Electromagnetic waves in a periodic nonstationary magnetoactive plasma

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Plane electromagnetic waves in a nonrelativistic electron plasma (or semiconductor) located in an external homogeneous magnetic field $\mathbf{H}=\mathbf{H}_0-\mathbf{H}_1\cos pt$ possessing an alternating component parallel to the constant component $\mathbf{H}_0$ are considered. Expressions for the natural waves are presented which are obtained by diagonalizing an infinite-dimensional matrix (without the application of perturbation theory in $\mathbf{H}_1$). The passage of an initially monochromatic wave of frequency ω (for $(p/\omega)^{1/2}<1$) through a layer of the given medium is considered. Expressions are obtained for the intensities of the transmitted wave of frequency ω and waves with new frequencies $\omega+np$ ($n=\pm1, \pm2,\ldots$). The dependence of these intensities on the amplitude $\mathbf{H}_1$ of the alternating component of the external magnetic field is investigated. The possibility is indicated of a significant reorganization of the modes of optical resonators when such media are used.

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Cyclotron resonance in an alternating magnetic field[1] has of late attracted attention as a method for investigating the structure of the conduction band of semiconductors,[2,3] as a method for measuring the amplitude of an alternating magnetic field in the microwave range,[3] etc. The effect in question was first considered by the present author in[1], where it was assumed that some volume of a nonrelativistic electron plasma or a semiconductor was located in a homogeneous magnetic field $\mathbf{H}=\mathbf{H}_0-\mathbf{H}_1\cos pt$ with the alternating component $\mathbf{H}_1$ parallel to the constant component $\mathbf{H}_0$. The electrical conductivity tensor of the medium in question with respect to an auxiliary weak homogeneous electric field $\mathbf{e}_2=\mathbf{e}_{20}\cos\omega t$ arbitrarily oriented with respect to the field H was computed, and it was established on this basis that the absorption of the energy of the monochromatic electric field (of frequency ω) is an oscillating function of the amplitude H_1 of the alternating component of the magnetic field. Similar oscillations in the form of oscillations in the magnetoresistance of a sample (at $\omega=0$) have been experimentally observed by Katz and Shekhter.[3]

In the present paper we consider the propagation of plane electromagnetic waves in a medium of the type indicated above (i.e., in a plasma located in a homogeneous magnetic field $\mathbf{H}=\mathbf{H}_0-\mathbf{H}_1\cos pt$) under the condition that

$$(p/\omega)^{1/2}\ll1, \tag{1}$$

where ω is the "central" frequency of the propagating wave (with a total spectrum $\Delta\omega\ll\omega$). It is easy, under the condition (1), to satisfy the homogeneity requirement for the magnetic field H in intervals containing many wavelengths $2\pi/\omega$.

Below we give for the natural waves in the medium under consideration expressions obtained by diagonalizing an infinite-dimensional matrix without the use of perturbation theory in H_1. By their structure, these waves are similar to Bloch functions in crystals, with the difference that it is necessary to exchange in the latter the roles of the space coordinate and the time and assign complex values to the energy. On the basis of the expressions given below we investigate the passage of a weak monochromatic wave of frequency ω through a layer of the medium under consideration. Expressions are found for the intensities of waves with new frequencies $\omega+np(n=\pm1,\pm2,\ldots)$ arising in the course of such a passage, and the dependence of the intensities of these waves on the amplitude H_1 of the alternating component of the magnetic field and on the thickness L of the layer of the medium is investigated.

To solve the formulated problem, let us write down in its general form the electric field $\mathbf{e}_2$ of the weak electromagnetic wave propagating from a monochromatic source of frequency ω:

$$\mathbf{e}_2(\mathbf{r},t)=\mathrm{Re}\sum_{n=-\infty}\mathbf{E}_n(\mathbf{r})e^{-i(\omega+np)t}. \tag{2}$$

