Asymmetry of the quasiparticle distribution in superconductors and normal metals

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The population asymmetry of the energy spectrum branches ($\xi > 0$ and $\xi < 0$; $\xi = v(p - p_F)$) that is produced in a superconductor S upon the tunnel injection of nonequilibrium quasiparticles into S is theoretically investigated. It is shown that such an asymmetry leads to the appearance in S of a gauge-invariant potential $\Phi = \varphi + (1/2)\dot{\chi}$ and to the appearance of a voltage potential in the measuring circuit. The steady-state value of the voltage potential is computed. It is also shown that if the electron-hole distribution in a normal metal N is made asymmetric through injection, then a potential difference arises between N and the measuring electrode if as the latter a superconductor $S_{\rm meas}$ is used.

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Clarke, Peterson, and Tinkham[1-3] have carried out experimental and theoretical investigations of the tunnel structure S′-S-N (Fig. 1). A current I was passed through the structure in such a way that quasiparticles were continuously injected from N into S. The neutrality in S was maintained as a result of the drift of Cooper pairs into S′. It was observed that a voltage potential $\mathscr{E}$ existed in the measuring circuit whenever the injecting current I was different from zero. As has been shown by Clarke and Tinkham, the appearance of the voltage potential is due to the asymmetry of the excitation distribution function $n(\xi)$ in the superconductor with respect to ξ ($\xi = v(p - p_F)$). This means that the number $N_>$ ($\xi > 0$) of particles on the n-type excitation branch is not equal to the number $N_<$ ($\xi < 0$) of particles on the p-type excitation branch. Tinkham computed the time for the establishment of the steady-state value of $Q = N_> - N_<$, and found that in the case of scattering of the quasiparticles by phonons this time is $\tau_Q \sim \Delta^{-1}$, since in the normal metal the collision integral leaves the quantity Q unchanged (in the normal metal Q is the difference between the number of electrons and the number of holes, and therefore the invariability of Q implies the conservation of the total number of particles). The time τ_Q determines the attenuation length of the longitudinal electric field E in a superconductor with a nonzero energy gap[4]. Tinkham also calculated the quantity $\mathscr{E}$, and found that $\mathscr{E} \sim Q^*\tau_Q$, where $Q \approx Q^*$ near the critical temperature T_c. He did not, however, take into account the appearance in S of the gauge-invariant potential

$$\Phi = \varphi + {}^1/_2\dot{\chi}$$

(φ is the electrical potential and χ is the phase of the order parameter), and the electrical neutrality of the superconductor S was not properly taken into account. Meanwhile, as will be shown below, the voltage potential $\mathscr{E}$ is due precisely to the presence of the potential Φ. Such a potential arises if the divergence of the supercurrent is different zero, which is the case when, for example, current is passed across an S-N junction[1)] [5,6].

In a previous paper by one of the present authors[9], the growth rate of Φ was found (in the $\varphi = 0$ gauge) without allowance for the collision integral. Here we shall find the steady-state values of Φ and $\mathscr{E}$, taking into account the inelastic collisions with phonons.

We shall also consider the tunnel structure N_1-N-N_2, and show that the electron and hole distributions in the metal N become asymmetric when a current is passed through such a structure (although the total number of electrons remains equal to the total number of holes, i.e., Q = 0). Such an asymmetry leads also to the appearance of a potential difference between N and the measuring electrode if as the latter a superconductor is used.

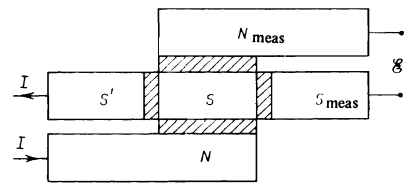

FIG. 1. The system under consideration: the hatched regions represent insulator layers, I is the injection current, and $\mathscr{E}$ is the voltage potential to be measured.

1. THE SYSTEM S′-S-N

For greater physical clarity, we use a computational method somewhat different from the one used earlier in[9]. It is convenient to express the Green functions with the aid of which the calculation in[9] was carried out in terms of the occupation number $n(\xi)$ of the quasiparticles, as is done by Aronov and Gurevich in[10].

