High-frequency impedance of thin metal plates

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We evaluate the surface impedance of thin metallic plates in a constant external magnetic field **H** in the range of high frequencies ω of the electromagnetic wave, where ωt_0 is much larger than the ratio r/δ of the radius of curvature of the electron trajectory to the skin-layer depth, (t_0 is the mean flight time of the charge carrier). We show that the size-effect cyclotron resonance, experimentally observed by Volodin, Khaĭkin, and Édel'man (1973) will diminish when ωt_0 increases while the logarithmic singularity at the resonance frequencies will occur not for the impedance, but for its derivative with respect to the magnetic field. The non-local character of the coupling between the current density Fourier components and those of the high-frequency electrical field plays under those conditions an essential role; if it is not taken into account, it is no longer possible to obtain the correct order of magnitude of the resonance curve width and of the amplitude of the effect. We analyze in detail the effect of the nature of the reflection on the shape of the resonance curve, we elucidate the role of the retardation effect on the size-effect cyclotron resonance, and we study the effect of the inclination of the magnetic field on this effect.

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A study of the resonance absorption of the energy of electromagnetic waves by thin conductors with a thickness d appreciably smaller than the electron mean free path l, but considerably larger than the skin depth δ, reveals new possibilities for studying the electron energy spectrum.[1] In a magnetic field H parallel to the surface of the plate, at $2r_{max} > d$, there appear new cyclotron resonance (CR) frequencies[1,3] determined by the electrons with orbital diameters equal to the thickness of the sample, instead of resonance frequencies which are cut off.[2] The resonance values of the magnetic field and the position of the sections of the Fermi surface on which resonance occurs are determined by the equations

$$\omega = n\Omega(p_1), \quad 2r(p_1) = d, \tag{1}$$

where $2r(p_z) = cD(p_z)/eH$; $D(p_z)$ is the diameter of the electron orbit in momentum space along the p_x axis; the y-axis is the normal to the surface of the plate; p_z is the component of the electron momentum along the direction of the magnetic field; $\Omega(p_z) = eH/m^*(p_z)c$ is the frequency of revolution of the electron in the magnetic field; $m^*(p_z)$ the effective mass of the charge carrier; e the electron charge; c the velocity of light; and n the number of the resonance.

As the resonance frequencies depend on an external parameter—the thickness of the conductor—we can, by varying it, obtain resonance on any section of the Fermi surface and determine the connection between the effective mass $m^*(p_z)$ and the orbital diameter for all p_z. When the magnetic field is inclined to the surface of the plate, the frequencies of the size-effect CR are due to the Doppler effect shifted by an amount which can determine also the drift velocity of the electrons on that section.[4]

At the present time such accurate experiments are fully realizable and Volodin, Khaĭkin, and Édel'man have observed this size-effect CR in bismuth films.[5] There is thus undoubtedly interest in a further theoretical study of this effect. Since the singularity in the impedance caused by the size-effect CR shows up more weakly than in the case of resonance in bulk samples, one uses for experimental observations very pure samples. At the same time, to obtain a maximum strength of the effect it is necessary, as we shall show in the present paper, to have not only large mean free paths, but also a high degree of the anomalous skin effect, viz,

$$r/\delta \gg \omega t_0. \tag{2}$$

When $r/\delta \ll \omega t_0$ the spread in the electron diameters which participate in the resonance $\Delta D \approx r/\omega t_0$ turns out to be much less than the skin depth δ and the selection of electrons by diameter is considerably more restricted. Under those circumstances the contribution from the resonance electrons to the current is small compared to the contribution from the other electrons on the Fermi surface, and the logarithmic singularity at frequencies satisfying condition (1) does not occur for the impedance, but for its derivative with respect to the magnetic field. The main resonance characteristics are then determined not only by the electron mean free path, as is the case normally for CR in bulk conductors, but also by the distribution of the high-frequency field in a thin sample (when the mean free path is infinite the width of the resonance curve is determined by the parameter δ/r).

Although the position of the resonance frequencies is not connected with the nature of the reflection of the carriers by the surface of the sample, nevertheless, the form of the resonance curve is sensitive to this electron scattering mechanism and its role in the size-effect CR, and we shall also study the role of the retardation effect in what follows.

1. STATEMENT OF THE PROBLEM

To find the surface impedance $Z_{\mu\nu}$ of the sample, which connects the electrical field at the metal surface

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with the total current,

$$E_\mu(0)=Z_{\mu\nu}\int_0^d J_\nu(y)\,dy \tag{3}$$

it is necessary to solve the Maxwell equations which we write down for the Fourier components of the high-frequency electrical current $j(k)$ and field $\mathscr{E}(k)$:

$$\begin{gathered} k^2\mathscr{E}_\mu(k)+2E_\mu'(0)=4\pi i\omega c^{-2}j_\mu(k),\\ \mathscr{E}_\mu(k)=2\int_0^d E_\mu(y)\cos ky\,dy. \end{gathered} \tag{4}$$

The prime indicates here diffentiation with respect to y; $\mu,\ \nu\equiv(x,\ z)$; we assume the electromagnetic field to be monochromatic and to be excited in the plate at the surface $y=0$ so that the amplitudes of the high-frequency electrical field $\mathbf{E}(y)$ and current $\mathbf{J}(y)$ decrease with increasing y.

We restrict our consideration of the problem to only the main approximation in the anomalous skin-effect parameter δ/d, and we have therefore dropped in Eqs. (4) terms proportional to $E_\mu(d)$ and $E'_\mu(d)$. In the same approximation we can put $E_y(y)=0$ and we can forget about the equation for the electrical neutrality which is used to determine that component of the field. When $\omega=n\Omega$ there appear electromagnetic field spikes in the metal at depths which are multiples of the orbital diameter of the resonance electrons and the quantity $E_\mu(d)$ may turn out to be large.[6] The field amplitude in a spike decreases slowly with increasing number of the resonance when the resonance electrons make the main contribution to the current and the spread in the diameters of their orbits is less than the skin depth, i.e., when $\omega_0 t\gg r/\delta$. In a thin sample ($2r_{\max}>d$) when $\omega t_0\gg r/\delta$ the electrons responsible for the CR make, as we shall show below, a small contribution to the electrical current and $E_\mu(d)$ is therefore negligibly small compared to the field amplitude in the skin layer. This justifies the use of Fourier's method, continuing the function $E_\mu(y)$ to be an even function and putting $E_\mu(y)=0$ for $|y|>d$.

