Interaction between a photon and an intense electromagnetic wave

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Nonlinear vacuum effects in the interaction of a photon with the field of a classical plane electromagnetic wave are discussed within the framework of operator diagram technique. The amplitude for the scattering of a photon by the field of a wave of general form is obtained. The case of a monochromatic plane wave is analyzed. A new representation is obtained for the probability of the creation of a pair of particles by a photon. The propagation of a photon in the field of the wave is investigated.

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1. INTRODUCTION

An operator diagram technique has been formulated in[1] for quantum electrodynamics in external fields based on the operator representation of the Green's function for a charged particle in a field. This means that the contribution of a particular diagram can be taken in the form which coincides with the form in which it is recorded for free particles, but in which the operator for the momentum of the particle is replaced according to $p_\mu = i\partial_\mu \to P_\mu = i\partial_\mu - eA_\mu$. An essential element of this approach is an appropriate transformation of the operator expressions entering into it after which the calculations turn out to be not complicated. Thus, an advantage of the technique being developed is both its universality, and also the relative simplicity of calculations. In[1,2] phenomena were investigated in a homogeneous and constant in time electromagnetic field, in[3] the specific features of investigating phenomena in the field of a plane electromagnetic wave were elucidated on the example of calculating the mass operator for a charged particle. In the present paper the same technique is applied to the study of the interaction between a photon and the field of a plane electromagnetic wave, and for this the contribution of the polarization of the vacuum by the external photon in the field of the wave is investigated to the first order in α. The corresponding physical realization is the interaction of an external photon with the field of a laser wave, when the latter can be represented as a classical electromagnetic field of the form

$$A_\mu(\varphi) = a_{1\mu}\psi_1(\varphi) + a_{2\mu}\psi_2(\varphi), \tag{1.1}$$

where $\varphi = \kappa x = \kappa^0 x^0 - \boldsymbol{\kappa}\cdot\mathbf{x}$, ψ_1, ψ_2 are certain functions, and

$$\varkappa^2 = 0, \quad a_1\varkappa = a_2\varkappa = a_1 a_2 = 0. \tag{1.2}$$

Phenomena in the field of the wave (1.1) are characterized by the invariant intensity parameter

$$\xi_{1,2}^2 = -e^2 a_{1,2}^2/m^2, \tag{1.3}$$

with the expansion in series in powers of $\xi_{1,2}^2$ being an expansion in terms of the number of interactions with the field of the wave (1.1). In the region where $\xi_{1,2}^2 \ll 1$ perturbation theory is applicable, while for $\xi_{1,2}^2 \sim 1$ the interaction with the field of the wave must necessarily be taken into account exactly. We note that there exist lasers for which $\xi_{1,2}^2 \sim 1$. The imaginary part of the amplitude of elastic scattering of an external photon in the forward direction is related to the total probability for the creation of a particle-antiparticle pair by a photon in the field of the wave. This problem has been studied in a number of papers[4-6]. In the present paper we have obtained a new representation for the probability of pair creation which is expressed in the case of a circularly polarized monochromatic wave by a single integral. Knowing the amplitude for the interaction of a photon with the field (1.1) enables us to solve the problem of the propagation of a photon in the "medium" represented by the field of the wave[1)].

In Sec. 2 we obtain the amplitude for the scattering of a photon by the field of the wave (1.1), we calculate the final form of the amplitude for the case of a monochromatic wave, we obtain different representations for the probability of creation of a pair of particles by a photon, and we trace out the transition to the case of a constant crossed field. In Sec. 3 we discuss the specific features of the propagation of a photon in the field of a laser wave.

2. SCATTERING OF A PHOTON BY THE FIELD OF A PLANE ELECTROMAGNETIC WAVE

The amplitude for the scattering of a photon of momentum k_1 by the field of a plane wave (k_1 + wave $\to k_2$) taking into account the polarization of vacuum by spinor particles has the form (cf., (1.14) of reference[1])

$$T = e_{1\mu}(k_1)e_{2\nu}^*(k_2)T^{\mu\nu}(k_1, k_2),$$
$$T_{\mu\nu}(k_1, k_2) = -\frac{e^2}{(2\pi)^4}\int d^4x\,\mathrm{Sp}\left\langle x\left|\frac{1}{\hat{P}-m}\gamma_\mu e^{-ik_1x}\frac{1}{\hat{P}-m}\gamma_\nu e^{ik_2x}\right|x\right\rangle, \tag{2.1}$$

where $|x\rangle$ is the eigenvector of the operator for the X coordinate. Since a change in k_1 is determined by the interaction with the wave, then we always have $k_2 = k_1 + c\kappa$, where c is a scalar. Taking into account this and the relations (1.2) we have

$$\varkappa k_1 = \varkappa k_2 \equiv \varkappa k, \quad a_1k_1 = a_1k_2 \equiv a_1k, \quad a_2k_1 = a_2k_2 \equiv a_2k, \tag{2.2}$$

i.e., when the scalar products for the vectors k_1 and k_2 are the same we shall use the notation k.