Let us find the rate of change, due to the injection, of the number of quasiparticles in S in terms of the functions G^{12} and G^{21}[10]:

$$\frac{\partial n}{\partial t} = \frac{1}{2\pi i}\frac{\partial}{\partial t}\int_0^\infty d\omega\,(G_\omega^{12} - G_{-\omega}^{21}). \tag{1}$$

Let us write down the equation for G[10,11]:

$$\left(i\frac{\partial}{\partial t_1} - \xi\right) G^{12}(t_1 t_2) = \Sigma^{11}G^{12} + \Sigma^{12}G^{22} \equiv A(t_1, t_2),$$
$$\left(i\frac{\partial}{\partial t_2} + \xi\right) G^{12}(t_1, t_2) = A^*(t_2, t_1), \tag{2}$$

where Σ is the self-energy part responsible for the BCS interaction and the tunneling of quasiparticles from N into S[9]. Let us add the Eqs. (2) and carry out a Fourier transformation with respect to the difference variable $(t_1 - t_2)$. We obtain

$$i\frac{\partial}{\partial t} G_\omega^{12} = 2\,\mathrm{Re}\,A_\omega(t). \tag{3}$$

Similarly, for G_ω^{21} we find

$$-i\frac{\partial}{\partial t} G_\omega^{21} = 2\,\mathrm{Re}\,(\Sigma^{21}G^{11} + \Sigma^{22}G^{21})_\omega, \tag{4}$$

where $t = \frac{1}{2}(t_1 + t_2)$. Further, let us substitute the equilibrium values of Σ^{ik}_{tun} and G^{ik} into the right-hand sides of (3) and (4), as was done in[9] in the computation of the sources J_{S-N}. Using (1), we have

$$\frac{\partial n}{\partial t}=-\frac{\nu}{2}\left[\operatorname{th}\frac{\varepsilon+V}{2T}+\operatorname{th}\frac{\varepsilon-V}{2T}-2\operatorname{th}\frac{\varepsilon}{2T}+\frac{\xi}{\varepsilon}\left(\operatorname{th}\frac{\varepsilon+V}{2T}-\operatorname{th}\frac{\varepsilon-V}{2T}\right)\right]\equiv-\frac{1}{2}\left[\rho+\frac{\xi}{\varepsilon}\eta\right], \tag{5}$$

where the functions ρ and η, introduced in[9], are even functions of ξ and V is the voltage potential at the S-N junction (the charge e in included in V). The rate of change of n as a result of pair injection will be equal to zero, a fact which can be directly verified by computing the corresponding source. To the right-hand side of (5) must also be added the collision integral. Thus, in the steady-state case we have

$$\frac{1}{2}\left[\rho+\frac{\xi}{\varepsilon}\eta\right]=\frac{\pi\zeta_{ph}}{2\Theta_D^2}\int\omega^2\,d\omega\int d\xi'\Big\{[\delta(\tilde\varepsilon'-\tilde\varepsilon-\omega)[n'(1-n)\times(1+N_\omega)-n(1-n')N_\omega]+\delta(\tilde\varepsilon-\tilde\varepsilon'-\omega)[n'(1-n)N_\omega-n(1-n')(1+N_\omega)]]\left(1+\frac{\tilde\xi\tilde\xi'-\Delta^2}{\tilde\varepsilon\tilde\varepsilon'}\right)+\delta(\tilde\varepsilon+\tilde\varepsilon'-\omega)[N_\omega(1-n)(1-n')-nn'(1+N_\omega)]\left(1-\frac{\tilde\xi\tilde\xi'-\Delta^2}{\tilde\varepsilon\tilde\varepsilon'}\right)\Big\}, \tag{6}$$

where $\Theta_D = ps$, s is the velocity of sound, g is the matrix element of the interaction with phonons, defined by

$$g^2=2\pi^2\zeta_{ph}/pm\ [12],\quad N_\omega=(e^{\omega/T}-1)^{-1},\quad \tilde\varepsilon=\sqrt{(\xi+\Phi)^2+\Delta^2}\ [10],$$

the dimensionless constant ζ_{ph} being of the order of unity.