We can find the connection between $j_\mu(k)$ and $\mathscr{E}_\mu(k)$ by solving the kinetic Boltzmann equation with the boundary conditions for the electron distribution function, taking into account the nature of the reflection of the carriers by the sample boundaries. Such calculations are not difficult to perform for the case of a magnetic field parallel to the surface of the sample and, if we perform then, we obtain the following expression for the electrical current $j(k)$:

$$j_\mu(k)=\int_0^\infty Q_{\mu\nu}(k,k')\mathscr{E}_\nu(k')\,dk', \tag{5}$$

$$Q_{\mu\nu}(k,k')=Q_{\mu\nu}{}^0(k,k')+Q_{\mu\nu}^{\mathrm{coll}}(k,k')+K_{\mu\nu}(k,k'). \tag{6}$$

The last term in Eq. (6) takes into account the contribution to the current from electrons with trajectories which do not touch the sample boundary, the so-called volume electrons, and has the form

$$\begin{gathered} K_{\mu\nu}(k,k')=\frac{4e^3H}{\pi ch^3}\int_{-p_0}^{p_0}dp_z\theta(d-2r(p_z))\{1-\exp[i\omega^* T(p_z)]\}^{-1}\\ \times\int_0^T dt\,v_\mu(t)\int_{r+y(t)}^{d-r+y(t)}dy\cos ky\int_{t-T}^{t}dt'\,v_\nu(t')\exp[i\omega^*(t-t')]\cos k'(y+y(t')-y(t)), \end{gathered} \tag{7}$$

where $v=\partial\varepsilon/\partial p$ is the electronvelocity; $p_0=p_z^{\max}$ on the Fermi surface; $\omega^*=\omega+i/t_0$; h is Planck's constant; $\theta(x)=\frac{1}{2}\{1+\operatorname{sign}x\}$ is the Heaviside function; $T(p_z)=2\pi/\Omega(p_z)$; t is the time the charge moves in the magnetic field

$$y(t)=\int^t v_y(t)\,dt.$$

The kernels $Q_{\mu\nu}^{(0)}$ and $Q_{\mu\nu}^{\mathrm{coll}}$ describe the contribution to the current from electrons which collide with the sample boundaries, where

$$\begin{gathered} Q_{\mu\nu}{}^0(k,k')=\frac{4e^3H}{\pi ch^3}\int_{-p_0}^{p_0}dp_z\int_0^T dt\int_0^d dy\int_\lambda^t dt'\,v_\mu(t)v_\nu(t')\\ \times\exp[i\omega^*(t'-t)]\cos ky\cos k'(y+y(t')-y(t)) \end{gathered} \tag{8}$$

is independent of the nature of the reflection of the electrons and contributes only to the smooth dependence of the high-frequency current on the magnitude of the external magnetic field. Here λ is the moment when the electron is reflected by the sample boundaries $y_S=0$, d, i.e., the root of the equation

$$y(t)-y(\lambda)=y-y_S, \tag{9}$$

which approaches, but is less than, t.

The remaining part of the kernel $Q_{\mu\nu}$ can be written in the form

$$Q_{\mu\nu}^{\mathrm{coll}}=Q_{\mu\nu}^{(1)}+Q_{\mu\nu}^{(2)}+Q_{\mu\nu}^{(1,2)}, \tag{10}$$

where $Q_{\mu\nu}^{(1)}$ describes the contribution to the current from the electrons which collide with the plate surface $y=0$, and has the form

$$\begin{gathered} Q_{\mu\nu}^{(1)}(k,k')=\int_{-p_0}^{p_0}dp_z\int_0^{T/2}d\lambda\left\{\theta(d-2r(p_z))+\theta(2r(p_z)-d)\theta\left(\lambda-\frac{T}{2}+\tau\right)\right\}\\ \times\frac{q_1 v_y(\lambda)}{q_1\exp[i\omega^*(T-2\lambda)]-1}\varphi_\mu^k(\lambda,\omega^*;0)\varphi_\nu^{k'}(\lambda,-\omega^*;0), \end{gathered} \tag{11}$$

while the kernel $Q_{\mu\nu}^{(2)}$ is connected with the electrons which collide only with the surface $y=d$:

$$\begin{gathered} Q_{\mu\nu}^{(2)}(k,k')=\int_{-p_0}^{p_0}dp_z\int_0^{T/2}d\lambda\{\theta(d-2r(p_z))+\theta(2r(p_z)-d)\theta(\tau-\lambda)\}\\ \times\frac{q_2 v_y(\lambda)}{q_2\exp[i\omega^* 2\lambda]-1}\Phi_\mu^k(\lambda,\omega^*;d)\Phi_\nu^{k'}(\lambda,-\omega^*;d). \end{gathered} \tag{12}$$

One notes easily that when the specularity parameters q_1 and q_2 of the plate surfaces $y=0$ and $y=d$ are close to unity the kernels $Q_{\mu\nu}^{(1)}$ and $Q_{\mu\nu}^{(2)}$ have a resonance character.

The quantity $Q_{\mu\nu}^{(1,2)}$ takes into account the contribution to the high-frequency electrical current from electrons colliding with both surfaces of the plate. Like $Q_{\mu\nu}^{(1)}$ and $Q_{\mu\nu}^{(2)}$ this part of the kernel vanishes when the reflection of the electrons is diffuse, i.e., when $q_1 = q_2 = 0$, while for purely specular reflection it has the form

$$Q_{\mu\nu}^{(1,2)}(k,k') = \frac{4e^3H}{\pi ch^3}\int_{-p_0} dp_z\theta(2r(p_z)-d)\int_0^{T/2-\tau}\frac{v_y(\lambda)\,d\lambda}{\exp[2i\omega^*T_\lambda]-1}$$
$$\times\Big\{\int_\lambda^{\lambda+T_\lambda} C^{k'}(t',\lambda;0)[v_\mu(t')e^{i\omega^*t'}+v_\mu(-t')\exp\{i\omega^*(2T_\lambda+2\lambda$$
$$-t')\}]dt'\int_\lambda^{\lambda-T_\lambda} C^k(t,\lambda;0)v_\nu(t)e^{-i\omega^*t}\,dt \tag{13}$$
$$-\int_\lambda^{\lambda+T_\lambda} C^{k'}(t',\lambda;0)[v_\mu(t')\exp\{i\omega^*(t'-2\lambda)\}$$
$$+v_\mu(-t')e^{-i\omega^*t'}]dt'\int_\lambda^{\lambda+T_\lambda} C^k(t,\lambda;0)v_\nu(-t)e^{i\omega^*t}\,dt\Big\}.$$

