Theory of damping of magnetohydrodynamic waves in a high-temperature plasma

A. I. Akhiezer, V. I. Lapshin, and K. N. Stepanov

Khar'kov State University
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Absorption of electromagnetic waves with a frequency lower than or of the order of the cyclotron frequency of ions in a weakly collisional plasma whose pressure is small compared with the magnetic pressure is considered. Allowance is made for absorption of waves due to Cerenkov interaction between the waves and electrons, to friction between ions of various types and between electrons and ions, to electron viscosity, and to ion viscosity. The role of these processes in high-frequency plasma heating is discussed.

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1. INTRODUCTION

The idea of using high-frequency electromagnetic fields, in particular magnetohydrodynamic (MHD) waves for heating purposes to obtain thermonuclear temperatures was advanced and investigated many times.[1–13] The plasma heating is due in this case to the absorption of the MHD waves, the theory of which was developed in a number of papers (see[14–16] and the literature cited there). If a collisionless or strong-collision plasma is considered, then the theory of the absorption of such waves can be regarded as completely developed.[14–16] As to the allowance for the role of collisions in a high-temperature plasma, this process calls for additional study.

The present paper is devoted to a systematic investigation of the role of collisions in the absorption of MHD waves in a high-temperature plasma. The ratio β of the plasma pressure to the magnetic pressure is assumed to be low. We consider waves at frequencies lower than or of the order of the ion cyclotron frequencies, propagating at an arbitrary angle to the magnetic field. It is assumed that the plasma can contain ions of various sorts.

The following absorption mechanisms turn out to be essential in this case: 1) Cerenkov absorption of waves by the electrons, 2) absorption due to the friction force between ions of different sorts and between electrons and ions, 3) absorption connected with electron viscosity (with the gyrorelaxation effect for electrons), 4) absorption connected with ion viscosity (in particular, with the gyrorelaxation effect for the ions). Each effect plays a different role, depending on the relation between the parameters $\cos\theta$, β, m_e/m_i (m_e and m_i are the masses of electrons and ions, and θ is the angle between the direction of propagation of the wave and the magnetic field).

In the important particular case of the propagation of waves perpendicular to the magnetic field ($\theta=\pi/2$), the principal role is played by the electron viscosity and by the friction force between the ions of different sorts. In the case "oblique" propagation (θ not close to $\pi/2$), the principal role is played by Cerenkov absorption and by the force of friction between the ions. In a two-component plasma, the ion viscosity may turn out to be appreciable.

In the case of a two-component plasma, the damping of fast magnetosonic waves at $\theta=\pi/2$ was investigated earlier.[13,17] In[17] they determined the damping due to the electron viscosity, and the resultant heating of the plasma at high frequencies $\omega \gg \nu_{ei}$ (ν_{ei} is the frequency of the collisions between the electrons and the ions). In a paper devoted to the theory of plasma heating by these waves,[13] the damping due only to ion viscosity was determined in the frequency region $\omega \gg \nu_{ii} \sim \nu_{ei}(m_e/m_i)^{1/2}$. This analysis turned out to be valid in a narrow frequency range $(m_i/m_e)^{1/4}\nu_{ii} \gtrsim \omega \gg \nu_{ii}$; at $\nu_{ei} \gtrsim \omega \gtrsim (m_i/m_e)^{1/4}\nu_{ii}$, an appreciable contribution to the absorption is made by the electron viscosity, which can be taken into account in the hydrodynamic approximation.[16]

In the investigation of the damping of MHD waves, we start from the kinetic equations for the various plasma components; solution of these equations makes it possible to find the dielectric tensor of the plasma. Knowledge of this tensor enables us, in turn, to obtain from the dispersion equation the refractive index and the damping coefficient of the MHD waves with allowance for all the absorption mechanisms listed above.

As to heating, in the thermonuclear devices with small values of β, the absorption of MHD waves is weak. The principal role at high temperatures ($T\sim 10$ keV) is played by electron Cerenkov absorption, and during the initial stage of the heating ($T=0.1$–1 keV) absorption due to the friction between the ions of different sorts and due to electron viscosity may be significant. In traps with high plasma density (with very small β), the absorption of MHD waves is much stronger than in traps with small β. In this case, besides the Cerenkov absorption, an important role is played also by collision mechanisms—the electron viscosity and the force of friction between the ions.

2. DIELECTRIC TENSOR OF PLASMA

We assume that the phase velocity of the waves in question is large in comparison with the ion thermal velocity $v_{Ti}(\omega/k_\parallel \gg v_{Ti})$, and the transverse wavelength is much larger than the Larmor radius of the particles ($k_\perp v_{Ta}/\omega_{Ba} \ll 1$). In this case the dispersion equation for the considered electromagnetic waves ($\omega \lesssim \omega_{Bi}$) in a plasma with $\beta_a = 4\pi n_a T/B^2 \ll 1$ takes the form[7,14]

$$\begin{gathered}\cos^2\theta N^4 - (\varepsilon_{11}+\varepsilon_{22}\cos^2\theta)N^2+\varepsilon_{11}\varepsilon_{22}+\varepsilon_{12}^2 \\ = -(1/\varepsilon_{33})\{\varepsilon_{11}\sin^2\theta N^4+\varepsilon_{11}\varepsilon_{23}^2+[2\varepsilon_{12}\varepsilon_{23}\sin\theta\cos\theta \\ -\varepsilon_{23}^2\cos^2\theta-(\varepsilon_{11}\varepsilon_{22}+\varepsilon_{12}^2)\sin^2\theta]N^2\},\end{gathered} \tag{2.1}$$

where $N = ck/\omega$ is the complex refractive index, $\mathbf{k}$ is the wave vector, $k_\parallel = k\cos\theta$ and $k_\perp = k\sin\theta$ are its components parallel and perpendicular to the magnetic field $\mathbf{B}$, $\omega_{Ba} = e_a B/m_a c$ is the cyclotron frequency of particles of type a with charge e_a and mass m_a, ε_{ij} are the components of the dielectric tensor in a reference frame with Z axis parallel to the field $\mathbf{B}$ and X axis lying in the plane of the vectors $\mathbf{k}$ and $\mathbf{B}$. All the terms in the right-hand side of (2.1) are small in comparison with the principal terms in the left-hand side.