The expression for the density vector $\mathbf{I}_2$ of the current induced by this field has the same form:

$$\mathbf{I}_2(\mathbf{r},t)=\mathrm{Re}\sum_{n=-\infty}^{\infty}\mathbf{j}_n(\mathbf{r})e^{-i(\omega+np)t}, \tag{3}$$

the values of $\mathbf{j}_n$ and $\mathbf{E}_n$ being connected by the linear relation

$$\mathbf{j}_n=\sum_k\hat{\sigma}^{(n,k)}(\omega)\mathbf{E}_k. \tag{4}$$

In a Cartesian coordinate system (ξ,η,ζ) whose ζ axis coincides with the direction of the external magnetic field $\mathbf{H}(\mathrm{H}_\zeta=H_0-H_1\cos pt)$ the nonzero components of the $\hat{\sigma}^{(n,k)}$ tensors can be written in the form[1] (see also[2,4,5])

$$\sigma_{\xi\xi}^{(n,k)}={}^1/_2(D_{nk}^{+}+D_{nk}^{-}),\qquad \sigma_{\xi\eta}^{(n,k)}=\frac{i}{2}(D_{nk}^{+}-D_{nk}^{-}),$$
$$\sigma_{\eta\eta}^{(n,k)}=\sigma_{\xi\xi}^{(n,k)},\qquad \sigma_{\eta\xi}^{(n,k)}=-\sigma_{\xi\eta}^{(n,k)},$$
$$\sigma_{\zeta\zeta}^{(n,k)}=\frac{iNe^2}{m}\frac{1}{\omega+np+i/\tau}\delta_{nk}, \tag{5}$$

where

$$D_{nk}^{+}=\frac{iNe^2}{m}\sum_{s=-\infty}^{\infty}\frac{J_{s-k}(\Delta)J_{s-n}(\Delta)}{\omega+\omega_H+sp+i/\tau},\qquad D_{nk}^{-}=\frac{iNe^2}{m}\sum_{s=-\infty}^{\infty}\frac{J_{n-s}(\Delta)J_{k-s}(\Delta)}{\omega-\omega_H+sp+i/\tau},$$

$$\omega_H=|e|H_0/mc,\quad \Delta=|e|H_1/mc; \tag{6}$$

$J_n(\Delta)$ is the Bessel function of order n; N is the electron density; c is the velocity of light in vacuo; e and m are the electron charge and mass; and τ is the effective electron mean free time.

The equality (4) is a material equation for the medium in question in a field $\mathbf{e}_2$ of the arbitrary form (2). Supplemented by the Maxwell equations, it gives a closed system of equations describing stationary (in time) electromagnetic waves in the medium under consideration. For the case of plane waves, we must set $\mathbf{E}_n = \mathbf{E}_n(z)$ and $\mathbf{j}_n = \mathbf{j}_n(z)$, after which the indicated system of equations can be written as:

$$\frac{d^2E_{n\alpha}}{dz^2} = -\varkappa_n{}^2E_{n\alpha} - \frac{4\pi i}{c}\varkappa_n\sum_{k,\beta}\sigma_{\alpha\beta}^{(n,k)}E_{k\beta},$$

$$E_{nz} = -\frac{4\pi i}{c}\varkappa_n{}^{-1}\sum_{k,\beta}\sigma_{z\beta}^{(n,k)}E_{k\beta}. \tag{7}$$

Here $\varkappa_n = (\omega + np)/c$; $\alpha = x, y$; $\beta = x, y, z$; under (x, y, z) is meant an arbitrarily chosen Cartesian system of coordinates. Below we shall, for simplicity, consider the cases of longitudinal and transverse (with respect to the direction of the external magnetic field $\mathbf{H}$) wave propagations, i.e., we shall set $\xi = x$, $\eta = y$, and $\zeta = z$ in the first case; $\xi = z$, $\eta = x$, and $\zeta = y$ in the second.