We shall seek the solution to (6) in the form $n = n_0(\tilde\epsilon) + n_1$, where

$$n_0(\tilde\varepsilon)=[e^{\tilde\varepsilon/T}+1]^{-1}.$$

We shall, however, be interested not in the function n_1 itself, but in some integral of it. In fact, let us write down the expression for the change δN in the total number of particles in S[10], a change which should be equal to zero:

$$\delta N=\delta\int d\tau\,[u^2n+v^2(1-n)]=\frac{pm}{\pi^2}\int d\xi\left[\frac{\xi}{\varepsilon}n_1+\left(\frac{\partial n_0}{\partial\varepsilon}\left(\frac{\xi}{\varepsilon}\right)^2-\frac{\Delta^2}{2\varepsilon^3}\operatorname{th}\frac{\varepsilon}{2T}\right)\Phi\right]=0, \tag{7}$$

where Φ is the nonequilibrium potential that was discussed above. It can be seen that the integral of the odd part of n_1 over ξ (with the weight ξ/ϵ) determines the potential Φ, i.e., Φ is determined by the asymmetry with respect to ξ of the quasiparticle distribution function. Notice that if we compute the growth rate of Φ with the aid of (5) and (7), then the result coincides with the formula (23) of[9]. To find Φ, let us multiply (6) by ξ/ϵ and integrate over ξ. Then

$$\int d\xi\,(\xi/\varepsilon)^2\eta=-\frac{\pi\zeta_{ph}}{\Theta_D^2}\int d\omega\,d\xi\,d\xi'(\xi+\xi')\omega^2(\Delta^2/\tilde\varepsilon\tilde\varepsilon')\{\delta(\tilde\varepsilon'-\tilde\varepsilon-\omega)\times[n'(1-n)(1+N_\omega)-n(1-n')N_\omega]+\delta(\tilde\varepsilon-\tilde\varepsilon'-\omega)[n'(1-n)N_\omega-n(1-n')\times(1+N_\omega)]-\delta(\tilde\varepsilon+\tilde\varepsilon'-\omega)[N_\omega(1-n)(1-n')-nn'(1+N_\omega)]\}. \tag{8}$$

Linearizing (8), we find after simple transformations that

$$-\int d\xi\left(\frac{\xi}{\varepsilon}\right)^2\eta=\int d\xi\,n_1\frac{\xi}{\varepsilon}\nu_Q(\varepsilon)\equiv\frac{\pi^2}{pm}Q^*\nu_Q, \tag{9}$$

where

$$\nu_Q(\varepsilon)\equiv\Delta^2\frac{2\pi\zeta_{ph}}{\Theta_D^2}\int_\Delta^\infty\frac{d\varepsilon'}{\varepsilon'\sqrt{\varepsilon'^2-\Delta^2}}[\theta(\varepsilon-\varepsilon')(\varepsilon-\varepsilon')^3(1-n_0'-N_{\varepsilon-\varepsilon'})+(\varepsilon+\varepsilon')^3(n_0'+N_{\varepsilon+\varepsilon'})-(\varepsilon'-\varepsilon)^3(n'+N_{\varepsilon'-\varepsilon})\theta(\varepsilon'-\varepsilon)],\quad Q^*=\frac{pm}{\pi^2}\int d\xi\frac{\xi}{\varepsilon}n_1. \tag{9'}$$

Thus, the steady-state value of Q^* is determined by the frequency ν_Q, which vanishes in the normal metal. The time τ_Q, introduced by Clarke and Tinkham, characterizes the time necessary for the establishment of equilibrium between the branches of the spectrum (the branch-mixing time).

Let us now turn to the establishment of the relation between the observable voltage potential and the potential Φ. This relation can be found if the formula (5) of[9], as applied to the junctions $S\text{-}N_{meas}$ and $S\text{-}S_{meas}$ of the measuring circuit, is used. Integration over ξ makes the left-hand side of this formula vanish, since it is assumed that no current flows through the measuring junctions. Let us choose the gauge $\dot\chi = 0$, i.e., let us assume that $\Phi = \varphi$ in S. The potential of the electrode N_{meas} is then equal to $\mathscr{E}_1 + \Phi$, where $\mathscr{E}_1$ is the potential difference between S and N_{meas}. As will be shown, there arise between the superconductors S and S_{meas} a potential difference Φ and an order-parameter phase difference $\delta\chi$.