To determine the tensor ε_{ij} we start from a linearized kinetic equation for a small deviation of the distribution function $f_a^\sim$ of the particles of sort a from the equilibrium (Maxwellian) function f_{a0}:

$$\begin{gathered}\hat{L}f_a^\sim \equiv -i(\omega-\mathbf{kv}_a)f_a^\sim - \omega_{Ba}\frac{\partial f_a^\sim}{\partial\phi} \\ = \frac{e_a}{T}(\mathbf{Ev}_a)f_{a0}+\sum_b S_{ab}(f_{a0}+f_a^\sim, f_{b0}+f_b^\sim),\end{gathered} \tag{2.2}$$

where ϕ is the azimuthal angle in the space of the velocities $\mathbf{v}_a$. The collision integral S_{ab} contained in this expression is defined by[18]

$$S_{ab}(f_a, f_b) = 2\pi e_a^2 e_b^2 L\frac{\partial}{\partial p_{ai}}\int\left(\frac{\partial f_a}{\partial p_{aj}}f_b - \frac{\partial f_b}{\partial p_{bj}}f_a\right)\frac{u^2\delta_{ij}-u_i u_j}{u^3}d^3p_b. \tag{2.3}$$

where $\mathbf{u} = \mathbf{v}_a - \mathbf{v}_b$ and L is the Coulomb logarithm. In order of magnitude $S_{ab} \sim \nu_{ab} f_a^\sim$, where ν_{ab} is the frequency of the collisions of particles of sort a with particles of sort b. In the region of rare collisions, the only one which we shall consider, taking into account the inequality $\nu_{ab} \ll \max(\omega, k_\parallel v_{Ta})$, we can seek the solution of (2.2) in the form of the expansion $f_a^\sim = f_a^{(0)\sim} + f_a^{(1)\sim} + \ldots$, where $f_a^{(0)\sim}$ and $f_a^{(1)\sim}(|f_a^{(1)\sim}| \ll |f_a^{(0)\sim}|)$ are determined from the equations

$$\hat{L}f_a^{(0)\sim} = \frac{e_a}{T}(\mathbf{Ev}_a)f_{a0}, \tag{2.4}$$

$$\hat{L}f_a^{(1)\sim} = \sum_b S_{ab}(f_{a0}+f_a^{(0)\sim}, f_{b0}+f_b^{(0)\sim}). \tag{2.5}$$

Solving (2.4) and (2.5), we can easily obtain the density of the electric current in the plasma j and the dielectric tensor ε_{ij}:

$$j_i = \frac{\omega}{4\pi i}(\varepsilon_{ij}-\delta_{ij})E_j = \sum_a\int v_{ai}(f_a^{(0)\sim}+f_a^{(1)\sim})d^3p_a.$$

The tensor ε_{ij} can be represented in the form of the sum

$$\varepsilon_{ij} = \varepsilon_{ij}^{(0)}+\varepsilon_{ij}^{(c)}+\varepsilon_{ij}^{(e)}+\varepsilon_{ij}^{(i)}, \tag{2.6}$$

for $\varepsilon_{ij}^{(0)}$ is the dielectric tensor of the plasma in the absence of collisions and is determined by the function $f_a^{(0)\sim}$, while the terms $\varepsilon_{ij}^{(c)}$, $\varepsilon_{ij}^{(e)}$, $\varepsilon_{ij}^{(i)}$ take into account the presence of collisions in the plasma and are determined by the perturbed distribution function $f_{a\sim}^{(1)}$. The term $\varepsilon_{ij}^{(c)}$ takes into account the contribution of the collisions between the plasma particles corresponding to introduction into the equation of motion the force of friction between the particles in a direction perpendicular to the magnetic field, while the term $\varepsilon_{ij}^{(e)}$ takes into account the force of friction between the electrons and the ions in the magnetic field direction and the electron viscosity, and the term $\varepsilon_{ij}^{(i)}$ accounts for the ion viscosity (the last two terms take into also the gyrorelaxation effect[1)]).