We choose the origin of t such that $y(0) = y_{\min}$ and $y(T/2)$ corresponds to the maximum value of the coordinate y on the electron trajectory. The time T_λ it takes an electron to move from one surface of the plate to the other and the quantity τ are determined by the equations

$$y(\lambda+T_\lambda)-y(\lambda)=d,\qquad y(\tau)-y(0)=d. \tag{14}$$

The functions φ_μ^k and Φ_μ^k have the form

$$\varphi_\mu^k(\lambda,\omega^*;y_s)=\left(\frac{4e^3H}{\pi ch^3}\right)^{1/2}\int_\lambda^{T/2} dt\,C^k(t;\lambda;y_s)[v_\mu(t)e^{i\omega^*t}$$
$$+v_\mu(-t)\exp\{i\omega^*(T-t)\}],$$
$$\Phi_\mu^k(\lambda,\omega^*;y_s)=\left(\frac{4e^3H}{\pi ch^3}\right)^{1/2}\int_0^\lambda dt\,C^k(t,\lambda;y_s)[v_\mu(t)e^{i\omega^*t}+v_\mu(-t)e^{-i\omega^*t}]; \tag{15}$$
$$C^k(t,\lambda;y_s)=\cos k(y(t)-y(\lambda)+y_s).$$

The resonance singularity of $Q_{\mu\nu}^{(1,2)}$ is nevertheless considerably weaker than that of $K_{\mu\nu}$ due to the fact that the resonance denominator $\exp(2i\omega^* T_\lambda) - 1$ is additionally integrated over λ. As the period of the motion of the electrons near one of the surfaces of the plate also depends on λ, the singularities of the kernels $Q_{\mu\nu}^{(1)}$ and $Q_{\mu\nu}^{(2)}$ turn out to be in a number of cases, in particular when $\omega t_0 \gg kr$, less important when compared with the contribution to the resonance part of the current from the "volume" electrons.

We assume, in order to avoid unnecessary complications of the formulae, that the direction of the magnetic field is lying in the symmetry plane of the crystal. The generalization to an arbitrary case does not cause any difficulties in principle; the asymptotic behavior of $Q_{\mu\nu}(k, k')$ at large k and k' remains unaltered for any orientation of the magnetic field in the plane of the plate. This makes it possible to generalize Hartmann and Luttinger's method for solving the integral Eq. (4) in which the current $j_\mu(k)$ is written using (5), (6) to the case of an arbitrary dispersion law for the carriers and an arbitrary form of the reflection of them by the boundaries of a thin conductor. However, we restrict the detailed consideration to merely some of the most important particular cases.

2. INFLUENCE OF THE SURFACE PROPERTIES

1. We consider to begin with the simples case when the reflection of the conduction electrons from the sample boundaries is diffuse. In that case, only the electrons which do not collide with the surface of the conductor return many times to the skin layer, and only they can absorb in a resonant way the energy of an electromagnetic wave. Under anomalous skin-effect conditions, when the penetration depth δ of the electromagnetic field into the metal is a very small length parameter, the main contribution to the impedance comes from large $k \sim \delta^{-1}$, and to find the shape of the CR line it is sufficient to have the asymptotic expression for the high-frequency conductivity tensor of the metal at $kr \gg 1$. Integrating in Eq. (7), which describes the contribution to the current from the volume electrons, over y and after that integrating over t and t' using the stationary-phase method, we get the following expression for the resonance part of the tensor $Q_{\mu\nu}(k, k')$:

$$Q_{\mu\nu}^{\rm res}(k,k') \approx \frac{e^2}{h^3}\int_{-p_0}^{p_0} dp_z a_{\mu\nu}(p_z)\theta(d-2r(p_z))$$
$$\times\frac{(1+\exp[i\omega^*T(p_z)])(1+\exp[-i\omega^*T(p_z)])}{1-\exp[i\omega^*T(p_z)]}$$
$$\times(kk')^{-1/2}\left\{\frac{\sin[(k'-k)(d-2r(p_z))]}{k'-k}-\frac{1-\cos[(k'+k)(d-2r(p_z))]}{k'+k}\right\}, \tag{16}$$
$$a_{\mu\nu}(p_z)=v_\mu(p_z,0)v_\nu(p_z,0)m^*\left|\frac{\partial v_y(p_z,\varphi)}{\partial\varphi}\right|_{\varphi=0}^{-1},\qquad \varphi=\Omega t.$$

When $d = 2r(p_z)$ the integrand in this expression vanishes and this can weaken the singularity of the conductivity at $\omega = n\Omega(p_1)$. This is connected with the fact that in a thin sample the position of the center of the orbit of volume electron with $2r(p_z) = d$ is fixed. At the same time in bulk conductors electrons with the same orbital diameter contribute the same to the high-frequency current if the orbital center changes in a range $\Delta y_0 \lesssim \delta$.

It is necessary to take it into account that a contribution to CR comes from the whole region of electrons of width $\delta p_z \approx p_0(\omega t_0)^{-1}$ near the section $p_z = p_1$ for which the spread in frequencies $\delta\Omega(p_z)$ is less than the collision frequency. If for the majority of these electrons the difference between their orbital diameter and the thickness of the plate is not less than the skin depth, i.e., if inequality (2) is satisfied, the above-mentioned fact is of no importance whatever. The role of the sample boundary $y = d$ reduces in that case to a selection of the resonance frequency $\Omega(p_1)$, and the number of electrons which take part in the resonance turns out to be the same as in a bulk sample. We can under those conditions with sufficient accuracy replace in Eq. (16) the first term in the braces by $\pi\delta(k' - k)$, and the second term by its average value which is equal to unity:

$$Q_{\mu\nu}^{\rm res}(k,k') \approx \frac{\pi e^2}{h^3}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\theta(d-2r(p_z))$$
$$\times\frac{(1+\exp[i\omega^*T(p_z)])(1+\exp[-i\omega^*T(p_z)])}{1-\exp[i\omega^*T(p_z)]}(kk')^{-1/2}\left\{\delta(k'-k)-\frac{1}{\pi}\frac{1}{k'+k}\right\}. \tag{17}$$

In this limiting case the way the resonance kernel $Q_{\mu\nu}^{\rm res}$ depends on k and k' is the same as in bulk samples.[7] Thanks to the fact that the (k, k')- and the d-

dependence of the kernel $Q_{\mu\nu}^{\text{res}}$ are completely separated we can solve the corresponding integral equation which is obtained by substituting Eq. (17) into Eq. (4), using the method proposed by Hartmann and Luttinger.[8] As a result we get the following expression for the impedance:

$$Z_\mu = \frac{2^{5/3}\pi^{2/3}}{\sqrt{3}} e^{-i\pi/3} h \left(\frac{\omega}{e^2 c}\right)^{2/3} L_{\mu\mu}^{-1/3},$$

$$L_{\mu\nu} = \frac{1}{4}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\,\theta(d-2r(p_z))\,\frac{(1+\exp[i\omega^* T(p_z)])(1+\exp[-i\omega^* T(p_z)])}{1-\exp[i\omega^* T(p_z)]}. \tag{18}$$