Let us consider the junction $S\text{-}N_{meas}$. Then from the formula (5) of[9] we have

$$\operatorname{Re}\int d\xi\int d\tau\,e^{-i(\mathscr{E}_1+\Phi)}[\Sigma^R(\tau)G(-\tau)-\Omega(\tau)G^A(-\tau)]=\nu\operatorname{Im}\int d\xi\frac{d\omega}{2\pi}\left[G(\omega)-2\operatorname{th}\frac{\omega-\mathscr{E}_1-\Phi}{2T}G^R(\omega)\right]=0. \tag{10}$$

Here we have taken account of the fact that in the case when the junction is with a normal metal $\Sigma^R(\omega) = -i\nu$, and we have used the relation[9]

$$\Omega(\omega)=2\operatorname{th}\frac{\omega}{2T}\Sigma^R(\omega).$$

Let us represent $G(\omega)$ in the form

$$G(\omega)=2i\operatorname{th}\frac{\omega}{2T}\operatorname{Im}G_0^R+\delta G(\omega), \tag{11}$$

where $G_0^R(\omega)$ is the equilibrium retarded Green function and $\delta G(\omega)$ is the nonequilibrium correction to $G_0(\omega)$ due to the injection of quasiparticles. The expression for $\operatorname{Im}G_0^R(\omega)$ has the form

$$\operatorname{Im}G_0^R(\omega)=-\pi[u^2\delta(\omega-\varepsilon)+v^2\delta(\omega+\varepsilon)], \tag{12}$$

where $u^2 = \frac{1}{2}(1 + \xi/\epsilon)$ and $v^2 = \frac{1}{2}(1 - \xi/\epsilon)$. Substituting (11) and (12) into (10), we obtain

$$\frac{1}{2}\int d\xi\left[\operatorname{th}\frac{\varepsilon+\mathscr{E}_1}{2T}+\operatorname{th}\frac{\mathscr{E}_1-\varepsilon}{2T}\right]=\operatorname{Im}\int d\xi\,\delta G(t,t)+\int d\xi\frac{d\omega}{2\pi}\left[2\Phi\operatorname{Im}G_0^R(\omega)\frac{\partial}{\partial\omega}\operatorname{th}\frac{\omega}{2T}-2\operatorname{th}\frac{\omega}{2T}\operatorname{Im}(\tilde G_0^R(\omega)-G_0^R(\omega))\right], \tag{13}$$

where $\tilde G_0^R(\omega)$ coincides with the function $G_0^R(\omega)$ if we make the substitution $\xi \to \xi + \Phi$ in the latter.

In calculating $\mathscr{E}_1$, Tinkham[2] took only the first term on the right-hand side of (13) into account, since he assumed that only the quasiparticle distribution function changes during the injection and that $\Phi = 0$. Indeed, if we set $\Phi = 0$, then

$$\operatorname{Im}\delta G(t,t)=(\xi/\varepsilon)\delta(2n-1)=2(\xi/\varepsilon)n_1,$$

and Tinkham's result follows from (13):

$$\frac{1}{2}\int_{-\infty}^{\infty}d\xi\left[\operatorname{th}\frac{\varepsilon+\mathscr{E}_1}{2T}+\operatorname{th}\frac{\mathscr{E}_1-\varepsilon}{2T}\right]=2\int_0^\infty d\xi\frac{\xi}{\varepsilon}[n_1(\xi)-n_1(-\xi)].$$

As was pointed out in[9], however, the first term in (13) is the change in the total number of particles in S and is therefore equal to zero. In fact, the voltage potential $\mathscr{E}_1$ is due to the second term. Computing it with the aid of (12), we find

$$-\int d\xi\left[\operatorname{th}\frac{\mathscr{E}_1+\varepsilon}{2T}+\operatorname{th}\frac{\mathscr{E}_1-\varepsilon}{2T}\right]=4\Delta^2\Phi\int_\Delta^\infty\frac{d\varepsilon}{(\varepsilon^2-\Delta^2)^{1/2}}\frac{\partial}{\partial\varepsilon}\frac{\operatorname{th}(\varepsilon/2T)}{\varepsilon}. \quad (14)$$

The expression (14) coincides with one found earlier in[9].