The form of the tensor $\varepsilon_{ij}^{(0)}$ is known[7,14]:

$$\begin{gathered}\varepsilon_{11}^{(0)} = \varepsilon_1 \equiv -\sum_i\frac{\omega_{pi}^2}{\omega^2-\omega_{Bi}^2}, \\ \varepsilon_{12}^{(0)} = i\varepsilon_2 \equiv i\frac{\omega_{pe}^2}{\omega\omega_{Be}} - i\sum_i\frac{\omega_{pi}^2\omega_{Bi}}{\omega(\omega^2-\omega_{Bi}^2)}, \\ \varepsilon_{22}^{(0)} = \varepsilon_1+\varepsilon_{22}' = \varepsilon_1 + i\pi^{1/2}\frac{\omega_{pe}^2}{\omega_{Be}^2}\operatorname{tg}^2\theta\frac{w(z_e)}{z_e}, \\ \varepsilon_{23}^{(0)} = i\frac{\omega_{pe}^2}{\omega\omega_{Be}}\operatorname{tg}\theta\left[1+i\pi^{1/2}z_e w(z_e)\right], \quad \varepsilon_{33}^{(0)} = \frac{\omega_{pe}^2}{k_\parallel^2 v_{Te}^2}\left[1+i\pi^{1/2}z_e w(z_e)\right],\end{gathered} \tag{2.7}$$

where

$$\omega_{pa} = (4\pi e_a^2 n_a/m_a)^{1/2}, \quad v_{Ta} = (T/m_a)^{1/2},$$

$$w(z) = e^{-z^2}\left[\operatorname{sgn}k_\parallel + \frac{2i}{\pi^{1/2}}\int_0^z e^{t^2}dt\right], \quad z_e = \frac{\omega}{2^{1/2}k_\parallel v_{Te}}.$$

In the expressions for ε_1 and ε_2, the summation is over all the sorts of ions. Inasmuch as in the considered case of a low-pressure plasma the phase velocity of the waves, which is of the order of the Alfven velocity, is much larger than the thermal velocity of the ions, we have neglected in $\varepsilon_{ij}^{(0)}$ the exponentially small anti-Hermitian parts, which take into account the Cerenkov absorption of the waves by the ions.

The components $\varepsilon_{ij}^{(c)}$ are equal to

$$\begin{gathered}\varepsilon_{11}^{(c)} = \varepsilon_{22}^{(c)} = i\varepsilon_1' = i\sum_{a,b}\frac{\omega_{pa}^2\nu_{ab}}{\omega(\omega^2-\omega_{Ba}^2)}\left(\frac{\omega^2+\omega_{Ba}^2}{\omega^2-\omega_{Ba}^2} - \frac{e_b m_a}{e_a m_b}\frac{\omega^2+\omega_{Ba}\omega_{Bb}}{\omega^2-\omega_{Bb}^2}\right), \\ \varepsilon_{12}^{(c)} = \varepsilon_2' \equiv -\sum_{a,b}\frac{\omega_{pa}^2\nu_{ab}}{\omega^2-\omega_{Ba}^2}\left(\frac{2\omega_{Ba}}{\omega^2-\omega_{Ba}^2} - \frac{e_b m_a}{e_a m_b}\frac{\omega_{Ba}+\omega_{Bb}}{\omega^2-\omega_{Bb}^2}\right)\end{gathered} \tag{2.8}$$

where

$$\nu_{ab} = \frac{4(2\pi)^{1/2}e_a^2 e_b^2 n_b L}{3(m_a T^3)^{1/2}}\left(\frac{m_b}{m_a+m_b}\right)^{1/2}.$$

In (2.8), the summation is over all these sorts of particles.

The values of $\varepsilon_{ij}^{(e)}$ are

$$\varepsilon_{22}^{(e)}=i\frac{\omega_{pe}^2\nu_{ee}k_\perp^2}{\omega_{Be}^2\omega k_\parallel^2}\left[I_{22}^{(e)}(z_e')+\sum_i\frac{e_i^2n_i}{e^2n_e}I_{22}^{(i)}(z_e)\right].$$

$$\varepsilon_{23}^{(e)}=\frac{\omega_{pe}^2\nu_{ee}k_\perp}{\omega_{Be}k_\parallel^3v_{Te}^2}\left[I_{23}^{(e)}(z_e')+\sum_i\frac{e_i^2n_i}{e^2n_e}I_{23}^{(i)}(z_e)\right]. \qquad (2.9)$$

$$\varepsilon_{33}^{(e)}=-i\frac{\omega_{pe}^2\nu_{ee}}{k_\parallel^3v_{Te}^3}\left[I_{33}^{(e)}(z_e')+\sum_i\frac{e_i^2n_i}{e^2n_e}I_{33}^{(i)}(z_e)\right].$$

where

$$I_{22}^{(e)}(z_e')=\frac{3}{4\pi^{5/2}}\iint\frac{d^3\xi\,d^3\eta}{\xi^3}\frac{(\eta_y-\xi_y)^2}{z_e'-\eta_z+\xi_z}\exp(-\eta^2-\xi^2)$$
$$\times\left\{\left(\xi^2-3\xi_z^2+2\eta_z\xi_z\xi^2-2\xi\eta\xi\right)\frac{1}{z_e'-\eta_z+\xi_z}+\frac{2\xi_z(\eta_\perp\xi_\perp-\xi_\perp^2)}{(z_e'-\eta_z+\xi_z)^2}\right.$$
$$-\frac{\xi_\perp^2[(\eta_x-\xi_x)^2+(\eta_y-\xi_y)^2]}{(z_e'-\eta_z+\xi_z)^3}+[\xi^2(\eta_\perp^2+\eta_\perp\xi_\perp)-\eta\xi(\xi_\perp^2+\eta_\perp\xi_\perp)-\xi_\perp^2-\eta_\perp\xi_\perp]$$
$$\times\left(\frac{1}{z_e'-\eta_z+\xi_z}-\frac{1}{z_e'-\eta_z-\xi_z}\right)+\frac{1}{2}(\xi^2\eta_z-\eta\xi\xi_z-\xi_z)\left[\frac{(\eta_x-\xi_x)^2+(\eta_y-\xi_y)^2}{(z_e'-\eta_z+\xi_z)^2}\right.$$
$$\left.\left.-\frac{(\eta_x+\xi_x)^2+(\eta_y+\xi_y)^2}{(z_e'-\eta_z\xi_z^2)^2}\right]\right\}.$$