Near resonance, when the magnetic field is close to one of the values determined by Eqs. (1), the integral $L_{\mu\nu}$ can be evaluated by the saddle-point method:

$$L_{\mu\nu} \approx -\frac{i}{2\pi n\alpha}\tilde{a}_{\mu\nu}(p_1)\ln x, \qquad x = (\alpha/\beta - 1)\Delta + i/\omega t_0;$$
$$\tilde{a}_{\mu\nu}(p_1) = a_{\mu\nu}(p_1) + a_{\mu\nu}(-p_1), \qquad \alpha = \frac{1}{m^*}\frac{\partial m^*}{\partial p_z}\Big|_{p_z=p_1}, \tag{19}$$
$$\beta = \frac{1}{D}\frac{\partial D}{\partial p_z}\Big|_{p_z=p_1}, \qquad \Delta = (H - H_{\text{res}})/H_{\text{res}}$$

The impedance determined by Eqs. (18), differs therefore only by unimportant numerical factors of order unity from the one obtained earlier by us (see Eqs. (21) and (22) in [3]) in the approximation of a local coupling between the Fourier components of the current $j_\mu(k)$ and the field $\mathcal{E}_\mu(k)$.

In very pure samples, when the inequality

$$r/\delta \ll \omega t_0 \tag{20}$$

may be satisfied, the resonance curve is described by completely different formulae. In that case the resonance electrons form a very narrow layer, closely clamped to the surface of the plate. The spread in the diameters of these electrons is small compared to the thickness of the skin-layer and this leads to a weakening of the singularity of the electrical conductivity tensor caused by the size-effect CR.

For small values of the resonance detuning, $\Delta \ll \delta/r$, the characteristic denominator

$$1 - \exp[i\omega^* T(p_z)]$$

in Eq. (16) is appreciably "sharper" than the functions inside the braces. Integrating in this equation we get the following expression for $Q_{\mu\nu}^{\text{res}}$:

$$Q_{\mu\nu}^{\text{res}}(k, k') \approx \frac{4ie^2}{\pi n h^3}\frac{\tilde{a}_{\mu\nu}(p_1)\beta}{\alpha^2}\frac{r}{(kk')^{1/2}} x\ln x. \tag{21}$$

It is clear from this expression that the resonance conductivity is in this case only a small fraction of the total conductivity of the metal:

$$\left|\frac{Q_{\mu\nu}^{\text{res}}}{Q_{\mu\nu}^{\text{mon}}}\right| \approx \frac{r/\delta}{\omega t_0}\ln \omega t_0 \ll 1.$$

In the main approximation in the small parameter $r/\delta\omega t$, the impedance of the conductor also changes correspondingly smoothly when the magnetic field varies and the resonance behavior appears in the next approximation in that parameter.

To obtain the impedance in this case we must analyze the first term in Eq. (6) which together with the nonresonance part of the kernel $K_{\mu\nu}(k, k')$ determines the monotonic behavior of the electrical conductivity tensor:

$$Q_{\mu\nu}^{\text{mon}}(k, k') = A_{\mu\nu}(d/r)(kk')^{-1/2}\delta(k'-k)$$
$$+ B_{\mu\nu}\left(\frac{d}{r}\right)(kk')^{-1/2}\frac{1}{k+k'} + C_{\mu\nu}\left(\frac{d}{r}\right)\frac{\ln(k'/k)}{k'^2-k^2};$$
$$A_{\mu\nu}\left(\frac{d}{r}\right) = \frac{\pi e^2}{h^3}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\{\theta(d-2r(p_z))$$
$$\times(1-\exp[-i\omega^* T(p_z)]) + \theta(2r(p_z)-d)\},$$
$$C_{\mu\nu}\left(\frac{d}{r}\right) = -\frac{2e^2}{\pi h^3}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\{\theta(d-2r(p_z))(1+\exp[i\omega^* T(p_z)]) + 2\}$$
$$+ \exp[-i\omega^* T(p_z)]) - \theta(2r(p_z)-d)\},$$
$$C_{\mu\nu}\left(\frac{d}{r}\right) = -\frac{2e^2}{\pi h^3}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\{\theta(d-2r(p_z))(1+\exp[i\omega^* T(p_z)]) + 2\}. \tag{22}$$

As one should expect, in the main approximation in the anomalous skin-effect parameter δ/d the behavior of $Q_{\mu\nu}^{\text{mon}}(k, k')$ is similar to the k, k'-dependence of the electrical conductivity tensor in bulk conductors. The only difference lies in the fact that in a thin conductor the coefficients $A_{\mu\nu}$, $B_{\mu\nu}$, and $C_{\mu\nu}$ depend smoothly on the magnetic field.

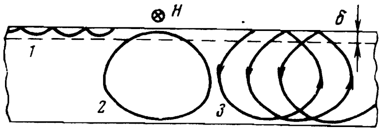
It is impossible to solve the Maxwell equations with $Q_{\mu\nu}^{\text{res}}(k, k')$ determined by Eqs. (21), (22) exactly, without making concrete assumptions about the form of the carrier energy spectrum. The resonance term in the impedance is, however, apart from a factor which varies very slowly with changes in the magnitude of H, not connected with the actual form of the conduction electron Fermi surface and has the form

$$\Delta Z_{\mu\nu}^{\text{res}} = Z_0 \tilde{C}_{\mu\nu}\left(\frac{d}{r}\right)\frac{r}{\delta} x\ln x, \tag{23}$$

where $|\tilde{C}_{\mu\nu}(d/r)| \sim 1$; Z_0 is the impedance of the plate when there is no magnetic field.