We construct the vectors

$$\Lambda_1^\mu = \frac{(kf_1)^\mu}{(\varkappa k)\sqrt{-a_1^2}}, \quad \Lambda_2^\mu = \frac{(kf_2)^\mu}{(\varkappa k)\sqrt{-a_2^2}},$$
$$\Lambda_3^\mu = \frac{\varkappa^\mu k_1^2 - k_1^\mu(\varkappa k)}{(\varkappa k)\sqrt{k_1^2}}, \quad \Lambda_4^\mu = \frac{\varkappa^\mu k_2^2 - k_2^\mu(\varkappa k)}{(\varkappa k)\sqrt{k_2^2}}, \tag{2.3}$$
$$\Lambda_1^2 = \Lambda_2^2 = \Lambda_3^2 = \Lambda_4^2 = -1,$$

where

$$f_{1,2}^{\mu\nu} = \varkappa^\mu a_{1,2}^\nu - \varkappa^\nu a_{1,2}^\mu, \quad (kf_{1,2})^\mu = k_\nu f_{1,2}^{\nu\mu}. \tag{2.4}$$

The sets

$$k_1^\mu/\sqrt{k_1^2},\ \Lambda_1^\mu,\ \Lambda_2^\mu, \Lambda_3^\mu \text{ и } k_2^\nu/\sqrt{k_2^2},\ \Lambda_1^\nu,\ \Lambda_2^\nu,\ \Lambda_4^\nu$$

represent orthonormalized sets[2] in terms of which any vector of the problem can be expanded.

The amplitude $T^{\mu\nu}(k_1, k_2)$ (2.1) is gauge-invariant (strictly speaking, after regularization has been carried out). Then in virtue of the foregoing it can be expanded in terms of the vectors (2.3)

$$T^{\mu\nu}(k_1, k_2)=c_1\Lambda_1^\mu\Lambda_2^\nu+c_2\Lambda_2^\mu\Lambda_1^\nu+c_3\Lambda_1^\mu\Lambda_1^\nu+c_4\Lambda_2^\mu\Lambda_2^\nu+c_5\Lambda_3^\mu\Lambda_4^\nu. \tag{2.5}$$

The coefficients in front of the other possible combinations constructed from the vectors Λ_n^μ vanish in virtue of the Furry theorem. The further problem consists of evaluating the coefficients $c_1 - c_5$. We make use of the fact that in the expression $\int d^4x\mathrm{Sp}\langle x|\ldots|x\rangle$ one can cyclically permute the operators within the brackets, and we move in (2.1) the operator $\hat{P}+m$ towards the right hand eigenfunction. After this we move the operator e^{-ik_1X} to the left, and the operator e^{ik_2X} to the right, taking into account the fact that they are displacement operators in momentum space, and then we move the matrix γ^ν to the right:

$$T^{\mu\nu}(k_1, k_2)=-\frac{e^2}{(2\pi)^4}\int d^4x \exp\{i(k_2-k_1)x\}\mathrm{Sp}\left\langle x\left|\frac{1}{(\hat{P}+\hat{k}_1)^2-m^2}\right.\right.$$
$$\left.\left.\times\gamma^\mu\frac{1}{\hat{P}^2-m^2}(\hat{P}+m)\left[2P^\nu+\gamma^\nu\hat{k}_2+(m-\hat{P})\gamma^\nu\right]\right|x\right\rangle. \tag{2.6}$$

In the expression obtained above the term with $(m-\hat{P})$ within the square brackets is brought to the form

$$\frac{e^2}{(2\pi)^4}\int d^4x \exp\{i(k_2-k_1)x\}\mathrm{Sp}\left[\left\langle x\left|\frac{1}{\hat{P}^2-m^2}\right|x\right\rangle\gamma^\mu\gamma^\nu\right], \tag{2.7}$$

using which it is not difficult to show (cf., below), that this term does not depend on the field of the wave. In carrying out the regularization it drops out, and therefore in future we shall not write it out. In the remaining expression we carry out the exponential parametrization of the propagators:

$$T^{\mu\nu}(k_1, k_2)=\frac{e^2}{(2\pi)^4}\int_0^\infty dt\int_0^\infty ds \exp\{-im^2(s+t)\}\tilde{T}^{\mu\nu}, \tag{2.8}$$

where

$$\tilde{T}^{\mu\nu}=\int d^4x \exp\{i(k_2-k_1)x\}\mathrm{Sp}\langle x|\exp\{it(\hat{P}+\hat{k}_1)^2\}$$
$$\cdot\gamma^\mu\exp\{is\hat{P}^2\}(\hat{P}+m)(\gamma^\nu\hat{k}_2+2P^\nu)|x\rangle. \tag{2.9}$$

For transformation of the terms appearing in (2.9) we utilize the formulas of (A.26), (3.7) of reference[3], and then $\tilde{T}^{\mu\nu}$ can be rewritten in the form

$$\tilde{T}^{\mu\nu}=\int d^4x \exp\{i(k_2-k_1)x\}\langle x|\exp\{it(P+k_1)^2\}\exp\{isP^2\}B^{\mu\nu}|x\rangle, \tag{2.10}$$

where

$$B^{\mu\nu}=\mathrm{Sp}\left[\gamma^\mu(1+e^+\hat{\varkappa}\hat{a})(\hat{P}+m)(\gamma^\nu\hat{k}_2+2P^\nu)(1+e^-\hat{\varkappa}\hat{a})\right],$$
$$e^+(s)=\frac{er^+(s)}{2(\varkappa P)},\quad r^+(s)=\psi(\varphi+2(\varkappa P)s)-\psi(\varphi), \tag{2.11}$$
$$e^-(t)=-\frac{er^-(t)}{2\varkappa(P+k)},\quad r^-(t)=\psi(\varphi-2\varkappa(P+k)t)-\psi(\varphi).$$

These formulas can be utilized directly for a linearly polarized wave. However, they can also be used in the general case of elliptical polarization, with the understanding that the compact form of recording has been utilized: $a\psi = a_1\psi_1 + a_2\psi_2$, i.e., $ar = a_1r_1 + a_2r_2$.

