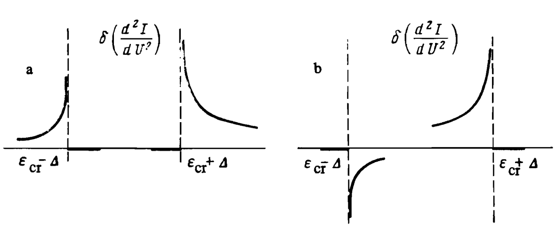

FIG. 2. The shape of the d^2I/dU^2 curves in the vicinity of the singular points $\epsilon_{cr} + \Delta$ and $\epsilon_{cr} - \Delta$ for S–I–N junctions: a) in the case of the appearance of a new constant-energy recess; b) in the case of a topological transition from closed to open constant-energy surfaces.

new constant-energy recess; the investigation of the other types of topological changes can be carried out in precisely the same fashion.

For a film of thickness d the values of the momentum components along the direction of the normal to the barrier are discrete: $q_\parallel^{(n)} = \pi\hbar n/d$ (n is an arbitrary integer). Here it is assumed that the electron mean free path is large compared to the film thickness and that the reflection of the electrons from the boundary is largely of specular nature. After some transformations of the basic expression (1), we find the nonlinear correction to the conductivity of the N-I-N junction, in which the second coating is the thin film under investigation:

$$\delta\sigma_{NN}(U)=\frac{2CD_{cr}\pi^2\hbar m_\perp}{m_\parallel d}\int_{-\infty}^{\infty}d\varepsilon\left[-\frac{\partial f}{\partial\varepsilon}(\varepsilon-U)\right]\sum_{n=1}^{\infty}q_\parallel^{(n)}\theta\left(\varepsilon-\varepsilon_{cr}-\frac{q_\parallel^{(n)2}}{2m_\parallel}\right), \quad (16)$$

where $m_\parallel = m_1\alpha_1^2 + m_2\alpha_2^2 + m_3\alpha_3^2$, the quantity $m_\perp$ is determined by the formula (7), and the remaining designations are as defined in (3).

Using the Poisson summation formula, we obtain that

$$\sum_{n=1}^{\infty}q_\parallel^{(n)}\theta\left(\varepsilon-\varepsilon_{cr}-\frac{q_\parallel^{(n)2}}{2m_\parallel}\right)=\left[\frac{m_\parallel d}{\pi\hbar}(\varepsilon-\varepsilon_{cr})+\frac{1}{\pi}(2m_\parallel(\varepsilon-\varepsilon_{cr}))^{1/2}\sum_{n=1}^{\infty}\frac{\sin(ns(\varepsilon-\varepsilon_{cr})^{1/2})}{n}-\frac{\hbar}{\pi d}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\left(\frac{ns}{2}(\varepsilon-\varepsilon_{cr})^{1/2}\right)\right]\theta(\varepsilon-\varepsilon_{cr}). \quad (17)$$

Here $s = 2d(2m_\parallel)^{1/2}/\hbar$; for films of thickness $d \sim 300$ Å and $m_\parallel \sim m_0$ (m_0 is the free-electron mass), $s^2 \sim 100$ meV. The ratio of the second term to the third in (17) $\sim s(\epsilon - \epsilon_{cr})^{1/2}$, i.e., $\sim s(U - \epsilon_{cr})^{1/2}$, at sufficiently low temperatures. It can easily be seen from the above-presented estimates that for the usual biases $U - \epsilon_{cr}$ of the order of tens of millielectronvolts the product $s(U - \epsilon_{cr})^{1/2} \lesssim 100$, and therefore the third term in (17) can be neglected.

The first term in (17), when substituted into (16), yields the volume correction already obtained above, (6); the second term, which is directly connected with the finite thickness of the film, determines the oscillating part $\delta\sigma_{osc}$. It is of greatest interest in the present case, but the observation of the corresponding oscillations in thin metallic layers is usually difficult because of the experimentally unavoidable nonuniformity in thickness of the film, i.e., the inconstancy of the quantity d, which has a stochastic character (for greater details about the structure of tunnel films, see[6]).