When the detuning Δ increases the "sharpness" of the resonance denominator in Eq. (16) decreases and when $\Delta \gtrsim \delta/r$ the resonance conductivity and correspondingly the impedance again are described by Eqs. (18), (19). The characteristic magnitude of the detuning Δ_0 or, in other words, the width of the resonance curve is thus in the case (20) considered here of the order of magnitude of $\Delta_0 = \delta/r$.[1)]

2. We now show how the results obtained change when the reflection of the electrons by the sample boundaries is close to specular reflection. When the nature of the scattering of the carriers by the surface of the conductor is almost specular a considerable contribution to the current comes from electrons which collide many times at small angles with the sample boundaries. The orbits of these "glancing" electrons which break away by almost specular reflections from the surface move completely in the skin-layer (see figure) and this



Different electron trajectories for the case when the carriers are specularly reflected by the conductor surface: 1) trajecjectory of a "glancing" electron; 2) trajectory of a volume electron; 3) trajectory of a resonance electron which collides with the sample boundaries ($\varphi_0 \to 0$).

leads to a strong increase in the monotonic electrical conductivity of the metal. It is clear that the glancing electrons are unifluenced by the presence of the plate boundary $y=d$ as long as $d \gg \delta$ and the monotonic electrical conductivity of the plate is thus for $1-q \ll 1$ the same as $Q_{\mu\nu}^{\text{mon}}$ in bulk samples ($2r_{\max} < d$), which was obtained by Meĭerovich[9] and by Zherebchevskiĭ and Kaner.[10] One can verify this by separating in Eq. (11) the contribution to the integral from the stationary phase points $t = t' = \lambda = T/2$ which also exactly corresponds to the contribution to the current from the abovementioned electrons. Integrating, we get for the monotonic part of the electrical conductivity tensor the following expression:

$$Q_{\mu\nu}^{\text{mon}}(k,k') = -\frac{4e^2}{\pi h^3}\frac{1+q}{1-q}J_{\mu\nu}\frac{\ln(k'/k)}{k'^2-k^2}, \qquad n\left(\frac{\delta}{r}\right)^{1/2} \ll 1-q \ll 1, \tag{24}$$

$$Q_{\mu\nu}^{\text{mon}}(k,k') = \frac{i\sqrt{\pi}\,e^2}{\omega^* h^3}\left(\frac{eH}{c}\right)^{1/2} d_{\mu\nu}(kk')^{-1/2}[(k-k')^{-1/2}-(k+k')^{-1/2}],$$
$$1-q \ll n(\delta/r)^{1/2}, \tag{25}$$
$$J_{\mu\nu} = \int_{-p_0}^{p_0} a_{\mu\nu}(p_z)\,dp_z, \qquad d_{\mu\nu} = \int_{-p_0}^{p_0} v_\mu(p_z)v_\nu(p_z)\left|\frac{m^*(p_z)}{\partial v_y/\partial\varphi}\right|^{1/2} dp_z,$$

where $q \equiv q_1$, while it will become clear from what follows that q_2, the specularity parameter at the plate surface $y=d$, completely drops out of the final formulae.

The changes in the resonance part of the electrical conductivity turn out to be unimportant. At first sight this is not immediately obvious as electrons which collide with the boundaries return many times to the skin layer when $1-q \ll 1$ and the nature of their motion reminds us of the motion of volume electrons when the angle at which they leave the surface is close to zero. However, as we noted earlier, the resonance denominators in Eqs. (11) to (13) contain for colliding electrons a dependence not only on p_z, but also on the emission angle $\varphi_0 = \Omega\lambda$ because of the λ-dependence of the frequency of the electron revolution. When $\omega t_0 \gg (r/\delta)^{1/2}$ these denominators are appreciably "sharper" than cosines. In that case the resonance denominator is once more integrated over λ and this leads to the appearance in the conductivity of small terms of the kind $(\omega t_0)^{-2}\ln\omega t_0$ or even (ωt_0) if the electron collides with both surfaces.

If the skin-layer is extremely narrow, viz.,

$$(r/\delta)^{1/2} \gg \omega t_0, \tag{26}$$

the resonance denominator can, on the other hand, be taken out of the integral with respect to λ. In that case the change in the frequency of motion of the electron $\Omega(\lambda, p_z)$ is, when its orbital center is shifted by the maximum allowed amount $\pm\delta$, much smaller than the collision frequency and hence $\Omega(\lambda, p_z)$ depends effectively only on p_z. One sees easily that for electrons with an orbital diameter larger than but close to the thickness of the conductor the deviation of the period $T_\lambda(p_z)$ from its value for $p_z = p_1$ turns out to be proportional to $|p_z - p_1|^{1/2}$. The resonance electrical conductivity of these electrons turns thus out to be less than that of the volume electrons by a factor ωt_0. Only the contribution from electrons with orbital diameters less than d which collide with the surface $y=0$ turns out to be in the case (26) of the same order of magnitude as the contribution from the volume electrons to the electrical conductivity. Evaluating the corresponding integral in Eq. (11) we get for the conductivity of these electrons the following expression:

$$\tilde{Q}_{\mu\nu}^{\text{res}}(k,k') = \frac{4\pi e^2}{h^3}\int_{-p_0}^{p_0} dp_z\,a_{\mu\nu}(p_z)\theta(d-2r(p_z))(1-\exp[i\omega^* T(p_z)])^{-1}$$
$$\times\left\{(kk')^{-1/2}\left[\delta(k'-k)+\frac{1}{\pi}\frac{1}{k'+k}\right]-\frac{2}{\pi^2}\frac{\ln(k'/k)}{k'^2-k^2}\right\}. \tag{27}$$

The total resonance conductivity is given by the sum of Eqs. (27) and (17). Bearing in mind that close to resonance we can replace the function $\exp(\pm i\omega^* T)$ in the numerator of Eq. (17) by unity, we have

$$Q_{\mu\nu}^{\text{res}}(k,k') = \frac{8\pi e^2}{h^3}\int_{-p_0}^{p_0} dp_z\,a_{\mu\nu}(p_z)\theta(d-2r(p_z))(1-\exp[i\omega^* T(p_z)])^{-1}$$
$$\times\left\{(kk')^{-1/2}\delta(k'-k)-\frac{1}{\pi^2}\frac{\ln(k'/k)}{k'^2-k^2}\right\}. \tag{28}$$

This expression is valid when the following inequalities are simultaneously satisfied:

$$(r/\delta)^{1/2} \gg \omega t_0, \qquad 1-q \ll (\Omega t_0)^{-1}. \tag{29}$$

The resonance current thus turns out to be either completely independent of the nature of the reflection of the electrons by the surface of the conductor, or in the case (29) this dependence is very weak so that taking into account the contribution from the term $\tilde{Q}_{\mu\nu}^{\text{res}}$ to the electrical conductivity leads only to giving a more exact value of the numerical factor of order unity in the equations for the impedance. Therefore, even for purely specular scattering electrons with an orbital diameter equal to the plate thickness are "isolated" to the same degree as when $q=0$, and the nature of the singularity in the electrical conductivity at $\omega = n\Omega(p_1)$ remains unchanged. The sensitivity of the shape of the resonance curve to the nature of the scattering of the electrons by the sample boundary $y=0$ is completely caused by the large magnitude of the monotonic component of the high-frequency electrical current when $1-q \ll 1$. At the same time the singularity in the electrical conductivity caused by the size-effect CR is logarithmic or even weaker. When the condition $1-q \ll n/\ln\omega t_0$, which is practically equivalent to $1-q \ll 1$, is satisfied the monotonic component of the electrical current thus exceeds the resonance component and for any ratio of the parameters δ/r and ωt_0 the oscillations in the impedance which are connected with the size-effect CR are a small part of its average value. The resonance term in the impedance on the other hand, is determined just by the ratios of the above-mentioned parameters and by how close the specularity parameter is to unity.