Let us consider the junction S-S_{meas}. Proceeding in the same way as in the determination of the source J_{S-S} in[9], we can derive from the formula (5) of[9] the following expression for the phase difference $\delta\chi$ for the order parameter in S and S_{meas}:

$$\begin{aligned}-\int d\xi\frac{d\omega}{2\pi}\left[\Sigma_{12}{}^R(\omega)F_0{}^*(-\omega)+\Omega_{12}(\omega)F_0{}^{A*}(-\omega)\right]\sin\delta\chi\\=\operatorname{Re}\int d\xi\frac{d\omega}{2\pi}\{\Sigma^R(\omega)G(\omega)-\Omega(\omega)G^A(\omega)-\Sigma_{12}{}^R(\omega)F^*(-\omega)\\+\Omega_{12}(\omega)F^{A*}(-\omega)\},\end{aligned} \quad (15)$$

where in the expression on the left-hand side of the equality figure the equilibrium functions F_0, while in the expression on the right figure the nonequilibrium functions G and F. The integral on the left-hand side is proportional to the Josephson current. Let us denote it by I_J. Let us express the functions G and F figuring in the expression on the right-hand side of (15) in terms of Im G^R and Im F^R and take into account the fact that the integral of the second term in the curly brackets vanishes on account of the fact that this term is an odd function of ω. Carrying out the computations, we find

$$-I_J\sin\delta\chi=2\nu\int d\xi\, n_1(\xi)\operatorname{sgn}\xi. \quad (16)$$

Thus, the asymmetry of the quasiparticle distribution leads to the appearance in the system S-S_{meas} of a current that is canceled by the pair current. As a result of this, the phase difference $\delta\chi$ arises and, furthermore, a potential difference exists between S and S_{meas}, since in computing (15) we assumed the potential of S_{meas} to be equal to zero. The observable voltage potential is made up of Φ and $\mathscr{E}_1$: $\mathscr{E}=\mathscr{E}_1+\Phi$.

Let us find the dependence $\mathscr{E}(V, T)$ for $T\to T_c$. It follows from (14) that, near the critical temperature, $\mathscr{E}_1\sim(\Delta/T)^2\Phi\ll\Phi$, and therefore the observable voltage potential is due to the potential difference between S and S_{meas}. In the expression (9′) for $\nu_Q(\epsilon)$ only the first and second terms are important near T_c. Computing them, we obtain

$$\nu_Q(\varepsilon)=\pi^2\zeta_{ph}(\varepsilon^2\Delta/\Theta_D{}^2)\operatorname{cth}(\varepsilon/2T). \quad (17)$$

The expression (17) coincides up to a numerical factor with the expression found by Tinkham[2]. In the integral (9), which contains $\nu_Q(\epsilon)$, the characteristic scale of the variation of n_1 for $T\to T_c$ is the temperature; therefore, ν_Q is determined by the expression (17) with the energy ϵ replaced by T^*, where the "temperature" T^* is of the order of T and should be determined from the exact solution to the kinetic equation (8). From Eq. (7) we find the relation between Φ and Q^*:

$$\Phi=\frac{\pi}{pm}Q^*\equiv\int d\xi\, n_1\frac{\xi}{\varepsilon}=-4(\nu/\nu_Q)V.$$

Substituting ν_Q from (17) into this expression, we finally find

$$\mathscr{E}=-\frac{4\nu}{\pi^2\zeta_{ph}}\left(\frac{T^*}{\Delta}\right)\left(\frac{\Theta_D}{T^*}\right)^2\operatorname{th}\left(\frac{T^*}{2T}\right)V.$$

The dependence of $\mathscr{E}$ on (T_c-T) for $\Delta\ll T$ agrees qualitatively with the corresponding dependence observed experimentally in the case of Sn[3].