In the first case of longitudinal propagation, it is convenient to separate out in the system (7) the circularly polarized waves, i.e., to introduce the new variables

$$E_n{}^{\pm} = E_{nx} \pm iE_{ny}, \tag{8}$$

in terms of which the system (7) breaks up into two independent systems of equations. These systems, in their turn, can be written in the compact matrix form

$$\frac{d^2E^{\pm}}{dz^2} = Q^{\pm}E^{\pm}, \qquad Q^{\pm} = -\beta^2 - \frac{4\pi i}{c}\beta D^{\pm}. \tag{9}$$

Here E^+ is an infinite-dimensional column made up of the elements E_n^+ $(-\infty < n < \infty)$, similarly E^- is a column made up of the elements E_n^-, β is a matrix made up of the elements $\beta_{nk} = \varkappa_n\delta_{nk}$, and the matrix elements $D_{nk}^{\pm}$ are given by the equalities (6).

As can be seen, the problem consists in the diagonalization of the $Q^{\pm}$ matrices. In accordance with (1), below we shall set $\beta_{nk} = \varkappa\delta_{nk}$ $(\varkappa = \omega/c)$, after which the problem reduces to one of diagonalizing the $D^{\pm}$ matrices. It can be verified that these matrices can be reduced to the diagonal form by the change of variables[1)]

$$q_l{}^+ = \sum_{k=-\infty}^{\infty} J_{l-k}(\Delta)E_k{}^+, \qquad q_l{}^- = \sum_{k=-\infty}^{\infty} J_{k-l}(\Delta)E_k{}^-. \tag{10}$$

The Eqs. (9) can then be written in terms of the variables $q_l^{\pm}$ as follows:

$$d^2q_l{}^{\pm}/dz^2 = \Lambda_l{}^{\pm}q_l{}^{\pm}, \quad -\infty < l < \infty, \tag{11}$$

where the eigenvalues $\Lambda_l^{\pm}$ of the $Q^{\pm}$ matrices are given by the expressions

$$\Lambda_l{}^{\pm} = -\varkappa^2 + \frac{4\pi\varkappa Ne^2}{mc}\left(\omega \pm \omega_H + lp + \frac{i}{\tau}\right)^{-1}. \tag{12}$$

The relations (10)–(12) give the solution to the formulated problem in the considered case of longitudinal propagation. The corresponding expression for $\mathbf{e}_2(z, t)$ can clearly be written as:

$$e_{2x} = \frac{1}{2}\operatorname{Re}(M^+ + M^-), \qquad e_{2y} = \frac{1}{2}\operatorname{Im}(M^+ - M^-), \qquad e_{2z} = 0,$$

$$M^{\pm} = \sum_{l=-\infty}^{\infty} C_l{}^{\pm}\exp[-i(\omega + lp)t \mp i\Delta\sin pt + (\Lambda_l{}^{\pm})^{1/2}z], \tag{13}$$

where $C_l^{\pm}$ are arbitrary constants and $\operatorname{Re}(\Lambda_l^{\pm})^{1/2} < 0$.

According to (13), a plane wave in the medium under consideration is, in the general case, a superposition of an infinite number of natural waves defined by the functions of t and z attached to the constants $C_l^{\pm}$. As can be seen, these waves have, in contrast to the case of a stationary medium, a factor, $\exp(\pm i\Delta \sin pt)$, that is a periodic function of time, the period of this function being determined by the period of the variation of the external magnetic field. From this point of view the natural waves under consideration are similar to Bloch waves (in crystals) having as a factor a periodic function of the coordinates with a period equal to that of the spatial variation of the potential. Such an analogy is, of course, valid if the roles of the space coordinate and the time in the Bloch functions are interchanged and complex values are formally assigned to the energy.[2)]

On the basis of (13), and taking (1) into account, we can easily obtain the z dependence of the time-averaged intensity of the electric-field oscillations

$$W(z) = \overline{e_x^2} + \overline{e_y^2}$$

in the propagating wave. For each of the circularly polarized waves this intensity is determined by the equality

$$W^{\pm}(z) = \frac{1}{4}\sum_l |C_l{}^{\pm}|^2\exp(2\operatorname{Re}(\Lambda_l{}^{\pm})^{1/2}z), \tag{14}$$

which, in its turn, shows that as the wave propagates deep into the medium there occurs absorption of the energy of the natural waves at the individual characteristic wavelengths $z_l^{\pm} = [-2\operatorname{Re}(\Lambda_l^{\pm})^{1/2}]^{-1}$. In this case amplification of the waves does not occur.