$$I_{22}^{(i)}(z_e)=\frac{3\sqrt{2}}{4\pi}\int\frac{d^3\xi}{\xi^3}\frac{\xi_y^2}{(z_e-\xi_z)^2}\exp(-\xi^2)$$
$$\times\left[3\xi_\perp^2-2\xi^2+\frac{3\xi_z\xi_\perp^2}{z_e-\xi_z}-\frac{\xi_\perp^4}{(z_e-\xi_z)^2}\right].$$

$$I_{23}^{(e)}(z_e')=\frac{3}{4\pi^{5/2}}\iint\frac{d^3\xi\,d^3\eta}{\xi^3}(\eta_y-\xi_y)^2\exp(-\eta^2-\xi^2)F_1(z_e',\eta,\xi),$$

$$I_{23}^{(i)}(z_e)=\frac{3\sqrt{2}}{4\pi}\int\frac{d^3\xi}{\xi^3}\xi_y^2\exp(-\xi^2)F_2(z_e,\xi),$$

$$I_{33}^{(e)}(z_e')=\frac{3}{4\pi^{5/2}}\iint\frac{d^3\xi\,d^3\eta}{\xi^3}(\eta_z-\xi_z)\exp(-\eta^2-\xi^2)F_1(z_e',\eta,\xi),$$

$$I_{33}^{(i)}(z_e)=\frac{3}{4\pi}\int\frac{d^3\xi}{\xi^3}\xi_z\exp(-\xi^2)F_2(z_e,\xi),$$

$$F_1(z_e',\eta,\xi)=\frac{1}{z_e'+\xi_z-\eta_z}\left\{(\xi_z-\eta_z\xi^2+\xi_z\eta\xi)\right.$$
$$\left.\times\left[\frac{1}{(z_e'-\eta_z+\xi_z)^2}-\frac{1}{(z_e'-\eta_z-\xi_z)^2}\right]+\frac{2\xi_\perp^2}{(z_e'-\eta_z+\xi_z)^3}\right\}.$$

$$F_2(z_e,\xi)=\frac{1}{(z_e-\xi_z)^3}\left(\frac{\xi_\perp^2}{z_e-\xi_z}-\xi_z\right),$$

$$z_e'=2^{1/2}z_e,\quad \eta_\perp^2=\eta_y^2+\eta_x^2,\quad \xi_\perp^2=\xi_x^2+\xi_y^2,\quad \eta_\perp\xi_\perp=\eta_x\xi_x+\eta_y\xi_y.$$

In $I_{ij}^{(e,i)}$ the integration with respect to η_z and ξ_z is carried out along the real axis, with the singular point at which $z_e=\xi_z$ or $z_e'=\eta_z\pm\xi_z$, circled from below at $k_\parallel>0$ and from above at $k_\parallel<0$. The terms proportional to $I_{ij}^{(e)}$ take into account the collisions of the electrons with one another, while the terms proportional to $I_{ij}^{(i)}$ take into account the collisions of the electrons with the ions.

At $|z_e|\gg1$, the expressions for $\varepsilon_{ij}^{(e)}$ take the form

$$\varepsilon_{22}^{(e)}=i\frac{2}{5}(1+\lambda\sqrt{2})\frac{\omega_{pe}^2\nu_{ee}k_\perp^2v_{Te}^2}{\omega_{Be}^2\omega^3},\quad \varepsilon_{23}^{(e)}=\frac{2}{5}(2+\lambda\sqrt{2})\frac{\omega_{pe}^2\nu_{ee}k_\parallel k_\perp v_{Te}^2}{\omega_{Be}\omega^4}$$

$$\varepsilon_{33}^{(e)}=i\left(\frac{8}{5}\frac{k_\parallel^2v_{Te}^2}{\omega^2}+\lambda\sqrt{2}\right)\frac{\omega_{pe}^2\nu_{ee}}{\omega^3}.$$

$$\lambda=\sum_i(e_i/e)^2(n_i/n_e). \qquad (2.10)$$

The component $\varepsilon_{22}^{(e)}$ coincides with the expression obtained at $\theta=\pi/2$ in the case of a two-component plasma in[17],[2] The first and second terms (in the right-hand side of (2.10)) take into account the electron-electron and electron-ion collisions, respectively.

At $|z_e|\ll1$, the order of magnitude of $\varepsilon_{ij}^{(e)}$ is given by

$$\varepsilon_{22}^{(e)}\sim i\frac{\omega_{pe}^2\nu_{ee}}{\omega_{Be}^2\omega}\operatorname{tg}^2\theta,\quad \varepsilon_{23}^{(e)}\sim\frac{\omega_{pe}^2\nu_{ee}}{\omega_{Be}k_\parallel^2v_{Te}^2}\operatorname{tg}\theta,$$
$$\varepsilon_{33}^{(e)}\sim i\frac{\omega_{pe}^2\nu_{ee}\omega}{k_\parallel^4v_{Te}^4}. \qquad (2.11)$$