We analyze first of all case (2) when the electron mean free path is not too large. For small, but nevertheless finite values of the quantity $1-q$

$$n(\delta/r)^{1/2} \ll 1-q \ll 1$$

the resonance conductivity is given by Eqs. (17) or (28) and the monotonic part by Eq. (24) while the equation for the Fourier components of the current and the field reduces to Hartmann and Luttinger's equation. In the main approximation in the small parameter $1-q$ the impedance is independent of the magnitude of the magnetic field and in the next approximation the resonance contribution has the form

$$\Delta Z_\mu^{\text{res}} = \frac{4\sqrt{3}\,\pi^{1/3} e^{-i\pi/3}\omega}{c^2 k_\mu}(1-q)^{2/3}\left(\frac{L_\mu}{J_\mu}\right)^{2/3}, \qquad k_\mu = \left(\frac{16e^2\omega J_\mu}{(1-q)c^2h^3}\right)^{1/3}; \tag{30}$$

the index μ corresponds here to the axis in which the tensor $J_{\mu\nu}$ is diagonal.

If the scattering of the electrons by the surface is yet closer to being specular: $1-q \ll n(\delta/r)^{1/2}$ it is necessary for finding $\Delta Z_{\mu\nu}^{\text{res}}$ to solve the Maxwell Eqs. (4) with the current density (17), (25) or (28), (25). Using the relations

$$\mathscr{E}_\mu(k) = -2E_\mu'(0)\tilde{k}_\mu^{-2}F_\mu(\xi), \qquad k = \tilde{k}_\mu \xi$$

to introduce the dimensionless variables ξ, $F_\mu(\xi)$, we can write these equations in the form

$$\xi^2 F_\mu(\xi) - i\int_0^\infty Q_0(\xi,\xi')F_\mu(\xi')\,d\xi' - i\varepsilon_\mu \int_0^\infty Q_1(\xi,\xi')F_\mu(\xi')\,d\xi' = 1, \tag{31}$$

where

$$Q_0(\xi,\xi') = (\xi\xi')^{-1/2}[\,|\xi-\xi'|^{-1/2} - (\xi+\xi')^{-1/2}], \tag{32}$$

$$Q_1(\xi,\xi') = \begin{cases} (\xi\xi')^{-1/2}\delta(\xi'-\xi) - \dfrac{1}{\pi^2}\dfrac{\ln(\xi'/\xi)}{\xi'^2-\xi^2}, & \dfrac{1}{\omega t_0} \gg \dfrac{\delta}{r} \\ \dfrac{1}{2}(\xi\xi')^{-1/2}\left[\delta(\xi'-\xi) - \dfrac{1}{\pi}\dfrac{1}{\xi'+\xi}\right], & \dfrac{1}{\omega t_0} \ll \dfrac{\delta}{r} \end{cases} \tag{33}$$

$$\varepsilon_\mu = \rho_\mu \frac{\ln(1/x)}{(\tilde{k}_\mu R)^{1/2}}, \qquad \rho_\mu = \frac{4}{\sqrt{\pi}}\frac{\tilde{a}_\mu(p_1)\sqrt{p_0}}{m^*(p_1)\alpha d_\mu}, \qquad R = \frac{cp_0}{eH}, \tag{34}$$

$$\delta_\mu \equiv \tilde{k}_\mu^{-1} = \left(4\pi^{3/2}\frac{e^2}{(1/\omega t_0 - i)c^2h^3}\left(\frac{eH}{c}\right)^{1/2} d_\mu\right)^{-2/5} \tag{35}$$

(δ_μ is the skin-layer thickness for specular reflection of the electrons from the surface of the sample and the given polarization of the electromagnetic wave, while the index μ corresponds to the axes in which the tensor $d_{\mu\nu}$ is diagonal).

We shall look for a solution of Eq. (31) in the form of of a series in the small parameter ε_μ:

$$F_\mu(\xi) = F^{(0)}(\xi) + \varepsilon_\mu F^{(1)}(\xi).$$

Using the symmetry of the kernel $Q_1(\xi, \xi')$ under the substitution $\xi \rightarrow \xi'$ we easily get the following formula for the resonance contribution to the impedance:

$$\Delta Z_\mu^{\text{res}} = C\frac{8\omega}{c^2\tilde{k}_\mu}\varepsilon_\mu, \tag{36}$$

$$C = \int_0^\infty d\xi \int_0^\infty d\xi' F^{(0)}(\xi)F^{(0)}(\xi')Q_1(\xi,\xi'). \tag{37}$$

One can find the solution $F^{(0)}(\xi)$ of Eq. (31) with ε_μ equal to zero in Zherebchevskiĭ and Kaner's paper.[10]

We can use the same way of evaluating $\Delta Z_\mu^{\text{res}}$ also in the limiting case of very long electron mean free paths (inequality (20)). We can easily transform the Maxwell equation with the current density given by Eqs. (21), (24) or (21), (25) to the form (31). For specular reflection: $1-q \ll n(\delta/r)^{1/2}$ the kernel $Q_0(\xi, \xi')$ is given by Eq. (32) and when $n(\delta/r)^{1/2} \ll 1-q \ll 1$ we have for the quantity $Q_0(\xi, \xi')$

$$Q_0(\xi,\xi') = 2\frac{\ln(\xi'/\xi)}{\xi'^2-\xi^2}. \tag{38}$$

The formulae for the small parameter ε_μ have in the appropriate limiting cases the form

$$\varepsilon_\mu = \begin{cases} \dfrac{1}{\pi}\dfrac{\beta}{\alpha}\rho_\mu(\tilde{k}_\mu r)^{1/2} x\ln x, & 1-q \ll n\left(\dfrac{\delta}{r}\right)^{1/2}, \\ \dfrac{i\tilde{a}_\mu(p_1)\beta}{n\alpha^2 J}(1-q)k_\mu r x\ln x, & n\left(\dfrac{\delta}{r}\right)^{1/2} \ll 1-q \ll 1. \end{cases} \tag{39}$$

The resonance contribution to the impedance is described by Eq. (36) in which we can write, taking into account the simple form of the kernel $Q_1(\xi, \xi') = (\xi\xi')^{-1/2}$, the constant C in the form

$$C = \left(\int_0^\infty d\xi\, \xi^{-1/2}F^{(0)}(\xi)\right)^2. \tag{40}$$

As in the case of diffuse reflection, Eqs. (36), (39), (40) are valid when $\Delta \ll \delta/r$. In the wings of the resonance curve $\Delta \gtrsim \delta/r$ the impedance is again given by Eqs. (36), (34), (37). The nature of the scattering of the charge carriers by the surface of the conductor therefore shows up in the detailed structure of the resonance curve without, however, changing the position of the resonance frequencies or the order of magnitude of the width of the $Z_\mu(H)$ curve. Regardless of the degree of specularity, the width of the resonance curve is determined by the electron mean free path when $r/\delta \gtrsim \omega T_0$ and by the attenuation depth of the high-frequency field in the opposite limiting case.