2. THE SYSTEM N_1-N-N_2

The system (Fig. 1) in question also allows us to investigate the mechanism responsible for energy relaxation in the case of a normal metal. Then instead of the system S′-S-N, we should use the system N_1-N-N_2, where N_1 and N_2 may be of the same metal: It is only necessary that the frequencies ν_1 and ν_2, which are proportional to the product of the density of states of the metal N_1 (N_2) and the matrix element for tunneling through the N_1-N (N_2-N) junction, differ from each other. Then in the presence of a current flowing through the system N_1-N-N_2 the distribution function in N will become asymmetric in ξ, although, as follows from the neutrality condition, the total number of electrons will remain equal to the total number of holes. In the measuring circuit will then arise a potential difference $\mathscr{E}$. In fact, using Eq. (7) of[9] and the expression for the self-energy part describing the tunneling from S_{meas} into N,

$$\Sigma^R=-i\nu|\omega|[\omega^2-\Delta^2]^{-1/2}\theta(|\omega|-\Delta),$$

we obtain for the voltage potential $\mathscr{E}$ in the S_{meas}-N circuit the equation

$$\int d\xi\left(\operatorname{th}\frac{\varepsilon+\mathscr{E}}{2T}-\operatorname{th}\frac{\varepsilon-\mathscr{E}}{2T}\right)=-2\int d\xi\frac{\xi n_1(\xi)}{(\xi^2-\Delta^2)^{1/2}}\theta(|\xi|-\Delta), \quad (18)$$

where $\epsilon=(\xi^2+\Delta^2)^{1/2}$ and n_1 is the deviation of the distribution function in N from the equilibrium distribution function[2)]. It can be seen from (18) that if the measuring electrode is a normal metal, then the integral on the right-hand side (and, consequently, $\mathscr{E}$) will vanish because of the electrical-neutrality condition.

Let us find the quantity n_1. Let a current flow through the system, so that we have established at the N_1-N and N_2-N junctions the voltage potentials V_1 and V_2 respectively. The kinetic equation for n_1 has the form

$$\operatorname{sgn}\xi\frac{\partial n_1}{\partial t}=-\sum_{k=1,2}\nu_k\left(\operatorname{th}\frac{\xi+V_k}{2T}-\operatorname{th}\frac{\xi}{2T}\right)+I_{st}(n_1), \quad (19)$$

where the second term on the right-hand side is the source due to the injection of quasiparticles from N_1 and N_2, the frequencies ν_k are connected with the resistances of the junctions N_k-N[9], and I_{st} is the linearized collision integral for collisions with phonons. We shall restrict ourselves to the case of low temperatures ($T\ll\Delta$) and shall assume that $V_k<\Theta_D$. After integration over the angles Eq. (19) in the steady-state case assumes the form

$$\begin{aligned}\frac{1}{3}\xi^3n_1(\xi)-\theta(\xi)\int_\xi^\infty d\omega(\omega-\xi)^2n_1(\omega)+\theta(-\xi)\int_{-\infty}^{\xi}d\omega(\omega-\xi)^2n_1(\omega)\\=\frac{2\Theta_D{}^2}{\pi\zeta_{ph}}[\nu_1\theta(\xi)\theta(|V_1|-\xi)-\nu_2\theta(-\xi)\theta(\xi+V_2)],\end{aligned} \quad (20)$$

where $V_1<0$ and $V_2>0$. We seek the solution in the form

$$n_1(\xi)=n_>\theta(\xi)\theta(|V_1|-\xi)+n_<\theta(-\xi)\theta(\xi+V_2). \quad (21)$$

Then for the electron distribution function we obtain the equation

$$\frac{1}{3}\xi^3n_>-\int_\xi^{V_1}d\omega(\omega-\xi)^2n_>(\omega)=\varkappa_1\equiv\frac{2\Theta_D{}^2}{\pi\zeta_{ph}}\nu_1,\quad \xi>0. \quad (22)$$