For the evaluation of the coefficients $c_1 - c_5$ in (2.5) one can contract the tensor $T^{\mu\nu}(k_1, k_2)$ (2.8) with the pair combinations of the vectors $\Lambda_1^\mu - \Lambda_4^\mu$ (2.3). In this case in virtue of the gauge invariance of $T^{\mu\nu}(k_1, k_2)$ the terms in the vectors $\Lambda_3^\mu, \Lambda_4^\mu$, containing k_1, k_2, will vanish on contraction. All the remaining terms in the vectors $\Lambda_1^\mu - \Lambda_4^\mu$ contain either $f_{1,2}^\nu$, or κ^μ, which vanish on contraction with κ^μ. This means that in the trace of $B^{\mu\nu}$ appearing in formula (2.10) one can omit all the terms containing κ_μ, κ_ν, and this enables one subsequently to utilize the essentially simplified expression for $B^{\mu\nu}$:

$$B^{\mu\nu}=4\{2P^\mu P^\nu+P^\mu k_2^\nu+P^\nu k_2^\mu-g^{\mu\nu}(k_2P)+(e^+(s)+e^-(t))[g^{\mu\nu}(Pfk)$$
$$-k_2^\mu(Pf)^\nu+P^\mu(kf)^\nu]-(e^+(s)-e^-(t))[(Pf)^\mu(2P^\nu+k_2^\nu)+(kf)^\mu P^\nu]\} \tag{2.12}$$

where one should interpret all the combinations in the following manner: $e^\pm f = e_1^\pm f_1 + e_2^\pm f_2$.

We now take into account the fact that the tensor $B^{\mu\nu}$ contains the operator P^μ in vector form, and also in the combinations (κP), $(fP)^\mu$ and (k_2P). In accordance with the foregoing when the tensor $B^{\mu\nu}$ is contracted with the vectors (2.3) expressions such as (κP) and $(fP)^\mu$ are formed. The scalar product (κP) commutes with all the operators of the problem and, consequently, can be regarded as a c-number. The combinations $(fP)^\mu$ also commute with one another. Therefore, in fact the tensor $B^{\mu\nu}$(2.12) contains the single operator term (k_2P) which we can investigate separately, taking for it the initial expression

$$\int d^4x\left\langle x\left|\frac{1}{(P+k_1)^2-m^2}\frac{1}{P^2-m^2}(k_2P)\exp\{i(k_2-k_1)X\}\right|x\right\rangle \tag{2.13}$$

Utilizing the identity

$$2(k_2P)=[(P+k_2)^2-m^2]-(P^2-m^2)-k_2^2, \tag{2.14}$$

we rewrite (2.13) in the form

$$\frac{1}{2}\int d^4x\left\langle x\left|\frac{1}{(P+k_1)^2-m^2}\frac{1}{P^2-m^2}\{[(P+k_2)^2-m^2]\right.\right.$$
$$\left.\left.-(P^2-m^2)-k_2^2\}\exp\{i(k_2-k_1)X\}\right|x\right\rangle. \tag{2.15}$$

We transform the first term in the figure brackets in (2.15) by taking into account the fact that

$$[(P+k_2)^2-m^2]\exp\{i(k_2-k_1)X\}=\exp\{i(k_2-k_1)X\}[(P+k_1)^2-m^2],$$

and utilizing the possibility of cyclic permutation of operators. After this transformation the first and the second terms in the figure brackets in (2.15) cancel, and this means that we can replace in $B^{\mu\nu}$ of (2.12) the quantity (k_2P) by $-k_2^2/2$. After this there will be no operator terms remaining in the expression for the tensor $B^{\mu\nu}$, and this essentially simplifies the calculation of the average $\langle x|\ldots|x\rangle$ in formula (2.10).

We transform the expression

$$I=\langle x|\exp\{it(P+k_1)^2\}\exp\{isP^2\}|x\rangle$$
$$=\langle x|\exp\{ik_1X\}\exp\{itP^2\}\exp\{-ik_1X\}\exp\{isP^2\}|x\rangle, \tag{2.16}$$

by utilizing the result of the unfolding of the exponential operator expressions (cf., (A.20), (3.7) of[3]):

$$I=\left\langle x\left|\exp\left\{it\int_0^1\frac{dy}{a^2}[a(P+k)-ea^2r^-(ty)]^2\right\}\exp\{it(P+k_1)_\perp^2\}\right.\right.$$
$$\left.\left.\times\exp\{isP_\perp^2\}\exp\left\{is\int_0^1\frac{dy}{a^2}[aP-ea^2r^+(sy)]^2\right\}\right|x\right\rangle, \tag{2.17}$$

where the notation of (2.11) has been utilized. It is convenient to conduct the further investigation in the "special" reference system, where the vector κ is directed along the 3 axis, i.e., $\kappa^0 = \kappa^3$, and the vectors

$\mathbf{a}_1$, $\mathbf{a}_2$ lie in the plane of (1.2). We introduce the variables

$$\vartheta=\frac{x^0-x^3}{\sqrt{2}},\quad \nu=\frac{x^0+x^3}{\sqrt{2}},\quad p_\vartheta=i\frac{\partial}{\partial\vartheta},\quad p_\nu=i\frac{\partial}{\partial\nu},\tag{2.18}$$

and then have

$$\varphi=\sqrt{2}\,\varkappa^0\vartheta,\quad P^0=p^0=(p_\vartheta+p_\nu)/\sqrt{2},\quad (\varkappa P)=\sqrt{2}\,\varkappa^0 p_\nu,$$
$$P^3=p^3=(p_\vartheta-p_\nu)/\sqrt{2},\quad P_0{}^2-P_3{}^2=2p_\vartheta p_\nu.\tag{2.19}$$