The foregoing results allow us to consider the problem of the incidence of a plane monochromatic wave on the boundary of a semi-infinite medium of the type under consideration here, or to find the wave for $z \geq 0$ if at the boundary $z = 0$ is given the monochromatic field

$$\mathbf{e}_2 = \operatorname{Re}[(E_x^{(0)}, E_y^{(0)}, 0)e^{-i\omega t}].$$

Let us consider the second of these problems. Solving the infinite system of algebraic equations for the coefficients $C_l^{\pm}$, obtained from the boundary condition at $z = 0$, and substituting further the expressions for these coefficients into (13), we obtain

$$M^+ = (E_x^{(0)} + iE_y^{(0)})\sum_{n,l=-\infty}^{\infty} J_{l-n}(\Delta)J_l(\Delta)\exp[(\Lambda_l{}^+)^{1/2}z - i(\omega + np)t],$$

$$M^- = (E_x^{(0)} - iE_y^{(0)})\sum_{n,l=-\infty}^{\infty} J_{n-l}(\Delta)J_{-l}(\Delta)\exp[(\Lambda_l{}^-)^{1/2}z - i(\omega + np)t]. \tag{15}$$

The equalities (15) determine the evolution in the medium in question of a plane, initially-monochromatic wave of frequency ω. It can be seen from these equalities that, in contrast to the case of a stationary medium, where the z dependence of the field is exponential, in the present case the corresponding dependence is determined by a sum of an infinite number of exponential functions and, thus, strictly speaking, it cannot be described in the framework of the wave-vector (or refractive-index) concept. It can also be seen that the propagation of an initially monochromatic wave is generally accompanied by the appearance of new waves with frequencies $\omega + np$.

Let us consider in greater detail the indicated evolution under the condition that

$$p\tau \gg 1, \tag{16}$$

which means that the width of the cyclotron-resonance line in the constant magnetic field $\mathbf{H}_0$ does not exceed the frequency p, and let us restrict ourselves to the analysis of, for example, the quantity M^-, i.e., of one of the circularly polarized waves. Let, furthermore,

$$|\omega - \omega_H + kp| \lesssim 1/\tau, \tag{17}$$

where $k = 0, \pm 1, \pm 2, \ldots$ is a whole number given beforehand and satisfying the condition $|k| \ll \omega_H/p$. Then, according to (12) and (15), the intensity of the harmonic component of frequency ω after passing through a layer of thickness $L \gtrsim z_l^-$ of the medium is an oscillating function of Δ (for $\Delta \gtrsim |k|$), i.e., of the amplitude of the alternating component of the magnetic field (see (6)). The values of Δ at which this intensity assumes its maximum values are determined from the condition $J_k(\Delta) = 0$.

The specific nature of, and the mechanism underlying, these oscillations do not, however, amount to the previously-described[1] absorption oscillations manifested by the medium in relation to a weak homogeneous monochromatic field, since in the case of waves this mechanism is, as can be seen from (15), determined simultaneously by absorption and quite a complex interaction between the various frequency components in the medium under consideration. The latter leads, in particular, to the transformation of the energy of the initial wave into the energy of waves with frequencies $\omega + np$. For a fixed value of $n \neq 0 (|n| \ll \omega_H/p)$ and a given value of k, the intensity of the wave with frequency $\omega + np$ assumes appreciable values under conditions when $L \gtrsim z_k^-$, $\Delta \gtrsim |n|$, $|k|$ and is then an oscillating function of Δ. The minimum values of this intensity correspond to the condition $J_{n-k}(\Delta) J_{-k}(\Delta) = 0$.

The efficiency of the transformation of the energy of the initial wave into the energy of waves with other frequencies depends essentially on the relation between the quantities z_k^- and L. For $z_k^- \ll L$, the fraction of transformed energy is small virtually for any Δ. The change in the intensity of the initial wave of frequency ω is then also small. For $L \gtrsim z_k^-$ and $\Delta \gtrsim 1$, this fraction generally becomes substantial. Under these same conditions the change in the intensity of the wave with frequency ω becomes appreciable. In the last case the total width of the spectrum can be estimated to be $\Delta\omega \sim p\Delta$. It is also interesting to note that the total energy of the wave transmitted across the layer of the medium under consideration may, as can be seen from (14), be close to the energy of the initial wave even when $L \gg z_k^-$.