The quantities $\varepsilon_{ij}^{(i)}$ are equal to

$$\varepsilon_{11}^{(i)}=i\frac{2}{5}\frac{\omega_{pi}^2\nu_{ii}v_{Ti}^2}{\omega(\omega^2-\omega_{Bi}^2)^2}\left[3k_\parallel^2\frac{(\omega^2+\omega_{Bi}^2)^2+4\omega^2\omega_{Bi}^2}{(\omega^2-\omega_{Bi}^2)^2}\right.$$
$$\left.+k_\perp^2\frac{4\omega^4+31\omega^2\omega_{Bi}^2+28\omega_{Bi}^4}{(\omega^2-4\omega_{Bi}^2)^2}\right].$$

$$\varepsilon_{12}^{(i)}=-\frac{2}{5}\frac{\omega_{pi}^2\omega_{Bi}\nu_{ii}v_{Ti}^2}{(\omega^2-\omega_{Bi}^2)^2}\left[12k_\parallel^2\frac{\omega^2+\omega_{Bi}^2}{(\omega^2-\omega_{Bi}^2)^2}\right.$$
$$\left.+k_\perp^2\frac{19\omega^4+28\omega^2\omega_{Bi}^2+16\omega_{Bi}^4}{\omega^2(\omega^2-4\omega_{Bi}^2)^2}\right]. \qquad (2.12)$$

$$\varepsilon_{22}^{(i)}=i\frac{2}{5}\frac{\omega_{pi}^2\nu_{ii}v_{Ti}^2}{\omega(\omega^2-\omega_{Bi}^2)^2}\left[3k_\parallel^2\frac{(\omega^2+\omega_{Bi}^2)^2+4\omega^2\omega_{Bi}^2}{(\omega^2-\omega_{Bi}^2)^2}\right.$$
$$\left.+k_\perp^2\frac{3\omega^6+40\omega^4\omega_{Bi}^2+4\omega^2\omega_{Bi}^4+16\omega_{Bi}^6}{\omega^2(\omega^2-4\omega_{Bi}^2)^2}\right].$$

The expressions for $\varepsilon_{11}^{(i)}$ and $\varepsilon_{22}^{(i)}$ (2.12) at $\theta=0$ and $\omega\ll\omega_{Bi}$ coincide with those obtained in[17]. The formulas for $\varepsilon_{ij}^{(i)}$ obtained in[15,19] for the case $\omega\ll\omega_{Bi}$ contain some errors: a term $\propto k^2$ is missing from the formula for $\varepsilon_{11}^{(i)}$, and the expressions for $\varepsilon_{22}^{(i)}$ and $\varepsilon_{23}^{(i)}$ should be multiplied respectively by $\frac{1}{2}$ and -1.

3. DAMPING OF MHD WAVES

We proceed to an analysis of the dispersion equation (2.1). In the zeroth approximation, discarding small terms, we can put $\varepsilon_{11}=\varepsilon_{22}=\varepsilon_1$ and $\varepsilon_{12}=i\varepsilon_2$; as a result we obtain

$$\cos^2\theta N^4-\varepsilon_1(1+\cos^2\theta)N^2+\varepsilon_1^2-\varepsilon_2^2=0. \qquad (3.1)$$

This equation, at a given frequency, determines two refractive indices $N=N(\omega,\theta)$, the order of magnitude of which is

$$N\sim N_A=\frac{c}{v_A}=\left(\sum_i\omega_{pi}^2/\omega_{Bi}^2\right)^{1/2}. \qquad (3.2)$$

For a given wave vector $\mathbf{k}$, Eq. (3.1) determines the $(q+1)$st frequency $\omega=\omega_j(k,\theta)$, where $j=1,2,\ldots,q+1$, q is the number of ions of different sorts.

Taking into account the small terms proportional to

$\varepsilon_{ij}^{(c)}$, $\varepsilon_{ij}^{(e)}$, $\varepsilon_{ij}^{(i)}$, and the small terms in the right-hand side, we find, putting $N=N(\omega,\theta)+i\varkappa(\omega,\theta)$ ($\varkappa \ll N$), the following expression for the damping coefficient[3)]:

$$\varkappa=\varkappa_{fr}+\varkappa_v^{(i)}+\varkappa', \tag{3.3}$$

where

$$\varkappa_{fr}=\frac{\varepsilon_1'(1+\cos^2\theta)N^2-2\varepsilon_1'\varepsilon_1-2\varepsilon_2'\varepsilon_2}{2N[2\cos^2\theta N^2-\varepsilon_1(1+\cos^2\theta)]}, \tag{3.4}$$

$$\varkappa_v^{(i)}=\frac{(\varepsilon_{11}^{(i)}+\varepsilon_{22}^{(i)}\cos^2\theta)N^2-(\varepsilon_{11}^{(i)}+\varepsilon_{22}^{(i)})\varepsilon_1-2i\varepsilon_{12}^{(i)}\varepsilon_2}{2iN[2\cos^2\theta N^2-\varepsilon_1(1+\cos^2\theta)]}, \tag{3.5}$$

$$\begin{aligned}\varkappa'=\operatorname{Im}\{(\varepsilon_{22}'+\varepsilon_{22})(\cos^2\theta N^2-\varepsilon_1)+(1/\varepsilon_{33})[(\varepsilon_1^2-\varepsilon_2^2\\ -\varepsilon_1N^2)\sin^2\theta N^2+\varepsilon_{23}^2(\cos^2\theta N^2-\varepsilon_1)\\ -2i\varepsilon_2\varepsilon_{23}\sin\theta\cos\theta N^2]\}[4\cos^2\theta N^3-2\varepsilon_1(1+\cos^2\theta)N]^{-1}.\end{aligned} \tag{3.6}$$