Utilizing the completeness theorem (cf., (2.40) in[2])

$$\langle x|R(p)|x\rangle=\int d^4p\,R(p),\tag{2.20}$$

we can calculate the average over the state which depends on the 4-vector x_μ, component by component: $|x\rangle = |\nu, \vartheta, xa_1, xa_2\rangle$. In calculating $\langle\vartheta|\ldots|\vartheta\rangle$ we must retain in the average only the terms containing the operator $P_\perp^2$, since the operator (aP) does not act on $|\vartheta\rangle$, while in the explicit functions of the variable ϑ appearing in the expression (these are the functions $r^\pm$) the state $|\vartheta\rangle$ is an eigenstate, i.e., $\exp\{if(\vartheta)\}|\vartheta\rangle = |\vartheta\rangle\exp\{if(\vartheta)\}$. Taking this into account we have from (2.17), (2.20)

$$\langle\vartheta|\exp\{2it(p_\vartheta+k_{1\vartheta})(p_\nu+k_{1\nu})\}\exp\{2isp_\vartheta p_\nu\}|\vartheta\rangle$$
$$=\frac{\pi}{s+t}\exp\{2itk_{1\vartheta}(p_\nu+k_{1\nu})\}\delta\left(p_\nu+\frac{k_{1\nu}t}{s+t}\right).\tag{2.21}$$

Then the calculation of the average $\langle\nu|\ldots|\nu\rangle$ reduces to integration over the δ-function, in which

$$2(P+k)\varkappa\to2(\varkappa k)\frac{s}{s+t},\quad 2(\varkappa P)\to-2(\varkappa k)\frac{t}{s+t},\tag{2.22}$$

and the evaluation of the average over (xa_1), (xa_2) is carried out directly in accordance with formula (2.20) and reduces to Fresnel integrals. Finally we obtain for the average (2.16)

$$I=-\frac{i\pi^2}{(s+t)^2}\exp\{i\mu k_1{}^2\}\exp\{i(s+t)\beta\},\quad \beta=\beta_1+\beta_2,\tag{2.23}$$

where

$$\beta_{1,2}=e^2a_{1,2}^2\left[\int_0^1 dy\,\Delta_{1,2}^2(\mu y)-\left(\int_0^1 dy\,\Delta_{1,2}(\mu y)\right)^2\right],$$
$$\Delta_{1,2}(\mu y)=\psi_{1,2}(\varphi-2(\varkappa k)\mu y)-\psi_{1,2}(\varphi),\quad \mu=\frac{st}{s+t}.\tag{2.24}$$

In accordance with the arguments given above, after the replacement $(k_2P)\to-k_2^2/2$ all the operators appearing in the tensor $B^{\mu\nu}$ (2.10), (2.12), can be regarded as c-numbers. Then the evaluation of the averages reduces to taking quadratures of the same type which are encountered in evaluating the polarization operator in α-order for free particles utilizing the exponential parametrization of the propagators (cf., for example,[9]). In fact, for obtaining the result one should substitute into (2.10) the average (2.16). (2.23), and to carry out in the tensor $B^{\mu\nu}$ the replacements

$$P_\mu\to R_\mu=-k_{1\mu}\frac{t}{s+t}+ea_{1\mu}\int_0^1 dy\,\Delta_1(\mu y)+ea_{2\mu}\int_0^1 dy\,\Delta_2(\mu y),$$
$$P^\mu P^\nu\to R^\mu R^\nu+\frac{i}{2(s+t)}\left(\frac{a_1{}^\mu a_1{}^\nu}{a_1{}^2}+\frac{a_2{}^\mu a_2{}^\nu}{a_2{}^2}\right).\tag{2.25}$$

Substituting the tensor $\tilde{T}^{\mu\nu}$ obtained in this manner into (2.8) and contracting $T^{\mu\nu}(k_1, k_2)$ with the appropriate combinations of the vectors $\Lambda_1^\mu-\Lambda_4^\mu$, appearing in (2.25), we obtain explicit expressions for the coefficients c_1-c_5. The expression for the tensor $T^{\mu\nu}(k_1, k_2)$ obtained in this manner must be regularized. In order to do this we represent it in the form

$$T^{\mu\nu}(k_1,k_2)=(T^{\mu\nu}(k_1,k_2)-T^{\mu\nu}_{F=0}(k_1,k_2))+T^{\mu\nu}_{F=0}(k_1,k_2).\tag{2.26}$$