3. RETARDATION EFFECT

All that has been said above is valid when the phase of the electromagnetic wave changes little during the time the electron stays in the skin layer, i.e., when $(r\delta)^{1/2}/v \ll 2\pi/\omega$. However, for sufficiently pure metals in magnetic fields for which $2r > d$ the frequency of revolution of the resonance electrons may be considerably lower than ω and, hence, resonance peaks with large n may be experimentally resolved. When the condition $(r/\delta)^{1/2} \ll n \ll \omega t_0$ is satisfied it is no longer possible to neglect the change in phase of the electromagnetic wave when it has a resonance interaction with the electrons. This fact which is called the retardation effect in cyclotron resonance[11] leads to a weakening of the CR amplitude and has been studied in a rather detailed way in bulk conductors in Refs.[12,13]. Below we consider the role of the retardation effect in the size-effect CR.

In the case considered the resonance contribution to the current comes, independent of the nature of the scattering of the electrons by the surface of the conductor, from the "volume" electrons and the non-monotonic electrical conductivity of the metal is completely determined by the last term in Eq. (6). When we evaluate the asymptotic behavior of the integrals over t and t' in Eq. (7) for $K_{\mu\nu}(k, k')$ we must take into account the shift in the stationary phase points caused by the presence in the index of the exponent $\exp\{-i\omega^*(t'-t)\}$ of the large parameter $|\omega^*/\Omega| \approx n$. Integrating we get the following expression for $Q^{\text{res}}_{\mu\nu}(k, k')$

$$Q^{\text{res}}_{\mu\nu}(k,k') \approx \frac{4e^2}{h^2}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\,\theta(d-2r(p_z))\,(1-\exp[i\omega^* T(p_z)])^{-1}$$
$$\times\left\{\frac{1}{k'-k}\sin\left[(d-2r(p_z))(k'-k)-\frac{n^2}{2r(p_z)}\left(\frac{1}{k'}-\frac{1}{k}\right)\right]\right.$$
$$-\frac{1}{k'-k}\sin\left[\frac{n^2}{2r(p_z)}\left(\frac{1}{k'}-\frac{1}{k}\right)\right]+\frac{1}{k'+k}\cos\left[(d-2r(p_z))(k'+k)\right.\tag{41}$$
$$\left.\left.-\frac{n^2}{2r(p_z)}\left(\frac{1}{k'}+\frac{1}{k}\right)\right]-\frac{1}{k'+k}\cos\left[\frac{n^2}{2r(p_z)}\left(\frac{1}{k'}+\frac{1}{k}\right)\right]\right\},$$

which is valid when $(kr)^{1/2} \ll n \ll kr$.

It is rather obvious (and this is verified by further calculations) that under retardation conditions we can consider the magnetic field as a perturbation. This means that in the main approximation in the small parameter kr/n^2 the absorption of the electromagnetic wave by the conductor is independent of the magnitude of H. To find the resonance contribution to the impedance we use Chambers' formula

$$\Delta Z^{\text{res}}_{\mu} = \frac{8\pi\omega^2}{c^2[E_\mu'(0)]^2}\int_0^\infty dk\int_0^\infty dk'\, Q^{\text{res}}_{\mu\mu}(k,k')\,\mathscr{E}^{(0)}_\mu(k)\,\mathscr{E}^{(0)}_\mu(k').\tag{42}$$

In this expression the Fourier components of the field are found from Eq. (4) in which $j_\mu(k)$ is the current when there is no magnetic field which in a plate of thickness $d \gg \delta$ is the same as the current in the half-space. Substituting Eq. (41) into (42) we get after simple transformations

$$\Delta Z^{\text{res}}_{\mu} = \frac{16\omega}{c^2 k_\mu^0 J_\mu}\int_{-p_0}^{p_0} dp_z\, a_{\mu\nu}(p_z)\,\theta(d-2r(p_z))\,(1-\exp[i\omega^* T(p_z)])^{-1}$$
$$\cdot(I_1(p_z)+I_2(p_z)),\tag{43}$$

$$I_1(p_z)=\frac{1}{2\pi}\int_0^\infty\frac{d\xi}{\sqrt{\xi}}F_0(\xi)\int_0^\infty\frac{d\xi'}{\sqrt{\xi'}}F_0(\xi')\left\{\frac{1}{\xi'-\xi}\sin\left[k_\mu^0(d-2r(p_z))(\xi'-\xi)\right.\right.$$
$$\left.-\lambda_0(p_z)\left(\frac{1}{\xi'}-\frac{1}{\xi}\right)\right]+\frac{1}{\xi'+\xi}\cos\left[k_\mu^0(d-2r(p_z))(\xi'+\xi)\right.$$
$$\left.\left.-\lambda_0(p_z)\left(\frac{1}{\xi'}+\frac{1}{\xi}\right)\right]\right\},\tag{44}$$

$$I_2(p_z)=\frac{1}{2\pi}\int_0^\infty\frac{d\xi}{\sqrt{\xi}}F_0(\xi)\int_0^\infty\frac{d\xi'}{\sqrt{\xi'}}F_0(\xi')\left\{\frac{1}{\xi'-\xi}\sin\left[\lambda_0(p_z)\left(\frac{1}{\xi'}-\frac{1}{\xi}\right)\right]\right.$$
$$\left.-\frac{1}{\xi'-\xi}\cos\left[\lambda_0(p_z)\left(\frac{1}{\xi'}+\frac{1}{\xi}\right)\right]\right\},$$
$$\xi=\frac{k}{k_\mu^0},\quad \mathscr{E}^{(0)}_\mu(k)=-2E_\mu'(0)(k_\mu^0)^{-2}F_0(\xi),\quad \lambda_0(p_z)=\frac{n^2}{2k_\mu^0 r(p_z)},\tag{45}$$

$(k^0_\mu)^{-1}$ is the penetration depth of the electromagnetic field into the metal when $H=0$.