Treating $\Delta d(\mathbf{r}) = d(\mathbf{r}) - d_0$ (d_0 is the mean thickness and $\mathbf{r}$ is a two-dimensional radius vector) as a stochastic quantity with a Gaussian distribution, we obtain, after averaging, the following expression for the N-I-N junction:

$$\delta\sigma_{osc}(U)=2\pi CD_{cr}m_\perp\bar\beta\int_{\varepsilon_{cr}}^{\infty}d\varepsilon\left[-\frac{\partial f}{\partial\varepsilon}(\varepsilon-U)\right](\varepsilon-\varepsilon_{cr})^{1/2}\cdot\sum_{n=1}^{\infty}\frac{1}{n}\sin[n\bar s(\varepsilon-\varepsilon_{cr})^{1/2}]\exp[-{}^1\!/_2 n^2\bar s^2\sigma(\varepsilon-\varepsilon_{cr})], \quad (18)$$

where $\sigma = \langle(\Delta d)^2\rangle/d_0^2$; $\bar\beta$ is the mean value of the coefficient in (16): $\bar\beta \approx (2/m_\parallel)^{1/2}\hbar/d_0$ (for films of thickness $d_0 \sim 100$ Å the quantity $\bar\beta^2 \sim 1$ meV); $\bar s = 2d_0(2m_\parallel)^{1/2}/\hbar$. Below we shall drop the averaging sign.

It follows from (18) that for a clear observation of the oscillations the relation $sT \ll (U - \epsilon_{cr})^{1/2}$ should be fulfilled. Then, expanding $\sin[ns(\epsilon - \epsilon_{cr})^{1/2}]$ in (18) in powers of $\epsilon - U$, we obtain for $U - \epsilon_{cr} \gg T$ and $s(U - \epsilon_{cr})^{1/2} \gg 1$ the expression ($y = U - \epsilon_{cr}$):

$$\delta\sigma_{osc}(U)=CD_{cr}\,\pi^2 m_\perp\beta Ts\sum_{n=1}^{\infty}\frac{\sin(ns\sqrt{y})}{\mathrm{sh}(\pi nsT/2\sqrt{y})}\exp(-{}^1/_2 n^2s^2\sigma y). \quad (19)$$

If $U < \epsilon_{cr}$ and $|U - \epsilon_{cr}| \gg T$, then $\delta\sigma_{osc}$ decreases exponentially. As can be seen from (19), the oscillations are noticeable if $s^2\sigma(U - \epsilon_{cr}) \lesssim 1$. Thus, the region in which they can be detected is limited by the inequalities

$$s^2T^2<(U-\varepsilon_{cr})<1/s^2\sigma;$$

for sufficiently high T and σ such biases may, in general, not exist, and then the observation of the fine structure in $\delta\sigma(U)$ becomes impossible. The existence of these two boundaries for the oscillations close to the beginning of a new energy band is noticeable in the experimental d^2I/dU^2 curves for thin magnesium films at voltage potentials $U \lesssim 1.2$ eV, which corresponds to the beginning of the population of a new band in Mg[5]. Notice that at low T the maximum values of $\delta\sigma_{osc}$ are of the order of $10^{-2}(y/\epsilon_F)^{1/2}\sigma_0$, i.e., quite small. To increase them, we must increase the quantity β by decreasing either the thickness of the film or the magnitude of the effective mass $m_{\|}$.