The first term vanishes when the field of the wave $F = 0$, while the second term (independent of the field when $k_1 = k_2$) must be renormalized in the standard manner (cf.,[2]). After subtracting the term $T^{\mu\nu}_{F=0}$ we obtain the following expression for the coefficients c_1-c_5 in (2.25):

$$c_n=-\frac{i\alpha}{2\pi}\int_{-1}^1 dv\int_0^\infty\frac{d\tau}{\tau}\int d^4x\exp\{i(k_2-k_1)x\}\exp\left\{-im^2\tau\left(1-\frac{k_1{}^2(1-v^2)}{4m^2}\right)\right\}b_n,\tag{2.27}$$

where we have gone over to the new variables

$$s+t=\tau,\quad v=(s-t)/(s+t),\ \text{т. е. }\ \mu={}^1/_4\tau(1-v^2)\ \text{ and}$$

$$b_1=2\xi_1\xi_2m^2\left[D_1\int_0^1 dy\,\Delta_2(\mu y)-\frac{\Delta_2(\mu)}{1-v^2}\int_0^1 dy\,\Delta_1(\mu y)\right]e^{i\tau\beta},$$
$$b_2=2\xi_1\xi_2m^2\left[D_2\int_0^1 dy\,\Delta_1(\mu y)-\frac{\Delta_1(\mu)}{1-v^2}\int_0^1 dy\,\Delta_2(\mu y)\right]e^{i\tau\beta},$$
$$b_3=2m^2\left[\xi_1{}^2D_1\int_0^1 dy\,\Delta_1(\mu y)+\frac{\xi_2{}^2\Delta_2(\mu)}{1-v^2}\int_0^1 dy\,\Delta_2(\mu y)\right]e^{i\tau\beta}-\left(\frac{i}{\tau}+\frac{k_2{}^2}{2}\right)(e^{i\tau\beta}-1),$$
$$b_4=2m^2\left[\xi_2{}^2D_2\int_0^1 dy\,\Delta_2(\mu y)+\frac{\xi_1{}^2\Delta_1(\mu)}{1-v^2}\int_0^1 dy\,\Delta_1(\mu y)\right]e^{i\tau\beta}-\left(\frac{i}{\tau}+\frac{k_2{}^2}{2}\right)(e^{i\tau\beta}-1),$$
$$b_5=-{}^1/_2\sqrt{k_1{}^2k_2{}^2}(1-v^2)(e^{i\tau\beta}-1).\tag{2.28}$$

Here

$$D_{1,2}=\int_0^1 dy\,\Delta_{1,2}(\mu y)+\frac{v^2}{1-v^2}\Delta_{1,2}(\mu),\tag{2.29}$$

and the notation of (2.24) has been utilized. Since the coefficients b_n are functions of $\varphi=\kappa x$, then the interaction of the photon described by the tensor $T^{\mu\nu}(k_1, k_2)$ is in the general case inelastic ($k_1\neq k_2$), i.e., the plane wave of the form (1.1) is an optically active "medium" for an external photon.

In the case of an elliptically polarized monochromatic wave when

$$\psi_1=\cos\varphi,\quad \psi_2=\sin\varphi,\tag{2.30}$$

the integrals appearing in b_n (2.28) can be evaluated without difficulty. As a result we obtain for the coefficient c_n in the expression for $T^{\mu\nu}(k_1, k_2)-T^{\mu\nu}_{F=0}(k_1, k_2)$ (cf. (2.25))

$$c_n=-i(2\pi)^4m^2\frac{\alpha}{\pi}\int_{-1}^1 dv\int_0^\infty\frac{d\rho}{\rho}\exp\left\{-i\frac{2\rho}{|\lambda|(1-v^2)}\left[1-\frac{k_1k_2(1-v^2)}{4m^2}+A(\xi_1{}^2+\xi_2{}^2)\right]\right\}\left[\delta(k_1-k_2)d_n+\sum_{\substack{l=-\infty\\ l\neq0}}^{+\infty}\delta(k_1-k_2-2\varkappa l)g_n{}^l\right],\tag{2.31}$$

where

$$d_1=-d_2=2\rho\xi_1\xi_2A_0\left(\frac{1+v^2}{1-v^2}\right)J_0(z)\operatorname{sign}\lambda,$$
$$d_3=\left[A_1\xi_1{}^2-\sin^2\rho\frac{\xi_1{}^2-\xi_2{}^2}{1-v^2}\right](J_0(z)-iJ_0{}'(z))+\xi_1{}^2\sin^2\rho\frac{1+v^2}{1-v^2}J_0(z)-\frac14\left(\frac{k_1k_2}{m^2}+\frac{i|\lambda|(1-v^2)}{\rho}\right)(J_0(z)-e^{iy}),$$
$$d_4=d_3(\xi_1{}^2\leftrightarrow\xi_2{}^2),$$
$$d_5=-\frac{k_1k_2}{4m^2}(1-v^2)(J_0(z)-e^{iy});$$
$$g_1{}^l=\xi_1\xi_2\left[2A_0\rho\frac{1+v^2}{1-v^2}\operatorname{sign}\lambda-A_1\frac{l}{z}\right]i^lJ_l(z),$$

$$g_2^l = g_1^l(A_0 \to -A_0,\quad z \to z),$$

$$g_3^l = \left[\xi_1^2 A_1 + \sin^2\rho \frac{\xi_1^2 v^2 + \xi_2^2}{1-v^2}\right] i^l J_l(z) + \left[\xi_1^2 A_1 - \frac{\sin^2\rho}{1-v^2}(\xi_1^2 - \xi_2^2)\right] i^{l-1} J_l'(z)$$

$$-\frac{1}{4}\left(\frac{k_1 k_2}{m^2} + \frac{i|\lambda|(1-v^2)}{\rho}\right) i^l J_l(z), \qquad (2.32)$$

$$g_4^l = g_3^l(\xi_1^2 \leftrightarrow \xi_2^2)(-1)^l, \qquad g_5^l = -\frac{\sqrt{k_1^2 k_2^2}}{4m^2}(1-v^2) i^l J_l(z).$$