Let us now consider the case of transverse propagation ($H_y = H_0 - H_1 \cos pt$). As can be seen from the system (7) and the equalities (5), in this case the natural waves of the medium have the following polarizations: $(0, E_y, 0)$ and $(E_x, 0, E_z)$. The waves with polarization of the first type coincide with waves in a stationary, isotropic plasma (see[9]), and will therefore not be considered below. We shall consider the waves with the second type of polarization under the conditions (1) and (16) and under the additional requirements that

$$4\pi Ne^2\tau/m\omega \ll 1, \quad |\omega - \omega_H| \ll \omega. \tag{18}$$

The first of the inequalities (18) implies that the electrical conductivity of the medium is, like the conductivity of a slightly anisotropic (at any ω) medium, fairly low even under resonance conditions. Under this condition, as can be seen from (7), the component E_z of the wave under consideration can be neglected. Owing to the second of the conditions (18), we can drop the quantities D_{nk}^+ in the expressions (5) for $\hat{\sigma}^{(n,k)}$.

As a result, the system of equations for E_{nx} can be written as:

$$\frac{d^2 E_x}{dz^2} = -\left(\beta^2 + \frac{2\pi i}{c}\beta D^-\right) E_x, \tag{19}$$

where E_x is a column made up of the elements E_{nx} and D^- is the matrix (10) made up of the elements D_{nk}^-. The difference between the equations (19) and (9) for E^- amounts to an insignificant factor of $\frac{1}{2}$ attached to the matrix D^-; therefore, let us at once give the result:

$$e_{2x} = \mathrm{Re} \sum_{l=-\infty}^{\infty} C_l \exp[-i(\omega + lp)t + i\Delta \sin pt + \lambda_l z]. \tag{20}$$

Here

$$\lambda_l = i\varkappa - i\frac{\pi Ne^2}{mc}\left(\omega - \omega_H + lp + \frac{i}{\tau}\right)^{-1}, \tag{21}$$

and the C_l are arbitrary constants. When the first of the conditions (18) is fulfilled, the problem of the normal incidence of a plane wave at the boundary of the half-space $z \geq 0$, occupied by the medium under consideration, reduces directly to the prescription of the corresponding boundary conditions at $z = 0$, since the intensities of the reflected waves in this case are small.

Setting

$$\mathbf{e}_2|_{z=0} = \mathrm{Re}[(E_x^{(0)}, 0, 0) e^{-i\omega t}],$$

we find that

$$e_{2x} = \mathrm{Re}\left\{E_x^{(0)} \sum_{n,l=-\infty}^{\infty} J_{n-l}(\Delta) J_{-l}(\Delta) \exp[\lambda_l z - i(\omega + np)t]\right\}. \tag{22}$$

The expression (22) gives the same distinctive features of the evolution of the initial monochromatic wave in the medium as were considered above on the basis of the expression (15) for M^-.

Under the conditions (18), all the results can easily be generalized to the case when the z axis of the wave propagation is arbitrarily oriented with respect to the direction of the external magnetic field $\mathbf{H}$.