Let us now consider the oscillating terms in the conductivity of the same junction, but with a superconducting injector. In this case the formula (16) should be transformed in a manner similar to the transition from (3) to (11). For $U > \epsilon_{cr} + \Delta$ and $y = (U - \epsilon_{cr}) \gg T$, the quantity $\delta\sigma_{osc}$ can be represented in the form of a series of the modified cylinder functions $K_1(x)$[11]:

$$\delta\sigma_{osc}(U)=CD_{cr}\pi m_\perp\Delta\beta s\sum_{n=1}^{\infty}\mathrm{Re}\Big\{\Big[K_1\Big(i\frac{ns\Delta}{2\sqrt{y}}\Big)+\sum_{m=1}^{\infty}(-1)^m\Big(K_1\Big(\frac{m\Delta}{T}+i\frac{ns\Delta}{2\sqrt{y}}\Big)-K_1\Big(\frac{m\Delta}{T}-i\frac{ns\Delta}{2\sqrt{y}}\Big)\Big)\Big]\exp(ins\sqrt{y}-{}^1/_2n^2s^2\sigma y)\Big\}. \quad (20)$$

For $\Delta \to 0$, (20) goes over into the result (19) for N-I-N junctions.

In the opposite case, when $s\Delta \gg \sqrt{y}$ and $\Delta \gg T$, using the asymptotic representation of $K_1(x)$, we obtain in the case when $y \gg \Delta$:

$$\delta\sigma_{osc}(U)=CD_{cr}m_\perp\beta\pi^{1/2}s^{1/2}\Delta^{1/2}y^{1/4}\sum_{n=1}^{\infty}\frac{\cos(ns(y-\Delta)^{1/2}-\pi/4)}{n^{1/2}}\exp(-{}^1/_2n^2s^2\sigma y). \quad (21)$$

For reasonably smooth films, $\delta\sigma_{osc}$ at its maxima is of the order of $10^{-2}(yt/\epsilon_F)^{1/2}\sigma_0$, where $t = s\Delta/\sqrt{y}$ is a parameter, which was assumed above to be very large. Therefore, the contribution of $\delta\sigma_{osc}$ to σ_0 can, in contrast to the case of N-I-N junctions, be quite considerable and, consequently, the possibility of observing the oscillations is even greater than in the case of N-I-N junctions.

5. The contribution of tunneling investigations to the development of our ideas about superconductivity is now universally acknowledged, but for the study of the properties of normal metals this technique is used rather rarely. As can be seen from the above-presented results, we can extract from the tunneling data important information about the electronic spectrum of metals: to wit, information about the presence and the location of the singular points ϵ_{cr} relative to the Fermi level ϵ_F. The use of thin films will allow us to determine the magnitudes of the effective masses near these points and the directivity of the effect itself, i.e., the location of the corresponding singularities in **k** space. Such experiments would facilitate further refinement of the band-structure calculations, as well as the elucidation of the effect of external influences on them. In this connection, experiments with the use of pressure on those metals in which hydrostatic compression changes the topology of the Fermi surface[8] are of extreme importance; in this case we can directly observe the change of sign of the difference $\epsilon_{cr} - \epsilon_F$. Finally, the tunneling method of investigating the band structure of metals is applicable not only to pure materials, but also to alloys[6], the electronic spectrum of which we still know very little about.

It should be emphasized that above we considered a rather idealized situation when the role of impurities and other defects are unimportant. In practice, the measured dependences will, of course, be somewhat smeared. Another limitation is connected with the fact that we used the tunneling matrix element in the WKB approximation. The opposite and less real case of a rapidly varying potential barrier gives singularities in $\delta\sigma$ that are half an order of magnitude higher[3]. Therefore, the experimentally observed singularities may be somewhat different from the computed singularities. There also exist purely experimental difficulties: the relatively small height of the potential barrier of the insulating layer, the intricacy of the preparation of films with a given orientation of the grains, etc. Nevertheless, the presently available few experimental results[5,6] inspire a definite optimism about the development of this direction of research. The improvement of the experimental technique-technological and measuring- and the broadening of the range of experimental materials will guarantee further progress in the use of the tunneling effect for the study of the electronic characteristics of metals.

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