The quantity $\varkappa_{fr}$ takes into account the damping due to the friction force between the ions of different sorts and between the ions and electrons when these particles move in the field of the wave in a direction perpendicular to the magnetic field. The contribution of the collisions between the ions of different sorts to $\varkappa_{fr}$ and of the collisions between the electrons and ions is determined by the expressions

$$\varkappa_{fr}^{(i)}\sim(\nu_{ab}/\omega)N,\quad \varkappa_{fr}^{(e)}\sim(\nu_{ei}/\omega)(m_e/m_i)N. \tag{3.7}$$

Comparison of the quantities $\varkappa_{fr}^{(i)}$ and $\varkappa_{fr}^{(e)}$ shows that the collisions between ions of two sorts, having a density of the same order of magnitude $n_a\sim n_b\sim n_e$ and $e_a\sim e_b\sim e$ turn out to be more significant than collisions between the electrons and ions, $\varkappa_{fr}^{(e)}/\varkappa_{fr}^{(i)}\sim(m_e/m_i)^{1/2}\ll 1$. The collisions between the ions may turn out to be significant also in the case when the concentration of ions of one of the sorts is small ($n_b\ll n_e$), if the charge of the ions is large ($e_b=Z_be$, $Z_b\gg 1$), since $\nu_{ab}\sim Z_b^2 n_b$.

The contribution of the viscosity of the ions to the damping is determined by the expression

$$\varkappa_v^{(i)}\sim\frac{\nu_{ii}}{\omega}\frac{k^2v_{Ti}^2}{\omega^2}N\sim\frac{\nu_{ii}}{\omega}\beta_iN. \tag{3.8}$$

Since $\beta_i=(v_{Ti}/v_A)^2\ll 1$, the damping due to the ion viscosity turns out to be negligibly small in comparison with the damping due to the friction force between the ions of the different sorts. This type of damping may turn out to be significant only in the case when the plasma consists of ions of one sort. We note that formula (3.5) at $\theta=\pi/2$ coincides with the result obtained in[13].

The quantity $\varkappa'$ takes into account the electron Cerenkov absorption of the wave (the terms proportional to ε_{22}', $\varepsilon_{23}^{(0)}$, $1/\varepsilon_{33}^{(0)}$), the friction force between the electrons and the ions in the direction of the magnetic field, and the electron viscosity (terms proportional to $\varepsilon_{ij}^{(e)}$).

If $|z_e|\gg 1$, then the Cerenkov damping is exponentially small. In this case the main contribution to the damping $\varkappa'$ is made by the friction force that arises when electrons move in the field of the wave relative to the ions parallel to the magnetic field, and by the electron viscosity:

$$\varkappa'=\varkappa_{fr\parallel}^{(e)}+\varkappa_v^{(e)}, \tag{3.9}$$

where

$$\varkappa_{fr\parallel}^{(e)}=\frac{\lambda}{\sqrt{2}}\frac{\nu_{ee}\omega\sin^2\theta}{\omega_{pe}^2}\frac{(\varepsilon_1N^2-\varepsilon_1^2+\varepsilon_2^2)N}{2\cos^2\theta N^2-\varepsilon_1(1+\cos^2\theta)}, \tag{3.10}$$

$$\begin{aligned}\varkappa_v^{(e)}=\frac{\nu_{ee}k_\perp^2v_{Te}^2}{|\omega_{Be}|\omega^2}[2\cos^2\theta N^3-\varepsilon_1(1+\cos^2\theta)N]^{-1}\\ \times\left(\frac{1+\lambda\sqrt{2}}{5}\frac{\omega_{pe}^2}{\omega|\omega_{Be}|}(\cos^2\theta N^2-\varepsilon_1)-\frac{4+7\lambda\sqrt{2}}{5}\varepsilon_2N^2\cos^2\theta\right).\end{aligned} \tag{3.11}$$

The damping coefficient $\varkappa_{fr\parallel}^{(e)}$ coincides in order of magnitude with $\varkappa_{fr}^{(e)}$. Expression (3.11) is a generalization of the result obtained in[17] for a two-component plasma, at the lower frequencies $\omega\ll\omega_{Bi}$, to the case of a plasma containing ions of different sorts and to higher wave frequencies $\omega\gtrsim\omega_{Bi}$. The damping due to the electron viscosity is of the order of

$$\varkappa_v^{(e)}\sim\frac{\nu_{ei}}{\omega}\frac{k_\perp^2v_{Ti}^2}{\omega^2}N\sim\frac{\nu_{ei}}{\omega}\beta_iN. \tag{3.12}$$

Comparison of expressions (3.8) and (3.12) shows that at $|z_e|\gg 1$ the electron viscosity leads to a much larger damping of the MHD waves and the ion viscosity, $\varkappa_v^{(i)}/\varkappa_v^{(e)}\sim(m_e/m_i)^{1/2}\ll 1$.

We note that for $|z_e|\approx 2$–3 the electron Cerenkov damping is still small, therefore $\varkappa'$ can be determined by the same collision effects as at $|z_e|\gg 1$. But at these values of z_e the asymptotic expressions for $\epsilon_{ij}^{(0)}$ and $\varepsilon_{ij}^{(e)}$ no longer hold. In this case the quantity $\varkappa'$, even at $\theta\approx\pi/2$, is determined not only by the component $\varepsilon_{22}^{(e)}$ but also by $\varepsilon_{23}^{(e)}$ and $\varepsilon_{33}^{(e)}$ (at $|z_e|\gg 1$, $\theta\approx\pi/2$ the quantity $\varkappa'$ is proportional to $\operatorname{Im}\varepsilon_{22}^{(e)}$).