For the hole distribution function we obtain an equation coinciding with (22) if we make the substitutions $\xi \to -\xi$, $V_1 \to V_2$, and $\varkappa_1 \to \varkappa_2$ in the latter. Differentiating (22) three times with respect to ξ, we obtain

$$\frac{1}{3}\frac{\partial^3(\xi^3 n_>)}{\partial\xi^3}+2n_>=0. \quad (23)$$

The solution to (23) is the power function

$$n_>\xi^3=C_1\xi^{-1}+\mathrm{Re}\,(C_2+iC_3)\,\xi^{2+i\sqrt{2}}.$$

Let us determine the constants from the boundary conditions

$$n_>(V_1)=3\varkappa_1|V_1|^{-3},\ (n_>\xi^3)_\xi'=(n_>\xi^3)_\xi''=0 \text{ for } \xi=|V_1|,$$

which follow from Eq. (22) and its derivatives. The solution then assumes the form[3)]

$$n_>=\frac{18}{11}\frac{\varkappa}{|V_1|^3}\left\{\left(\frac{V_1}{\xi}\right)^4-\frac{\sqrt{33}}{6}\frac{|V_1|}{\xi}\sin\left(\sqrt{2}\ln\frac{\xi}{|V_1|}-\varphi_0\right)\right\}, \quad (24)$$

where $\sin\varphi_0 = 5/\sqrt{33}$.

Let us substitute the solutions $n_>(\xi, \varkappa_1)$ and $n_<(\xi, \varkappa_2) = n_>(-\xi, \varkappa_2)$ into (21) and n_1 from (21) into (18). Then, assuming that $E < T$, we obtain

$$\mathscr{E}=\frac{1}{2\sqrt{2\pi}}\left(\frac{T}{\Delta}\right)^{1/2}e^{\Delta/T}\left\{\int_\Delta^{V_2}d\xi\frac{\xi n_>(\xi,\varkappa_2)}{(\xi^2-\Delta^2)^{1/2}}-\int_\Delta^{|V_1|}d\xi\frac{\xi n_>(\xi,\varkappa_1)}{(\xi^2-\Delta^2)^{1/2}}\right. \quad (25)$$

$$=-(9/22\sqrt{2\pi})\,(T/\Delta)^{1/2}e^{\Delta/T}(\varkappa_1/\Delta^2)\mathscr{F}(\alpha,\gamma),$$

where

$$\mathscr{F}(\alpha,\gamma)=\frac{\sqrt{\alpha^2-1}}{\alpha^2}-\sqrt{(\alpha/\gamma)^2-1}\frac{\gamma^2}{\alpha^2}+\arccos\frac{1}{\alpha}$$
$$-\arccos\frac{\gamma}{\alpha}-\frac{\sqrt{33}}{3\alpha^2}\int_1^\alpha dx\,(x^2-1)^{-1/2}\sin\left(\sqrt{2}\ln\frac{x}{\alpha}-\varphi_0\right)$$
$$+\frac{\sqrt{33}\gamma^2}{3\alpha^2}\int_1^{\alpha/\gamma}dx\,(x^2-1)^{-1/2}\sin\left(\sqrt{2}\ln\frac{x\gamma}{\alpha}-\varphi_0\right),$$

$\alpha = |V_1|/\Delta$, $\gamma = \varkappa_2/\varkappa_1 = R_1/R_2$, and $R_{1,2}$ are the resistances of the junctions $N_{1,2}$-N. In this case in deriving (25) we used the neutrality condition $\nu_1 V_1 = \nu_2 V_2$, which follows from Eq. (19) if we integrate this equation (in the steady-state case) over ξ.