Here $J_l(z)$ are Bessel functions, the following notation has been used

$$A = \frac{1}{2}\left(1 - \frac{\sin^2\rho}{\rho^2}\right), \quad A_0 = \frac{1}{2}\left(\frac{\sin^2\rho}{\rho^2} - \frac{\sin 2\rho}{2\rho}\right),$$

$$A_1 = A + 2A_0, \quad z = \frac{2\rho(\xi_1^2 - \xi_2^2)}{|\lambda|(1-v^2)} A_0, \quad \lambda = \frac{\varkappa k}{2m^2}, \qquad (2.33)$$

$$y = 2\rho(\xi_1^2 + \xi_2^2)\frac{A}{|\lambda|(1-v^2)},$$

and the replacement $\tau = 2\rho/|\lambda|(1 - v^2)m^2$ has been carried out. The expressions for c_n (2.31) are sums of two terms. One of them (with coefficients d_n) describes the elastic scattering of a photon by the field of the wave, the other (containing g_n) describes inelastic scattering accompanied by emission (absorption) of the "photons" of the wave. In the case of a circularly polarized monochromatic wave $\xi_1^2 = \xi_2^2 = \xi^2$ the coefficients appearing in (2.31) are essentially simplified. In this case the tensor $T^{\mu\nu}$ (2.5) has the form (we subtract the tensor $T^{\mu\nu}$ for $F = 0$)

$$\frac{T^{\mu\nu}(k_1, k_2) - T^{\mu\nu}_{F=0}}{i(2\pi)^4} = \Pi^{\mu\nu}_{(0)}\delta(k_1 - k_2) + \Pi^{\mu\nu}_{(-)}\delta(k_1 - k_2 - 2\varkappa) + \Pi^{\mu\nu}_{(+)}\delta(k_1 - k_2 + 2\varkappa),$$

$$\Pi^{\mu\nu}_{(0)} = (\Lambda_1^\mu \Lambda_2^\nu - \Lambda_2^\mu \Lambda_1^\nu)\alpha_1 + (\Lambda_1^\mu \Lambda_1^\nu + \Lambda_2^\mu \Lambda_2^\nu)\alpha_3 + \Lambda_3^\mu \Lambda_3^\nu \alpha_5, \qquad (2.34)$$

$$\Pi^{\mu\nu}_{(\mp)} = (\Lambda_1^\mu \mp i\Lambda_2^\mu)(\Lambda_1^\nu \mp i\Lambda_2^\nu)\alpha_0,$$

where (cf., also (2.33))

$$\alpha_n = -\frac{\alpha}{2\pi} m^2 \int_{-1}^{1} dv \int_0^\infty \frac{d\rho}{\rho} \exp\left\{-\frac{2i\rho}{|\lambda|(1-v^2)}\left[1 - \frac{k_1 k_2}{4m^2}(1-v^2) + 2A\xi^2\right]\right\}\omega_n,$$

$$\omega_0 = \xi^2 A_1, \quad \omega_1 = 4\xi^2 A_0 \rho \frac{1+v^2}{1-v^2}\operatorname{sign}\lambda,$$

$$\omega_3 = 2\xi^2 \sin^2\rho \frac{1+v^2}{1-v^2} - \left[1 + \frac{k_1 k_2}{4m^2}(1+v^2)\right](1 - e^{iy}), \qquad (2.35)$$

$$\omega_5 = -\frac{k_1^2}{2m^2}(1-v^2)(1-e^{iy}).$$

Thus, from a circularly polarized wave only two photons[3)] can be emitted (absorbed), and this is determined by the fact that the "photons" of the wave have in this case a definite chirality and transitions are possible both without a change in the chirality of the incident photon ($l = 0$) and with a reversal of chirality ($l = \pm 1$). This can be seen from (2.34), since in the inelastic terms characteristic combinations of chiral eigenvectors have been formed.

We introduce the "diagonal" polarization operator $\Pi_e^{\mu\nu}(k_1)$ related to the amplitude for the elastic scattering of a photon in the following manner:

$$T_e^{\mu\nu}(k_1, k_2) = i(2\pi)^4 \delta(k_1 - k_2)\Pi_e^{\mu\nu}(k_1). \qquad (2.36)$$

Its imaginary part determines the probability of creation of an electron-positron pair by a photon in the field of a wave (cf., formula (3.14)[2]):

$$W = \frac{e^\mu e^{\nu *}}{k_1^0}\operatorname{Im}\Pi_{\mu\nu}^e(k_1), \qquad (2.37)$$

where k_1^0 is the photon energy. Taking into account formulas (2.35), (2.5), (2.31), (2.34) we obtain from this for the probability of creation of a pair by an unpolarized real photon ($k_1^2 = 0$) in the field of a circularly polarized wave:

$$W^{cr} = -\frac{g^{\mu\nu}\operatorname{Im}\Pi_{\mu\nu}^{cr}}{2k_1^0} = \frac{\alpha m^2}{4\pi k_1^0}\operatorname{Im}\int_1^\infty \frac{du}{u\sqrt{u(u-1)}}$$

$$\times \int_0^\infty \frac{d\rho}{\rho}\{e^{-iz_1^{cr}}[1 - 2\xi^2(2u-1)\sin^2\rho] - \exp\{-2i\rho u/|\lambda|\}\}$$

$$= -\frac{\alpha m^2}{4k_1^0}\operatorname{Im}\int_0^\infty \frac{d\rho}{\rho} e^{-i\eta}\{(1 + 2\xi^2\sin^2\rho)\eta[H_0^{(2)}(\eta) + iH_1^{(2)}(\eta)] - 2i\xi^2 H_0^{(2)}$$

here $Z_1^{cr} = 2u\eta$,

$$\eta = \frac{\rho}{|\lambda|}\left[1 + \xi^2\left(1 - \frac{\sin^2\rho}{\rho^2}\right)\right], \qquad (2.38)$$