Comparison of $\varkappa_v^{(e)}$ with $\varkappa_{fr}^{(i)}$ in the presence of ions of different sorts of comparable density $n_a\sim n_b\sim n_e$ and $e_a\sim e_b\sim e$ shows that at $\beta>(m_e/m_i)^{1/2}$ the main contribution to the damping is made by the electron viscosity, and if the opposite inequality holds the main contribution is made by the friction between ions of different sorts.

In a two-component plasma, the effect of electron viscosity turns out to be decisive in comparison with the force of friction between the electrons and ions, if $\beta>(m_e/m_i)$. The friction between the electrons and the ions turns out to be significant in comparison with the electron viscosity only in a plasma with very small β ($\beta\lesssim m_e/m_i$).

Inasmuch as $\omega\sim kv_A$ for the waves under consideration, the condition $|z_e|\gg 1$ means that $|\cos\theta|\ll v_A/v_{Te}$. If θ is not too close to $\pi/2$, the condition $|z_e|\gg 1$ means that $v_A\gg v_{Te}$. In this case the electron viscosity always makes a negligibly small contribution to the damping in comparison with the contribution due to the force of friction between the electrons and the ions. At $v_A \ll v_{Te}$, the condition $|z_e| \gg 1$ is satisfied, if the angle θ is close to $\pi/2$, and it is precisely in this case that the electron viscosity plays the decisive role in comparison with the friction.

For strongly decelerated waves that propagate at an angle θ not too close to $\pi/2$, when $v_A \ll v_{Te}$, the conditions $|z_e| \ll 1$ is satisfied. In this case the condition for the applicability of the expressions for $\varepsilon_{ij}^{(e)}$ takes the form $\nu_{ee} \ll k_\parallel v_{Te}$. Comparing the anti-Hermitian parts in $\varepsilon_{22}^{(0)}$, $\varepsilon_{23}^{(0)}$, $\varepsilon_{33}^{(0)}$ with $\varepsilon_{ij}^{(e)}$, which take into account the electron viscosity, we find that the ratio of the latter to the former is of the order of $\nu_{ee}/k_\parallel v_{Te} \ll 1$. This means that at $|z_e| \lesssim 1$ the effects of electron viscosity can be neglected in comparison with Cerenkov absorption of the waves by the electrons.

The damping coefficient due to the Cerenkov absorption is equal to

$$\varkappa_{\mathrm{Ch}}^{(e)} = \frac{\pi^{1/2}}{4}\frac{\sin^2\theta}{z_e N}\exp(-z_e^2)\left[\frac{\omega_{pe}^2(\cos^2\theta N^2-\varepsilon_1)}{\omega_{Be}^2\cos^2\theta} + \frac{c^2k^2}{\omega_{pe}^2}\frac{\varepsilon_1 N^2-\varepsilon_1^2+\varepsilon_2^2}{|1+i\pi^{1/2} z_e w(z_e)|^2}\right][2\cos^2\theta N^2-\varepsilon_1(1+\cos^2\theta)]^{-1}. \quad (3.13)$$

This expression is a generalization of the previously obtained result (see[7]) to include the case of a plasma containing different sorts of ions. In order of magnitude we have

$$\varkappa_{\mathrm{Ch}}^{(e)} \sim \frac{m_e}{m_i}\frac{v_{Te}}{v_A}\frac{\sin^2\theta}{\cos\theta}\exp(-z_e^2)N \quad (3.14)$$

($v_A \gg v_{Ti}\cos\theta$, $v_{Te}\cos\theta \gtrsim v_A$). The waves most strongly absorbed are those propagating in an angle θ such that $\cos\theta \sim v_A/v_{Te}$ (that is, $z_e \sim 1$), for which $\varkappa_{\mathrm{Ch}}^{(e)} \sim \beta N$. Comparing (3.14) with the expression for $\varkappa_{fr}$, we find that the Cerenkov absorption plays the decisive role if the parameter β is not too small. Damping due to ion viscosity turns out to be insignificant in comparison with Cerenkov damping, if $\nu_{ii}/\omega \ll z_e \sim v_A/v_{Te}\cos\theta$. If this inequality is not satisfied, then ion viscosity may turn out to be significant in a plasma containing ions of one sort.

4. HIGH FREQUENCY HEATING OF PLASMA

We discuss now the possibility of using the MHD wave absorption mechanisms discussed in a preceding section for plasma heating. At frequencies $\omega \lesssim \omega_{Bi}$ these waves can be easily excited in a dense plasma of both small and large dimensions by using external current-carrying loops or special decelerating systems. On the other hand, this range of frequencies has significant technical advantages, since the development of RF generators of the required power in this band entails no significant difficulty. In this case the efficiency of using MHD waves is determined principally by the magnitude of their damping.