All functions in the integrand in Eq. (43) have different scales over which they vary. The resonance denominator changes in a characteristic interval $\delta p_z = p_0(\omega t_0)^{-1}$, the interval of change of the function $I_1(p_z)$ is equal to $\delta p_z' = p_0\delta/r$, while that of the function $I_2(p_z)$ equals $\delta p_z'' = p_0/\lambda_0$. When inequality (2) is satisfied the quantity $I_1(p_z)$ is "sharper" than the resonance denominator and its role is reduced to a contraction to the limiting value of the function

$$\frac{1}{\xi'-\xi}\sin\lambda_0\left(\frac{1}{\xi'}-\frac{1}{\xi}\right),$$

which is equal to $-\pi\delta(\xi'-\xi)$ as $\lambda_0 \to \infty$.

The asymptotic behavior of $\Delta Z^{\text{res}}_\mu$ when $\lambda_0 \gg 1$ is then given by the quantity $I_2(p_z)$. In the case (20) when the "sharpest" function of p_z is the resonance denominator, the asymptotic behavior of $\Delta Z^{\text{res}}_\mu$ is for $\lambda_0 \gg 1$, on the other hand, determined by the function $I_1(p_z)$. Using the saddle-point method to integrate in Eq. (43) we get the following expressions for $\Delta Z^{\text{res}}_\mu$:

$$\Delta Z^{\text{res}}_\mu = \begin{cases} \dfrac{-8i\omega}{\pi c^2 k_\mu^0 n}\dfrac{\tilde a_{\mu\mu}(p_1)}{\alpha J_\mu}\ln x\, I_2(\lambda_0(p_1)), & kr \gtrsim \omega t_0,\\[2ex] -\dfrac{16i\omega}{\pi c^2 k_\mu^0 n}\dfrac{\beta\tilde a_{\mu\mu}(p_1)}{\alpha^2 J_\mu}k_\mu^0 rx\ln x\, J(\lambda_0(p_1)), & kr \ll \left|\dfrac{1}{\omega t_0}-i\Delta\right|^{-1},\end{cases}\tag{46}$$

$$I_2(\lambda_0(p_z))=\frac{i}{4\pi}\int_\infty^{\lambda_0(p_z)} d\lambda'\,[g^-_{1/2}(\lambda')+ig^+_{1/2}(\lambda')]^2,$$

$$J(\lambda_0(p_z))=\frac{i}{4\pi}[g^+_{1/2}(\lambda_0(p_z))+ig^-_{1/2}(\lambda_0(p_z))]^2,$$

$$g_p^{\pm}(\lambda_0)=\int_0^\infty d\xi\, F_0(1/\xi)\,\xi^{-p}\exp[\pm i\lambda_0\xi].\tag{47}$$

To evaluate the integral $g^{\pm}_p$ it is necessary to know the actual form of the function $F_0(\xi)$, i.e., to solve the Maxwell equations with $H=0$. This calculation is given in the classical paper by Reuter and Sondheimer[14] for any form of the scattering of the electrons by the surface of the conductor. $F_0(\xi)$ has the simplest form in the case of specular reflection:

$$F_0(\xi)=\xi/(\xi^3-i).$$

We have then for the functions $I_2(\lambda_0)$ and $J(\lambda_0)$ which describe the fast decrease in the amplitude of the resonance peaks with number n, as a result of the integration

$$I_2(\lambda_0)=-\frac{\pi}{18}\exp\left[i\frac{\pi}{3}-\lambda_0(1+i\sqrt{3})\right]-\frac{1}{10\pi}\Gamma^2(^{11}/_2)\lambda_0^{-10}$$

$$-\frac{2}{3}\Gamma(^{11}/_2)\lambda_0^{-11/2}\exp\left[-\lambda_0\frac{1+i\sqrt{3}}{2}\right],$$

$$J(\lambda_0)=\frac{\pi}{9}\exp[-\lambda_0(1+i\sqrt{3})]+\frac{1}{\pi}\Gamma^2(^9/_2)\lambda_0^{-9}$$

$$+\frac{2}{3}\Gamma(^9/_2)\lambda_0^{-9/2}\exp\left[-\lambda_0\frac{1+i\sqrt{3}}{2}\right], \tag{48}$$

$\Gamma(x)$ is Euler's gamma-function. These formulae are valid also when $q\neq 1$, provided $1-q\ll 1/\lambda_0$. In the case of a rough boundary (when $1-q$ $1/\lambda_0$) the resonance amplitude decreases somewhat more slowly:

$$I_2(\lambda_0)\sim\lambda_0^{-8},\quad J(\lambda_0)\sim\lambda_0^{-7},\quad \lambda_0\to\infty. \tag{49}$$

The expressions we have obtained for the impedance describe the shape of the resonance line in all limiting cases and give a comprehensive solution of the problem of the size-effect resonance in a magnetic field parallel to the surface of the plate. The universal role of inequality (2) shows up when we consider both the effect of the properties of the surface on the size-effect CR and the retardation effect. If inequality (2) is satisfied, it is clear from the formulae given above that the impedance might be obtained by "splitting off" from the resonance current the electrons with $2r(p_z)>d$ in the appropriate formulae of the CR theory.[7, 9, 10, 13]

In the opposite limiting case (20) there occurs to a full extent a specific size-effect CR and it is important to note that even to determine the order of magnitude of the width of the resonance curve it is necessary to take the non-local relations between the Fourier components of the high-frequency field and of the current into account rigorously. This inequality which in the range of magnetic fields $2r_{max}\gtrsim d$ is equivalent to the condition $d^2\ll nl\delta$ determines the behavior of the size-effect resonance also when the magnetic field is inclined relative to the surface of the plate (see[4]). In such thin conductors where $d^2\ll nl\delta$ there is a range of angles $d^2/nl^2\ll\varphi\ll\delta/l$ where the size-effect CR occurs only in the subsequent approximations in the small parameter $d^2/n\varphi l^2$. At the same time the CR in bulk samples is completely insensitive to such angles of inclination of the magnetic field. Calculations which we shall not give here show that for such angles the impedance is described by Eqs. (23), (36) with, in general, other dimensionless constants, provided the right-hand side of these formulae is multiplied by the quantity $d^2/n\varphi l^2$.

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1)It follows from what has been said above that the results of[3], obtained in the approximation of a local coupling between the Fourier components of the current density and the electrical field, have a limited region of applicability: they are valid when $kr\gtrsim\omega t_0$. When $kr\ll\omega t_0$, Eqs. (21) and (22) from[3] describe the behavior of the derivative of the impedance with respect to the magnetic field near the size-effect CR frequencies, rather than that of the impedance itself. The form of the resonance curves shown in the figure occurs only when $kr\ll\omega t_0$, if we bear in mind that along the ordinate axis we plot not R, but $\partial R/\partial H$, while the electrical field vector of the linearly polarized wave is at right angles to the external magnetic field.

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