Let us consider a numerical example, having in mind, for definiteness, a semiconductor plasma in which the charge-carrier density has the value $N \sim 10^{11}-10^{12}$ cm^{-3}. Assuming the value of the effective mass of the electron in the conduction band to be of the order of the mass of the free electron, and assuming further that $H_0 = 6\times10^4$ Oe, we find that $\omega_H = 10^{12}$ rad/sec, which, in its turn, corresponds under the condition (17) to a wavelength $2\pi c/\omega \sim 0.2$ cm for the propagating wave. At a frequency $p \sim 10^{10}$ rad/sec, satisfying the condition (1), the results obtained above are applicable in a layer of thickness $L \ll 20$ cm, in which case $z_k \lesssim 1$ cm (in particular, $z_k \lesssim L$) for $\tau \lesssim 10^{-10}$ sec. Furthermore, for $H_1 \gtrsim 5\times10^2$ Oe, we have $\Delta \gtrsim 1$. As can be seen, the above-considered values of the parameters of the system are entirely realizable. Notice also that in some semiconductors the value of the effective mass of the change carriers is much less than the free-electron mass; for example, in the semiconductor n-InSb the indicated values differ by two orders of magnitude. This allows us to lower the values of H_0 and H_1 (for the same values of the remaining parameters) to 6×10^2 and 5 Oe respectively. This circumstance has already been used in experiments[3] on the observation of the magnetoresistance of a sample, in which values of the parameter $\Delta \gg 1$ were attained at a value of $H_1 \sim 30$ Oe.

Let us now briefly discuss another physical example to which the above results are applicable. Expressions of the same form as the formulas (5) are, under certain conditions, valid for the susceptibility of a two-level quantum-mechanical system. It can be verified by direct calculations with the aid of the equation for the density matrix with a collision term that if the off-diagonal matrix element $\mathbf{d}_{12}$ and the difference $\mathbf{d}_{22}-\mathbf{d}_{11}$ between the corresponding diagonal elements are different from zero, then there arises in an external electric field $\mathbf{e}_1 = \mathbf{e}_{10}\cos pt$, where

$$p \ll \omega_{21}, \quad |\mathbf{d}_{12}\mathbf{e}_{10}| \ll \hbar\omega_{21}, \quad \text{and} \quad \omega_{21} = (E_2 - E_1)/\hbar$$

is the frequency of transitions in the unperturbed system, owing to the linear Stark effect, a modulation of the transition frequency that is similar to the cyclotron-frequency modulation arising in a plasma on account of the alternating component of the magnetic field. The susceptibility of the medium with respect to a weak field $\mathbf{e}_2 = \mathbf{e}_{20}\cos\omega t$ is then determined by expressions similar to the expressions (5) and (6) for $\sigma_{\xi\xi'}^{(n,k)}$ (the argument of the Bessel functions is then the quantity $\Delta = (\mathbf{d}_{22} - \mathbf{d}_{11})\mathbf{e}_{10}/\hbar p$). Therefore, the results obtained above are applicable (mutatis mutandis) in the case of the two-level medium.

The use of such a medium may afford us a practical opportunity for observing the effects considered in this paper in the optical range of frequencies ω. Indeed, the quantity Δ attains values exceeding unity when, for example, $p \sim 10^9$ rad/sec, $|\mathbf{d}_{22}-\mathbf{d}_{11}| \sim 0.3\times10^{-18}$ cgs esu, and $e_{10} \gtrsim 3$ cgs esu. If a layer of a medium of the indicated type is placed in an optical resonator, then at values of the frequency p close to the frequency interval between two axial (not necessarily neighboring) types of oscillations of the unperturbed resonator, as for example when $p \sim 10^9-10^{10}$ rad/sec and $\Delta \gtrsim 1$, $L/z_k \gtrsim 1 - r$ (where r is the coefficient of specular reflection), there will occur a considerable reconstruction of the modes of this resonator, which, in turn, can be used to control the generation regimes of lasers. In the particular case when the frequency p coincides exactly with one of the axial intermode intervals and the mode losses are all the same, the modes will, as a result of the reconstruction, be determined by the natural waves of the medium.

1)The inverse relations look like:

$$E_k^+ = \sum_{l=-\infty}^{\infty} J_{l-k}(\Delta)\, q_l^+, \qquad E_k^- = \sum_{l=-\infty}^{\infty} J_{k-l}(\Delta)\, q_l^-.$$

2)According to (13), the appearance of a periodic nonstationarity of the medium results in a reconstruction of the natural waves of the medium. The general form of the natural waves in the periodically nonstationary medium (i.e., the kind of reconstruction under consideration) and the formulation of the problem of finding these waves as a problem of the diagonalization of the Q matrix are contained in the author's papers.[6-8]

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