Let us consider examples illustrating the contribution of the discussed MHD wave damping mechanisms to plasma heating. For a thermonuclear reactor based on the tokomak[20] we have $n_e \sim 10^{14}$ cm^{-3}, $B \sim 50$ kG, $T \sim 10$ keV, minor radius of plasma $a \sim 100$ cm, and energy lifetime of the plasma $\tau_E > 1$ sec. In this case $\beta \sim 4\cdot 10^{-3}$, $v_{Te} \sim 3\cdot 10^9$ cm/sec, $v_{Ti} \sim 5\cdot 10^7$ cm/sec, $m_i \sim 3.5\cdot 10^{-24}$ g, $v_A \sim 10^9$ cm/sec, $\omega_{Bi} \sim 2\cdot 10^8$ sec^{-1}, and $\nu_{ei} \sim 50\nu_{ii} \sim 5\cdot 10^3$ sec^{-1}. Since the time of energy exchange between the electrons and ions is $\lesssim 1$ sec, the plasma component to which the RF field energy is transferred is immaterial. If the funadmental radial mode of the oscillations with frequency $\omega \sim v_A/a \sim 10^7$ sec$^{-1} \sim (1/20)\omega_{Bi}$ is excited, then at $k_\parallel \sim k_\perp \sim 1/a \sim 10^{-2}$ cm^{-1} (the spatial period of the exciting system is $L_\parallel = 2\pi/k_\parallel \sim 600$ cm) we have $z_e \sim 0.2$ and $\gamma_{\mathrm{Ch}}^{(e)}/\omega \sim 10^{-3}$, that is, the equality factor of the apparatus is $Q = \omega/2\gamma \sim 500$. The effects of wave absorption due to the friction force between the ions are negligibly small, $\gamma_{fr}^{(i)}/\omega \sim 10^{-5}$. During the initial heating stage at $T \lesssim 1$ keV, the damping due to the Cerenkov absorption becomes weaker, and the damping due to the friction force between the ions increases, $\gamma_{\mathrm{Ch}}^{(e)}/\omega \lesssim \gamma_{fr}^{(i)}/\omega \sim 3\cdot 10^{-4}$, that is, $Q \sim 5\cdot 10^3$. At lower values of the temperature, the Cerenkov absorption becomes insignificant and the principal role is assumed by the force of friction between the ions, and then the Q of the system decreases, $Q < 4\cdot 10^3$.

If waves of frequency $\omega \sim \omega_{Bi}$ are used, the excitation turns out to be effective if $k_\parallel \lesssim (\omega^2/av_A^2)^{1/3}$ [10] in the considered example $k_\parallel \lesssim 0.1$ cm^{-1}. In this case the collision absorption is negligibly small in comparison with the Cerenkov absorption, which leads to a value $\gamma_{\mathrm{Ch}}^{(e)}/\omega \sim 2\cdot 10^{-3}$ at $T \sim 10$ keV ($z_e \sim 0.5$), but at $T \sim 1$ keV the Cerenkov damping becomes much weaker, $\gamma/\omega \sim 10^{-4}$, and the Cerenkov absorption becomes of the same order as the absorption due to the friction between the ions.

Thus, in the considered example of a plasma with small values of β, at thermonuclear temperatures ($T \sim 10$ keV), the principal role is played by Cerenkov absorption by electrons,[4)] which leads to a relatively strong wave absorption both in the case of low frequencies $\omega \sim v_A/a \ll \omega_{Bi}$, when use can be made of the resonance of the buildup of the natural oscillations, and in the case of high frequencies $\omega \sim \omega_{Bi}$, when the exciting system operates in the radiation regime ($k_\perp a \gg 1$). We note that absorption of a wave with a frequency lying between the cyclotron frequencies of the ions in a plasma and containing ions of two sorts can be greatly increased when the wave propagates in the interior of the plasma in the region, where $\epsilon_1 \approx N^2\cos^2\theta$, and the wave under consideration is transformed into so-called slow waves.[21]

We consider now an example of a plasma with a relatively large value of β ($n \sim 10^{16}$ cm^{-3}, $B \sim 100$ kG, $T \sim 10$ keV, $a = 20$ cm, $\beta \sim 0.1$). In this case upon excitation of the fundamental radial mode $\omega \sim v_A/a \sim 10^7$ sec^{-1} $\sim (1/50)\omega_{Bi}$ at $k_\parallel \sim k_\perp \sim 1/a$ we have $\gamma_{\mathrm{Ch}}^{(e)}/\omega \sim 3\cdot 10^{-2}$ ($z_e \sim 1$), that is, $Q \sim 20$. The collision effects in this case exert no influence on the absorption. On the other hand, if

$k_{\parallel} < 1/4a$, then $z_e > 4$ and the Cerenkov absorption turns out to be negligibly small, in which case the absorption is due mainly to the electron viscosity, which leads to a value $Q \sim 100$. At lower values of the temperature, $T \sim 1$ keV, the absorption is due to the electron viscosity and friction between the ions ($Q \sim 10$). This example shows that in a dense plasma the effects of electron viscosity and of the friction forces between the ions can lead to a strong absorption of the waves and can be used to obtain thermonuclear temperatures of a dense plasma, whereas the absorption due to ion viscosity turns out negligibly small in the case of high temperatures.

1) In the region of frequent collisions ($\omega \ll \nu_{ii}$), the gyrorelaxation effect[3,4] is taken into account in the equations of motion and the viscosity force[16] perpendicular to the magnetic field.

2) When expression (2.10) for $\varepsilon_{22}^{(e)}$ is compared with the result in[17], it must be borne in mind that the values of L in[17] are twice as large as those used here, and the values of ν_{ee} are larger by a factor $\sqrt{2}$.

3) The temporal damping decrement γ is connected with by the relation $\gamma = \varkappa(\omega/c)\partial\omega/\partial k$.

4) The possibility of using electron Cerenkov absorption of waves with $\omega \lesssim \omega_{Bi}$ for plasma heating in a tokomak reactor was considered also in[21,22].

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