$H_{0,1}^{(2)}$ is a Hankel function, and a replacement of the variables $u = 1/(1 - v^2)$ has been carried out. In a similar manner one obtains the probability of pair creation by polarized photons. Formula (2.38) is a new representation for the probability of pair creation analogous to formulas (3.36) and (3.35) of[3] for the probability of emission of a photon by an electron in the field of a wave. Utilizing the generating function for the Bessel functions and the power series expansion of the square of the Bessel function we obtain from (2.38) the well-known representation[5] for W^{cr}:

$$W^{cr} = \frac{\alpha m^2}{4k_1^0}\sum_{\substack{n \\ (u_n > 1)}} \int_1^{u_n} \frac{du}{u\sqrt{u(u-1)}}\{2J_n^2(z)$$

$$+ \xi^2(2u-1)[J_{n+1}^2(z) + J_{n-1}^2(z) - 2J_n^2(z)]\}, \qquad (2.39)$$

where

$$z = \frac{2\xi}{\lambda}\sqrt{1+\xi^2}\sqrt{u(u_n - u)}, \qquad u_n = \frac{n\lambda}{1+\xi^2},$$

the inequality $u_n \geq 1$ $(2(\varkappa k)n \geq 4m^2(1 + \xi^2))$ is the condition for the threshold of pair creation when n "photons" are absorbed from the wave taking into account the change in the mass of the particles in the field of the wave.

The limit $\xi \gg 1$ corresponds to the transition to processes in a constant and homogeneous field $\mathbf{E} \perp \mathbf{H}$, $|\mathbf{E}| = |\mathbf{H}|$, or, what is the same thing, a transition to the quasiclassical approximation for a photon propagated in an external field. In this case $\psi(\varphi) = \varphi$ and it is convenient to use formulas (2.5), (2.27) for a linearly polarized wave ($\xi_2 = 0$). The polarization operator obtained in this manner in the quasiclassical approximation $\Pi_{\mu\nu}^{(a)}$ (cf., (2.36)) can be utilized, in particular, for the calculation of pair creation by a photon. For example, for unpolarized photons we obtain the probability

$$W^{(q)} = -\frac{g^{\mu\nu}\operatorname{Im}\Pi_{\mu\nu}^{(q)}}{2k_1^0} = \frac{\alpha m^2}{3\sqrt{3}\pi k_1^0}\int_0^1 dv\left(\frac{9-v^2}{1-v^2}\right)K_{2/3}\left(\frac{8}{3\chi_p(1-v^2)}\right), \qquad (2.40)$$

which depends on the single parameter $\chi_p = \xi(\varkappa k)/m^2$. This result agrees with the one obtained earlier in the quasiclassical approximation (cf.,[10], p. 174).

The calculation of the contribution of particles of spin 0 to the polarization of the vacuum is completely analogous to the one carried out above, while the calculations themselves are considerably simpler. The result can be represented in the form (2.5), (2.27), where the coefficients b_n are given by

$$b_1^{(0)} = -\xi_1\xi_2 m^2 \int_0^1 dy\, \Delta_2(\mu y)\left[\int_0^1 dy\, \Delta_1(\mu y) - \Delta_1(\mu)\right] e^{i\tau\beta},$$

$$b_2^{(0)}=b_1^{(0)}(\xi_1\leftrightarrow\xi_2,\Delta_1\leftrightarrow\Delta_2),$$

$$b_3^{(0)}=-\xi_1^2m^2\int_0^1 dy\,\Delta_1(\mu y)\left[\int_0^1 dy\,\Delta_1(\mu y)-\Delta_1(\mu)\right]e^{i\tau\beta}+\frac{i}{2\tau}(e^{i\tau\beta}-1), \tag{2.41}$$

$$b_4^{(0)}=b_3^{(0)}(\xi_1\leftrightarrow\xi_2,\Delta_1\leftrightarrow\Delta_2),\quad b_5^{(0)}=-\sqrt{k_1^2k_2^2}\,\frac{v^2}{4}(e^{i\tau\beta}-1).$$

3. PROPAGATION OF A PHOTON IN THE FIELD OF A WAVE

For an external photon the field of a wave can be regarded as a material "medium." The propagation of a photon in this medium is described by solutions of the Maxwell equations, which on taking into account the interaction with the field of a plane wave can be written in the momentum representation in the form

$$k_1^2e^\mu(k_1)=\frac{1}{i(2\pi)^4}\int d^4k_2T^{\mu\nu}(k_1,k_2)e^\nu(k_2), \tag{3.1}$$

where $e_\mu(k)$ is the photon polarization vector. We examine this equation in the case of a circularly polarized wave utilizing the tensor (2.34). The polarization vector in (3.1) can be expanded in terms of the orthonormal set $k_1^\mu/\sqrt{k_1^2}$, Λ_1^μ, Λ_2^μ, Λ_3^μ. Correspondingly, we obtain four solutions; one longitudinal one and three transverse ones. The longitudinal solution can be written in the form

$$e_\mu^l(k)=f(k)\delta(k^2)k_\mu, \tag{3.2}$$

where $f(k)$ is an arbitrary function. One of the transverse solutions is

$$e_\mu^{(1)}=f_1(k)\delta(k^2+\alpha_5)\Lambda_{3\mu}, \tag{3.3}$$

since $(k^2+\alpha_5)e_\mu^{(1)}=0$. We seek the other two solutions in the form

$$e^\mu=f_2\Lambda_+^\mu+f_3\Lambda_-^\mu,\quad \Lambda_\pm^\mu=(\Lambda_1^\mu\pm i\Lambda_2^\mu). \tag{3.4}$$