Let us give the asymptotic expressions for $\mathscr{F}(\alpha, \gamma)$, in the case when $\gamma \gg 1$:

$$\mathscr{F}(\alpha,\gamma)=2\sqrt{\alpha^2-1} \text{ при } \alpha\to 1,$$
$$\mathscr{F}(\alpha,\gamma)=\alpha[\pi/2-{}^{11}/_3\sqrt{(\alpha/\gamma)^2-1}] \text{ при } \alpha\to\gamma.$$

The form of the function (α, γ), obtained by a numerical integration, is shown in Fig. 2 for several values of γ. Let us estimate the magnitude of the effect, setting $\Theta_D/\Delta \sim 10^2$, $\nu_1 \sim 10^6\ \mathrm{sec}^{-1}$[9], and $\mathscr{F}(\alpha, \gamma) \sim 1$. We obtain $\mathscr{E} \gtrsim 10\sqrt{T/\Delta}\,e^{\Delta/T}\ \mu V$. Thus, by measuring the function $\mathscr{E}(V_1)$ we can make judgments about the distribution function of the nonequilibrium electrons and about the mechanism responsible for its relaxation. Allowance for the electron-electron collisions leads to the appearance in (20) of terms of the type $(\xi^2/\epsilon_F)n_1$. These terms can be neglected provided $\Delta \gg \Theta_D^2/\epsilon_F$. The flopover processes, which are neglected by us, will be unimportant if the Fermi surface does not get close to the Brillouin-zone boundaries.

3. CONCLUSION

Thus, the above-considered system allows us to investigate the asymmetry of the populations of the energy-spectrum branches of a superconductor and, in particular, determine the important characteristic τ_Q, the time for the establishment of equilibrium between the branches. This time determines the attenuation length of the longitudinal electric field in the superconductor. Notice that the gap $\delta\Delta$ also changes during the tunnel injection. The change in the gap is due to the first term on the left-hand side of (6). The time characterizing the establishment of the steady-state value of Δ is less (near T_c) than τ_Q and coincides in order of magnitude with the energy-relaxation time in the normal metal[13]. The change in Δ does not (so long as it is small) affect the magnitude of Φ, since $\delta\Delta$ does not enter into Eq. (7).

The experimental investigation of the characteristic function $\mathscr{E}(V_1, T)$ in the case of the system N_1-N-N_2 is also of interest, since such a dependence allows us to draw some conclusions about the mechanism responsible for energy relaxation in normal metals.

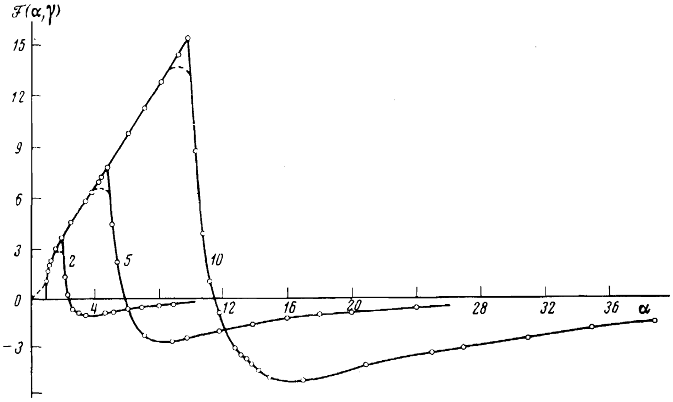

FIG. 2. Dependence of F (α, γ) on α for γ = 2, 5, and 10.

1)The potential Φ also arises near the core of a moving vortex [7,8].

2)The quantity n_1 is an electron distribution function for $\xi > 0$ and a hole distribution function for $\xi < 0$.

3)This solution diverges at small ξ. To obtain a finite $n_>$ at $\xi = 0$, we may take into account either the induced transitions or the nonlinear terms in I_{st} and in the generation terms in (20). We are, however, not interested in the region of small ξ, since the contribution to (18) is made by $\xi \geqslant \Delta$. Notice that in the case of the model matrix element $g^2 \sim q^{-2}$ of the interaction with the phonons, it is possible to solve exactly the nonlinear kinetic equation with allowance for recombination of nonequilibrium electrons and holes. It then turns out that the exact distribution function differs from the approximately determined function at energies $\xi \lesssim (\nu_1 V_1)^{1/2}$. In the case of the matrix element used by us the nonlinear effects are important at $\xi \lesssim (\Theta_D^2\nu_1 V_1)^{1/4}$.

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