Substituting this expression into the equation, we find that the functions $f_2(k)$, $f_3(k)$ satisfy the system of equations

$$\begin{gathered}(k_1^2+\alpha_3+i\alpha_1)f_2+2\alpha_0f_3^+=0,\\ 2\alpha_0f_2^-+f_3(k_1^2+\alpha_3-i\alpha_1)=0,\end{gathered} \tag{3.5}$$

where the functions having the subscript ± depend on $k_1\pm2\kappa$, and the other functions depend on k_1. This system has solutions under the condition of vanishing of the determinant

$$\begin{vmatrix} k^2+\alpha_3+i\alpha_1 & 2\alpha_0 \\ 2\alpha_0^+ & (k+2\varkappa)^2+\alpha_3^+-i\alpha_1^+ \end{vmatrix}=0, \tag{3.6}$$

which represents the dispersion equation for these two solutions.

The analysis of the solutions appears particularly simple in the domain $|\lambda|\ll1$, $k^2\ll m^2$ ($\xi^2\lesssim1$), with this inequality being satisfied for lasers in the visible part of the spectrum, if the energy of the external photon $k_1^0\leq10^{10}$ eV. In this region, the coefficients α_n (cf. (2.35)) with an accuracy up to terms $\sim\lambda^2$ have the form

$$\alpha_0=\frac{1}{60}\frac{\alpha}{\pi}\xi^2\frac{(\varkappa k)^2}{m^2},\quad \alpha_3=\frac{11}{90}\frac{\alpha}{\pi}\xi^2\frac{(\varkappa k)^2}{m^2},\quad \alpha_1=\alpha_5=0. \tag{3.7}$$

With the same accuracy the roots of equation (3.6) are

$$k^2=-\alpha_3,\quad (k+2\varkappa)^2=-\alpha_3. \tag{3.8}$$

If we introduce the "index of refraction" for the wave n:

$$k_1^2=k_{10}^2(1-n^2),$$

then we have from (3.8) for a photon incident in the opposite direction to the wave

$$n^2=1+\frac{22}{45}\frac{\alpha}{\pi}\left(\frac{F}{F_0}\right)^2, \tag{3.9}$$

where F is the intensity of the field of the wave, $F_0=m^2/e$ is the critical field. Thus, the index of refraction in this limit depends neither on the photon energy nor on the frequency of the wave. It follows from formula (3.9) that for $\lambda\ll1$ the effects will be quite weak. They will become considerably more noticeable when $\lambda\sim1$. It is just the region relatively close to the threshold of the two-particle reaction that is of the greatest interest for the investigation of the effects of intensity in the field of the wave. The same conclusion follows also from an analysis of the imaginary part of the forward scattering amplitude of (2.37) (cf. also[4-6]).

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[1)]After the present work had been completed we became aware of the paper of Becker and Mitter [7] in which the same set of questions has been investigated using the explicit form of the Green's function for an electron in the field of the wave obtained by Schwinger [8]. The results of both investigations coincide in the overlapping region. We are grateful to V. I. Ritus who drew our attention to [7].

[2)]Completeness relations hold, for example:

$$g^{\mu\nu}=\frac{k_1^\mu k_1^\nu}{k_1^2}-\Lambda_1^\mu\Lambda_1^\nu-\Lambda_2^\mu\Lambda_2^\nu-\Lambda_3^\mu\Lambda_3^\nu.$$

[3)]This circumstance has been noted by Becker and Mitter [7].

[1] V. N. Baĭer, V. M. Katkov, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 67, 453 (1974) [Sov. Phys.-JETP 40, 225 (1975)].

[2] V. N. Baĭer, V. M. Katkov, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 68, 405 (1975) [Sov. Phys.-JETP 41, 198 (1975)].

[3] V. N. Baĭer, V. M. Katkov, A. I. Mil'shteĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. 69, 783 (1975) [Sov. Phys.-JETP 42, 000 (1976)].

[4] A. I. Nikishov and V. I. Ritus, Zh. Eksp. Teor. Fiz. 46, 776 (1964) [Sov. Phys.-JETP 19, 529 (1964)].

[5] N. B. Narozhnyĭ, A. I. Nikishov, and V. I. Ritus, Zh. Eksp. Teor. Fiz. 47, 930 (1964) [Sov. Phys.-JETP 20, 622 (1965)].

[6] I. I. Gol'dman, Zh. Eksp. Teor. Fiz. 46, 1412 (1964) [Sov. Phys.-JETP 19, 954 (1964)].

[7] W. Becker and H. Mitter, Preprint Universitäts Tübingen, 1974.

[8] J. Schwinger, Phys. Rev. 82, 664 (1951).

[9] N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh poleĭ (Introduction to the Theory of Quantized Fields), Nauka, 1973.

[10] V. N. Baĭer, V. M. Katkov, and V. S. Fadin, Izluchenie relyativistskikh elektronov (Radiation from Relativistic Electrons), Atomizdat, 1973.

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